Pinning Control of Complex Network by a Single Controller

Qunwei Wang
School of Business, Soochow University, 50 Donghuan Road, Suzhou 215021, China
Shilong Ge
College of Management and Engineering, Nanjing University of Finance and Economics, 3 Wenyuan Road, Nanjing 210046, China
Lin Jia
Center of Nonlinear Science, Jiangsu University, 301 Xuefu Road, Zhenjiang 212013, China

***Abstract*—Complex networks have attracted increasing attention from various fields of science and engineering today. In this paper, with assuming irreduciblity and symmetry of the couplings, we prove that a single controller can pin a coupled complex network to a homogenous solution, which is investigated for both continuous-time and discrete-time cases. Sufficient conditions are presented to guarantee the convergence of the pinning process locally and globally. The efficiency of the derived-results are illustrated by numerical simulation.**

***Index Terms*—Complex networks; Pin control; Continuous-time; Discrete-time**

Ⅰ. INTRODUCTION

Complex networks are shown to exist in various fields of real world [1–4], such as in the Internet, the World Wide Web (WWW), food webs, scientific citation web, etc., and thus become an important part of our daily life. Analysis and control of complex behaviors in complex networks consisting of a large number of dynamical nodes have attracted the attention of researchers from different fields. The nature of complex networks is their complexity, including dynamical evolution, topological structure, connection or node diversity, meta-complication, etc. [2]. The complex nature leads to difficulty in anatomizing them. So most of the existing work focus on networks with completely regular topological structures [5-9], such as chains, grids, lattices, and full-connected graphs.

In past few decades, synchronization and control problems are being widely studied in complex networks. In [10]–[14], the local stability of the synchronization manifold was studied via the transverse stability to the synchronization manifold. The synchronizability based on the topology of the complex network was discussed in detail especially focusing on the complex networks with small-world and scale-free properties. In [15]–[17], a distance was defined from the collective spatial states of the coupled system to the synchronization manifold. In particular, in [18], the author pointed out that chaos synchronization can be obtained if and only if the topology of the network has a spanning tree.

Also, the problem of chaos control has been a research subject, which attracts increasing attention (see [19]–[23] for references). Recently, the object of chaos control has been transferred from single or several nodes to a dynamical networks especially complex network (see [22], [23]). In particular, in [24]–[27], the authors studied pinning control problem on dynamical networks. Namely, controllers are only pinning on a very few fraction of nodes. In [24], [25], the authors investigated pinning control for linearly coupled networks and found that one can pin the coupled networks by introducing fewer locally negative feedback controllers. They also compared two different pinning strategies: randomly pinning and selective pinning based on the connection degrees, and found out that the pinning strategy based on highest connection degree has better performance than totally randomly pinning.

In this paper,we assum that the coupling matrix is irreducible and symmetric, and for simplicity, we also assum that the inner coupling matrix is a diagnal matrix. Here, we want to stabilize a complex network, such as a scale-free network or a random network, onto some desired homogenous stationary states by controlling a single node of the network. With some lemmas, four theorems and some remarks are proposed for both continuous-time and discrete-time cases. From the theorems, we can see that the stabilization of such networks is determined by the dynamics of each node, the coupling matrix, the inner-coupling matrix, and the feed-back gain matrix of the network. At last we prove that the whole network can be controlled to stable state by a single node. And, we verify our conclusion by numerical simulation of the linearly network.

The rest of this paper is organized as followings. Model description and some preliminaries consisting of one definition and three lemmas are introduced in section 2. In section 3 the synchronization of the complex network is studied with a controller. And two theorems are obtained for the continuous-time complex network. Besides, we extend our results to the discrete-time complex network. We prove that a controller can also pin the discrete-time

Corresponding author. Shilong Ge,
E-mail address: ecsigsl@163.com

doi:10.4304/jsw.7.10.2258-2262

complex networks to synchronization in section 4. In section 5 we give a numerical simulation.Finally, in section 6 conclusions are presented.

