Directional Fuzzy Data Association Filter

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Abstract—In this paper, a new multi-target tracking algorithm based on fuzzy logic for tracking in clutter is developed, it is called directional fuzzy data association (DFDA) filter. The new algorithm incorporates the directional information of the targets for data association with the Mahalanobis distance. Firstly, the directional information, called pseudo-direction, is defined; the method of how to calculate the pseudo-direction has been introduced. Then the state incorporating with the pseudo-direction is updated using the cubature Kalman filter (CKF). At last the fuzzy logic inference method is used for data association. Simulation results are used to evaluate the performance of this new algorithm comparing with the nearest neighbor standard filter (NNSF) and joint probability data association filter (JPDAF), the final results show that the proposed DFDA filter an efficient and effective approach for real application.

Index Terms—data association, multi-target tracking, fuzzy logic, pseudo-direction

I. INTRODUCTION

Tracking of multiple targets is a problem that arises in a wide variety of fields, for example, radar based tracking of aircraft, video based identification and tracking of people for surveillance or security purposes, sonar based tracking of sea submarines, and so on. The most commonly used framework for tracking is that of Bayesian sequential estimation.

The application of the Bayesian sequential estimation framework to real world multi-target tracking problems is plagued by two difficulties. First, realistic models for the target dynamics and measurement processes are often nonlinear, so that no closed-form analytic expression can be obtained for the tracking recursions. The second difficulty is due to the fact that in most practical tracking applications the sensors yield unlabelled measurements of the targets. This leads to a combinatorial data association problem that is very challenging when targets have a small separation compared with the measurement errors. Furthermore, clutter measurements may arise due to multi-path effects, sensor errors, spurious objects, etc., further increasing the complexity of the data association problem.

Many strategies have been proposed in the literature to address the difficulties associated with multi-target tracking. We do not attempt to give an exhaustive summary here, but rather highlight some of the key contributions over the years. When tracking a single object closed-form expressions are generally not available for nonlinear models, and approximate methods are required. The extended KF (EKF) [1] is one of the most popular nonlinear filters to deal with the nonlinear problems. However, the performance of the EKF degrades rapidly as the non-linearities become more severe. To alleviate this problem the Unscented Kalman Filter (UKF) [2], [3] and the Cubature Kalman Filter (CKF) [4] maintain the second order statistics of the target distribution by recursively propagating a set of carefully selected sigma points. This method requires no linearization, and generally yields more robust estimates.

Data association is another essential part of track maintenance in the multi-target tracking problem. A large number of strategies are available to solve the data association problem. A variety of data association algorithms, such as the nearest neighbor standard filter (NNSF) method [5, 6, 7], the joint probability data association filter (JPDAF) algorithm [8, 9], the Multi-hypothesis Testing (MHT) algorithm [10, 11] have been developed. In the NNSF approach, the candidate measurement which falls inside the validation gate is closest to measurement prediction for the only true measurement, originating from the target. After that, the
state estimation of target was updated. In this respect the JPD AF is more appealing. At each time step infeasible hypotheses are pruned away using a gating procedure. A filtering estimate is then computed for each of the remaining hypotheses, and combined in proportion to the corresponding posterior hypothesis probabilities. The main shortcoming of the JPD AF is that, to maintain tractability, the final estimate is collapsed to a single Gaussian, thus discarding pertinent information. The MHT attempts to keep track of all the possible association hypotheses over time. The methods mentioned above are mainly based on the target dynamic model. We get the probability through various simplifications in advance, so what we get is just an approximation. Therefore, the approximation from those association methods could not represent the whole uncertainty completely.