II.MODEL DESCRIPTION AND PRELIMINARIES

Now we consider a general model of complex network consisting of N identical nodes, where each node of the network is an n-dimensional nonautonomous dynamical system. The state equations of the whole network are described by the following differential equations:

$$\dot{x}_i(t) = f(x_i(t),t) + c\sum_{j=1}^{N} a_{ij}\Gamma x_i(t)$$

$i = 1,...,N$, (1)

where $x_i(t) = (x_{i1}(t),...,x_{in}(t))^T$ are the state variables of node i, $f: R\times R^n \to R^n$ is continuously differentiable, the constant $c > 0$ is the coupling strength and the inner coupling matrix $\Gamma = diag(\gamma_1 ... \gamma_n)$ is a diagonal matrix with $\gamma_i > 0$. $A = (a_{ij})_{N\times N}$ is the coupling matrix of the network, where a_{ij} is defined as follows: if there is a connection between node i and node j ($j \neq i$), then $a_{ij} > 0$; otherwise $a_{ij} = 0$, and the diagonal elements of matrix $A = (a_{ij})_{N\times N}$ are defined by $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij} = -\sum_{j=1, j\neq i}^{N} a_{ji}$, $i = 1,2,...,N$.

In this paper we suppose that the network is connected in the sense that there are no isolate clusters. To obtain our main results, we need the following lemmas.

Lemma 1: (Chen [2]) If $A = (a_{ij})_{N\times N}$ is an irreducible matrix with Rank(A)= $N-1$ and satisfying $a_{ij} = a_{ji} \geq 0$, if $\sum_{j=1}^{N} a_{ij} = 0$, for $i = 1,2,...,N$. Then, all eigenvalues of the matrix

$$\tilde{A} = \begin{pmatrix} a_{11}-\varepsilon & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{pmatrix}$$

are negative, where $\varepsilon > 0$.

Lemma 2: If $\tilde{A}$ satisfies the conditions of lemma 1, then there exists a unitary matrix, $\Phi = (\phi_1, \phi_2, ..., \phi_N) \in R^{N\times N}$, such that $\tilde{A}\phi_i = \lambda_i \phi_i$, $i = 1,2,...,N$, where λ_i is the eigenvalue of $\tilde{A}$. The lemma 2 can be deduced using linear algebra theory such as in Ref[9].

III. PINNING CONTROL OF A COMPLEX NETWORK WITH A SINGLE CONTROLLER

Note that the model (1) is more general than typically studied for a complex network. It is said to achieve synchronization if

$$x_1(t) = x_2(t) = \cdots = x_N(t) = s(t) \quad \text{as } t \to \infty,$$

where $s(t) \in R^n$ is a solution of an isolate node, namely

$$\dot{s}(t) = f(s(t)) \quad .$$

(2)

We prove that if $\varepsilon > 0$ and c is chosen suitably , the coupled network with a single controller shown in (3) can pin the complex dynamical (1) to $s(t)$.

$$\begin{cases} \dot{x}_1(t) = f(x_1(t),t) + c\sum_{j=1}^{N} a_{1j}\Gamma x_j(t) - c\varepsilon\Gamma(x_1(t) - s(t)) \\ \dot{x}_i(t) = f(x_i(t),t) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t), i = 2,...,N \end{cases}$$

. (3)

Denote $e_i(t) = x_i(t) - s(t)$, then the system (1) can be rewritten as follows:

$$\dot{e}_i(t) = f(x_i(t),t) - f(s(t),t) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t)$$

$i = 1,2,...,N$, (4)

And the network with a single controller (3) is rewritten as follows:

$$\dot{e}_i(t) = f(x_i(t),t) - f(s(t),t) + c\sum_{j=1}^{N} \tilde{a}_{ij}\Gamma e_j(t) \quad ,$$

(5)

where $\tilde{a}_{11} = a_{11} - \varepsilon$, $\varepsilon > 0$ and $\tilde{a}_{ij} = a_{ij}$ otherwise.

With lemma 1 given above, we prove two theorems. Theorem 1 addresses local synchronization. Theorem 2 addresses global synchronization.