In the research of artificial intelligence, the fuzzy theory has been extensively applied in describing some uncertain knowledge or physics phenomena within a nonlinear complicated system [12-29]. The fuzzy inference technique [12-23] and fuzzy clustering technique [24-29] are used to deal with the target-tracking problem recently. Actually, the data association in target tracking can be considered as a process to classify a given set of measurements according to some class rules. So many data association methods based on fuzzy clustering have been proposed. Unfortunately, because the cluster centers must be adjusted to ensure eventual convergence to an optimal solution through iteration, these algorithms have a heavy computational load. The fuzzy logic data association method normally using the residual of distance, azimuth and elevation, or sometimes maybe velocity as the input variables for the fuzzy inference system, but the residual of azimuth and elevation couldn’t reflect the target trajectories directly. It was found that the target direction is an important parameter which was used to carry out the track association because the directional information of the target could reflect the flight direction of the target. Based on this point, a fuzzy logic data association filter is proposed in this paper. This filter uses the Mahalanobis distance and the difference in angle between the pseudodirection and the predicted direction as the input variables of the fuzzy logic inference system for the data association.

The remainder of the paper is organized as follows. Section II introduces the definition and calculation method of the pseudodirection. Section III discusses the proposed directional fuzzy data association filter (DFDAF). Simulation results that compare the performances of the existing algorithms are presented in Section IV. Finally, some conclusions are provided in Section V.

II. THE DEFINITION OF THE PSEUDODIRECTION

Because the directional information of the target is not explicitly measurable, the pseudodirection is introduced. Pseudodirection is computed using prediction by connecting validated measurements with the previously estimated target position. In this paper, the reference direction is the positive direction of the y axis. \( h_{k-1} \) is direction of the target, \( c_1^k \) and \( c_2^k \) is the pseudodirection as shown in Fig. 1.

As Fig. 1 showing, the radar locates at the origin, \( T \) denotes the target trajectory, \( z_2^k \) (considering the \( z_2^k \) for example) denotes the radar measurement, \( r_2^k \) is the distance between the radar and the target, \( \alpha_2^k \) is the azimuth measured by radar. \( z_1^k \) and \( z_2^k \) are the two radar measurements at the \( k \)-th scan, \( \hat{x}_{k-1|k-1} \) denotes the \( k \)-th estimate state, \( x_{k|k} \) and \( y_{k|k} \) is the target position at the x direction and y direction, \( \dot{x}_{k|k} \) and \( \dot{y}_{k|k} \) is the velocity.

\[
\hat{x}_{k-1} = [x_{k-1|k-1} \ y_{k-1|k-1} \ \dot{x}_{k-1|k-1} \ \dot{y}_{k-1|k-1}]^T
\]

denotes the prediction state.

Then, the pseudodirection and the direction are defined as follows:

\[
c_1^k = \arctan \left( \frac{r_2^k \cos(\alpha_2^k) - x_{k-1|k-1}}{r_2^k \sin(\alpha_2^k) - y_{k-1|k-1}} \right), \quad (1)
\]

\[
h_k = \arctan \left( \frac{y_{k|k-1}}{x_{k|k-1}} \right). \quad (2)
\]

III. THE DATA ASSOCIATION BASED ON FUZZY LOGIC

In this paper, the Mahalanobis distance and pseudodirection will be used as the input variables of the fuzzy logic inference system, and the algorithm is as follows:

**Step 1:** Calculation of the measurements predictions and covariance of innovations

Suppose \( t \) targets have been gained by the track initiation, and \( m_k \) measurements also gained at the \( k \)-th moment. In order to use the tracking thresholds and fuzzy logic for data association, firstly, the nonlinear filter—
cubature Kalman filter (CKF) is used to calculate the prediction and innovation covariance.

\[
P_{\xi,k-1} = \sum_{i=1}^{m} \mathbf{Z}_{i,k-1} \mathbf{S}_{i,k-1}^T \left( \mathbf{S}_{i,k-1}^T \mathbf{S}_{i,k-1} \right)^{-1},
\]

(3)

\[
\mathbf{Z}'_{i,k-1} = \mathbf{S}_{i,k-1}^T \mathbf{v}_i + \mathbf{\hat{x}}_k \xi_{i,k-1},
\]

(4)

\[
\mathbf{Z}_{i,k-1}^h = \mathbf{f} \left( \mathbf{Z}_{i,k-1} \mathbf{u}_k \right),
\]