Let $e(t) = (e_1(t), e_2(t)..., e_N(t))$. Differentiating (5) along $s(t)$ gives

$$\dot{e}(t) = Df(s(t),t)e(t) + c\Gamma e(t)\tilde{A}^T \quad .$$

(6)

According to the theory of Jordan canonical forms, there exist eigenvectors

$$\Phi = (\phi_1, \phi_2, ..., \phi_N) \in R^{N\times N},$$

satisfying $\tilde{A} = \Phi J \Phi^T$, where $\Phi^T\Phi = I$ and $J = diag(\lambda_1,...,\lambda_n)$, where $\lambda_1, \lambda_2 ..., \lambda_N$ are the eigenvalue of $\tilde{A}^T$ and $0 > \lambda_1 > ... > \lambda_N$, and define $\upsilon(t) = e(t)\Phi$. Then we have

$$\dot{\upsilon}_i(t)=[Df(s(t),t)+c\lambda_i\Gamma]\upsilon_i(t), i=1,...,N.$$
(7)

It seems that the analysis is similar to that of Ref [28]. However, it is different because we take the inner-matrix into accout.

Theorem1. Suppose that $u_i(t)$, $i=1,...,n$ are the eigenvalues of the matrix $\frac{1}{2}[Df(s(t),t)+D^T f(s(t),t)]$, $u(t)=\max_{1\le i\le n} u_i(t)$. If $u(t)<-c\lambda_1\gamma-\eta$ for all $t>0$, $\eta>0$, and $\gamma=\min_{1\le i\le n}\gamma_i$. Then, the coupled system with a controller (3) can be locally exponentially synchronized to $s(t)$.

Proof: Select the following Lyapunov-Krasovskii of the form

$$V_i(t)=\frac{1}{2}\upsilon_i^T(t)\upsilon_i(t).$$

$$\dot{V}_i(t)=\upsilon_i^T(t)[\frac{1}{2}(Df(s(t),t)+D^T f(s(t),t))+c\lambda_i\Gamma]\upsilon_i(t)$$
.

Under $u(t)<-c\lambda_1\gamma-\eta$ for all $t>0$ $\eta>0$ and $\gamma=\min_{1\le i\le n}\gamma_i$, we have

$$\dot{V}_i(t)\le-2\eta V_i(t),$$

which means that $V_i(t)=O(e^{-2\eta t})$.

Theorem 1 is proved**.**

Theorem 2. Suppose $0>\lambda_1>...>\lambda_N$ are the eigenvalue of $\tilde{A}$. If there are positive diagonal matrices $P=$ diag{ $p_1,...,p_n$ }, $\Delta=$ diag { $\Delta_1,...,\Delta_n$ } and a constant $\eta>0$, such that

$$(x-y)^T P(f(x,t)-\Delta x-f(y,t)+\Delta y)\le-\eta(x-y)^T(x-y)$$
,

and $\Delta_j+c\lambda_1\gamma_j<0$ for $j=1,...,n$. Then the controlled system (3) is globally exponentially synchronized to $s(t)$.

Proof: Select the following Lyapunov-Krasovskii of the form

$$\dot{V}(t)=\sum_{i=1}^N e_i^T(t)Pf(x_i(t),t)-\sum_{i=1}^N e_i^T(t)Pf(s(t),t)+\sum_{i=1}^N e_i^T(t)Pc\sum_{j=1}^N\tilde{a}_{ij}\Gamma e_j(t)$$

$$V(t)=\frac{1}{2}\sum_{i=1}^N e_i^T(t)Pe_i(t).$$

$$=\sum_{i=1}^N e_i^T(t)P[f(x_i(t),t)-f(s(t),t)-\Delta e_i(t)]+\sum_{i=1}^N e_i^T(t)P[c\sum_{j=1}^N\tilde{a}_{ij}\Gamma e_j(t)+\Delta e_i(t)]$$

Denote $\overline{e}_j(t)=(\overline{e}_{1j}(t),...,\overline{e}_{Nj}(t))^T$

$$\le-\eta\sum_{i=1}^N e_i^T(t)e_i(t)+\sum_{i=1}^N e_i^T(t)P[c\sum_{j=1}^N\tilde{a}_{ij}\Gamma e_j(t)+\Delta e_i(t)]$$

$$=-\eta\sum_{i=1}^N e_i^T(t)e_i(t)+\sum_{j=1}^n p_j\overline{e}_j^T(t)(c\gamma_j\tilde{A}+\Delta_j I)\overline{e}_j(t)$$
.