(5)

\[
\mathbf{\hat{x}}_k = \frac{1}{m} \sum_{i=1}^{m} \mathbf{Z}_{i,k-1}^h ,
\]

(6)

\[
P_{\xi,k-1} = \sum_{i=1}^{m} \mathbf{Z}_{i,k-1}^h \mathbf{Z}_{i,k-1}^h^T - \mathbf{\hat{x}}_k \mathbf{\hat{x}}_k^T + \mathbf{Q}_{k-1},
\]

(7)

\[
\mathbf{P}_{\xi,k-1} = \mathbf{S}_{k-1}^T \left( \mathbf{S}_{k-1} \right)^T, \quad \mathbf{Z}_{i,k-1} = \mathbf{S}_{k-1}^T \mathbf{v}_i + \mathbf{\hat{x}}_k \xi_{i,k-1},
\]

(8)

\[
\mathbf{Z}_{i,k-1}^h = \mathbf{h} \left( \mathbf{Z}_{i,k-1} \mathbf{u}_k \right),
\]

(9)

\[
P_{\xi,k-1} = \sum_{i=1}^{m} \mathbf{Z}_{i,k-1} \mathbf{Z}_{i,k-1}^T - \mathbf{\hat{x}}_k \mathbf{\hat{x}}_k^T + \mathbf{R}_k ,
\]

(10)

To limit the numbers of associations, firstly, only those measurements which lie in the validation region will be dealt with. In this paper, the eclipse threshold is used for data association. In the \( k \)th moment, the \( i \)th measurement, \( \mathbf{z}_i \) \((i = 1, \ldots, m_k)\), is validated for the \( j \)th target if its Mahalanobis distance from the predicted location of \( \mathbf{\hat{z}}_k \) of a \( j \)th target is less than a threshold, i.e.

\[
d_j^2 = \mathbf{v}_j^T \left( \mathbf{P}_{\xi,k-1}^{-1} \right)^{-1} \mathbf{v}_j \leq \gamma,
\]

(14)

where \( \mathbf{v}_j \) is the innovation, \( \mathbf{v}_j = \mathbf{z}_k - \mathbf{\hat{z}}_k \xi_{i,k-1} \), here \( \mathbf{\hat{z}}_k \xi_{i,k-1} \) denotes the first two variables of the prediction measurement. \( \mathbf{P}_{\xi,k-1}^{-1} \) is the innovation covariance of the \( j \)th target. The threshold \( \gamma \) is selected to produce a predetermined probability of erroneous rejection of the correct return. It is assumed for simplicity that the true return is a Gaussian random vector. If the \( j \)th measurement correctly corresponds to the \( j \)th target, then \( d_j^2 \) has a \( \chi^2 \) distribution with \( n \) degrees of freedom, where \( n \) is the order of the state vector.

**Step 3:** The final correlation with the fuzzy logic

In a clutter environment, the measurement may not be selected for the \( j \)th target because several measurements are probably validated for the tracking threshold of the \( j \)th target at the \( k \)th moment. At this situation, the NNSF algorithm choose the measurement ‘nearest’ to the predicted measurement as the only validated observation, and the remaining observations are rejected from consideration, while the JPDAF algorithm consider a set of validated measurements (those that fall within the initial validation gate) and compute a probability for each measurement to form a weighted equivalent measurement. In our method, we choose the final measurement for the correlation by the Mahalanobis distance and the difference in angle between the pseudodirection and the predicted direction. Suppose the measurement at the \( k \)th moment is \( \mathbf{z}_k = [\mathbf{r}_i \mathbf{c}_k^T]^T \), the \( i \)th measurement’s pseudodirection \( \mathbf{c}_k^T \) and the \( j \)th target’s direction \( \mathbf{h}_k^j \) are calculated using (1) and (2), and then the difference in angle between the pseudodirection and the predicted direction is defined as follows:

\[
\Delta h_j = \left| \mathbf{c}_k - \mathbf{h}_k^j \right|.
\]

(15)

The magnitude of the Mahalanobis distance \( d_j^2 \) and the difference in angle between the pseudodirection and the predicted direction \( \Delta h_j \) may vary from target to target. In order to design a general fuzzy association algorithm, it is necessary to normalize the \( d_j^2 \) and the \( \Delta h_j \) in form of \( d_j^{z} \) and \( \Delta h_j^{z} \). Suppose \( m_p \) measurements

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are validated for the tracking threshold of the \( j \)th target, we compute \( d_{ij}^2 \) and \( \Delta h_{ij} \) where
\[
\begin{align*}
\frac{d_{ij}^2}{\max_{i=1} d_{ij}^2}, \quad (16) \\
\frac{\Delta h_{ij}}{\max_{i=1} (\Delta h_{ij})}, \quad (17)
\end{align*}
\]

After the normalization, we define the membership functions by experience for the Mahalanobis distance \( d_{ij}^2 \), the difference in angle between the pseudodirection and the predicted direction \( \Delta h_{ij} \), and the output variables, which are shown in Fig. 2, 3, and 4. The values are mapped into some fuzzy sets, labeled in the linguistic terms of zero (ZE), small positive (SP), medium positive (MP), large positive (LP), very large positive (VLP), extremely positive (EP) and very extremely positive (VEP). For the Mahalanobis distance’s fuzzy sets, ZE, SP, MP, LP, VLP, EP and VEP have been chosen; for the difference in angle, ZE, SP, MP, LP and VP have been chosen; while the output variable’s fuzzy sets use ZE, SP, MP, LP and VP.

The time-varying overall output variables for the data association is inferred by a double-input single-output (DISO) fuzzy system, for which the fuzzy IF-THEN rule is represented by
\[
\text{IF } d_{ij}^2 \text{ AND } \Delta h_{ij} = \text{B THEN OUTPUT = C}
\]

Here A is one of the fuzzy sets of \( d_{ij}^2 \), B is one of the fuzzy sets of \( \Delta h_{ij} \), and C is one of the fuzzy sets of the output variables.

Since every such rule used by the proposed fuzzy association can be considered as an implication that defines a fuzzy association, they are concisely written as a linguistic map or fuzzy associative memory here, as shown in Table 1.

The rules are designed based on the principle that the smaller the two input variables, the higher the magnitude of the output variables, and vice versa.

The magnitude of the output variables are evaluated through the Max-Min compositional rule of inference technique and center of gravity (COG) defuzzification. The measurement with the most membership magnitude will be chosen as the validated measurement for the \( j \)th target.

**Step 4:** The updating of target state and covariance

In this step, the CKF is used to update the state and the covariance.

\[
P_{x|x|k-1} = \sum_{i=1}^{n} \alpha_i \chi_{x|k-1}^T \chi_{x|k-1} - \hat{x}_{k|k-1} \hat{x}_{k|k-1}^T, \quad (18)
\]
\[
W_{k} = P_{x|x|k-1}^{-1}, \quad (19)
\]
\[
\hat{x}_{k|k} = \hat{x}_{k|k-1} + W_{k} v_{k|k-1}, \quad (20)
\]
\[
P_{k|k} = P_{x|x|k-1} - W_{k} P_{x|x|k-1} W_{k}^T. \quad (21)
\]

<table>
<thead>
<tr>
<th>Output variables</th>
<th>( d_{ij}^2 )</th>
<th>( \Delta h_{ij} )</th>
</tr>
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<tbody>
<tr>
<td>ZE</td>
<td>SP</td>
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<td>ZE</td>
<td>ZE</td>
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- **Figure 2.** Membership function for the difference of distance
- **Figure 3.** Membership function for the difference of heading angle
- **Figure 4.** Membership function for the output variables
After the four steps, the $j$th target and the measurement matching to it are deleted from the target and measurement lists. We can do the association for the rest of the targets and measurements in the same way, till all the targets find their matching measurements. The flow chart of the DFDA filter is shown in Fig. 5.

**Figure 5.** The flow chart of the DFDA filter

After the four steps, the $j$th target and the measurement matching to it are deleted from the target and measurement lists. We can do the association for the rest of the targets and measurements in the same way, till all the targets find their matching measurements. The flow chart of the DFDA filter is shown in Fig. 5.

**IV. SIMULATION RESULTS AND ANALYSIS**

The aim in this section is to test the performance of the DFDA filter versus the JPDA filter and NNSF algorithm. For this purpose, a multiple targets tracking example with crossing targets is considered. For the sake of comparison, we will reproduce here the same example as that considered by Chang and Bar-Shalom in an attempt to tackle crossing targets. The targets are modeled as constant velocity objects in a plane with process noise (Gaussian zero mean) that accounts for slight changes in the velocities. More specifically, let a state vector $x = [x, y, v_x, v_y]^T$ represent both the $x$-$y$ coordinates and velocities of a given target, $T$ is the time-interval between two consecutive measurements. Then all targets have the same state transition matrix such that the state model is given by

$$x_k = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} x_{k-1} + \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix} v_{k-1},$$

where $v_{k-1}$ is zero mean Gaussian noises with known covariance matrix respectively $Q$. Radar is fixed at the origin of the plane and equipped to measure the range, $r$, and bearing, $\alpha$. Hence, the measurement equation is written as:

$$\begin{bmatrix} r_k \\ \alpha_k \end{bmatrix} = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \arctan \left( \frac{y_k}{x_k} \right) \end{bmatrix} + w_k,$$

where $w_k$ is zero mean Gaussian noises with known covariance matrix respectively $R$.

The initial states of the two targets are as follows: $x_1 = [1500m, 300m/s, 500m, 400m/s]^T$, $x_2 = [500m, 400m/s, 1500m, 300m/s]^T$. The time-interval $T=1s$. The noise covariance matrices are such that $R_i = 10m^2$ and $Q_i = 4$, where $R$ is a $2 \times 2$ matrix and $Q$ is a $4 \times 4$ matrix ($R_{ij} = Q_{ij} = 0$ for $i \neq j$). The set of measurements is created in the following way. First the true measurements are created using the true position added with zero mean Gaussian perturbation of covariance $R$. Clutter measurements, whose number is Poisson distributed with parameter $\lambda = 2 \times 10^6$ (number of false measurements per unit area (km$^2$)), are generated uniformly within the region of Fig. 6. This figure also shows the two original trajectories in a real clutter environment. 50 Monte Carlo runs have been performed.
Fig. 7 and Fig. 8 show the root mean square errors (RMSE) of positions for the two trajectories. From the two figures, it can be seen that the proposed method outperforms the NNSF in tracking accuracy. Fig. 8 shows the mean execution time of the three methods during 50 Monte Carlo runs; it can be seen that DFDAF outperforms the JPDAF in computational efficiency. Besides, in the 50 independent tracking, NNSF lost track 17 times, JPDAF lost track 3 times while DFDAF lost track 5 times. The simulation results show that DFDAF performs better in associating accuracy than the NNSF, and it has a lower computational load than the JPDAF. It means that the proposed method is an efficient and effective approach for real application.

V CONCLUSIONS

In this paper, a directional fuzzy data association filter is proposed for multi-target tracking. We analyze that the direction information is a very important parameter to differentiate the different batches of target. As we know, the Mahalanobis distance parameters are commonly used for the data association. So both the direction information and the distance are used for the data association based on the fuzzy logic. Firstly, the cubature Kalman filter (CKF) is used to predict the target positions for each trajectory. To the predicted locations as the center, the measurements whose Mahalanobis distances from the appointed predicted location are more than the threshold will be eliminated. Then the difference in angle between the pseudodirection and the predicted direction and the Mahalanobis distance are used as the input parameters of the fuzzy logic inference system for final correlation. At last, the targets’ state vectors will be updated using the CKF. The simulation results show that the proposed method performs excellently in data association, and it is an efficient algorithm for the real application.

ACKNOWLEDGMENT

The author sincerely thanks Professors Weixin Xie and Jianping Yu. They played a big part in my development.
during my study in Shenzhen University. They have been true professionals and very supportive.

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