Because $\Delta_j+c\lambda_1\gamma_j<0$ then $c\gamma_j\tilde{A}+\Delta_j I<0$, we have

$$\dot{V}(t)\le-\eta\sum_{i=1}^N e_i^T(t)e_i(t)\le-2\eta\frac{V(t)}{\min_i p_i}.$$

Therefore $V(t)=O(e^{-2\eta t/\min_i p_i})$.

Theorem 2 is proved completely**.**

Rmark 1. It is clear that if c is large enough, then the coupled network with a controller can pin the complex network to a solution $s(t)$.

Rmark 2. Although the coupled network with a single controller can pin the complex network to a solution $s(t)$. It does not mean that one must use one single controller to pin a coupled system. Theorem 2 also tells us that by adding any number of controllers can pin the coupled system. It is clear that the larger the number of the controllers is, the easier to pin a coupled system.

Ⅳ.EXTENSION TO DISCRETE-TIME NETWORKS

A discrete-time system is described using different equations while a continuous-time system is described using differential equations, hence they should be treated differently. Thus it is essential to extend our results to the discrete-time case.

Considering a general discrete-time dynamical network described by

$$x_i(k+1)=f(x_i(k))+c\sum_{j=1}^N a_{ij}\Gamma x_i(k)$$
$$i=1,2,...,N, \quad (8)$$

where $x_i(k), f, c, a_{ij}, \Gamma$ have the same meanings as those in the system (1).

Suppose that the network (8) is connected in the sense that there are no isolated clusters; that is A is an irreducible matrix. As before, the discrete-time nodes in (8) are said to achieve synchronization if

$$x_1(k)=x_2(k)=\cdots=x_N(k)=s(k) \text{ as } k\to\infty,$$

where $s(k)\in R^n$ is a solution of the coupled system, namely,

$$s(k+1)=f(s(k)).$$

We prove that if $\varepsilon>0$ and c is chosen suitably , the coupled network with a single controller shown in (9) can pin the dynamical complex (8) to $s(k)$.

$$\begin{cases} \dot{x}_1(k) = f(x_1(k)) + c\sum_{j=1}^{N} a_{1j}\Gamma x_j(k) - c\varepsilon\Gamma(x_1(k) - s(k)) \\ \dot{x}_i(k) = f(x_i(k)) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(k), i = 2,..., N \end{cases}.$$

(9)

Let $e_i(k) = x_i(k) - s(k)$, $i = 1,..., N$, then the system (9) can be rewritten as follows:

$$e_i(k+1) = f(x_i(k)) - f(s(k)) + c\sum_{j=1}^{N} \tilde{a}_{ij}\Gamma e_j(k)$$

$i = 1, 2,..., N$, (10)

where $\tilde{a}_{ij}$ have the same meanings as those in the system (5).

Linearizing the controlled network (10) on the homogenous stationary state $s(k)$ leads to

$$e(k+1) = e(k)D^T f(s(k)) + c\tilde{A}e(k)\Gamma ,$$

(11)

where $D(f(s(k)))$ is the Jacobian of f on $s(k)$, $e(k) = (e_1^T(k), e_2^T(k),..., e_N^T(k))^T \in R^{N\times n}$.

Let $0 > \lambda_1 > \lambda_2 > ... > \lambda_N$ be the eigenvalue of the matrix $\tilde{A}$. According to Lemma 2, there exists an orthogonal matrix, $\Phi = (\phi_1, \phi_2,..., \phi_N) \in R^{N\times N}$, such that

$$\tilde{A}\phi_i = \lambda_i\phi_i \qquad i = 1, 2,..., N .$$

(12)

By expanding each colume $e(t)$ on the basis Φ, we obtain

$$e(k) = \Phi\,\eta(k).$$

(13)

Then (11) can be expanding into the following equations:

$$\eta(k+1) = \eta(k)D^T(f(s(k))) + \Lambda\,\eta(k)\Gamma ,$$

(14)

where $\Lambda = diag(\lambda_1, \lambda_2,..., \lambda_N)$.

Furthermore, we can obtain

$$\eta_i(k+1) = D(f(s(k)))\eta_i(k) + \lambda_i\Gamma\,\eta_i(k)$$

$i = 1, 2,...N$, (15)

where $\eta_i^T(k)$ is the i th row of $\eta(k)$.

Hence, the stability problem of the $(N\times n)$-dimensional (9) is converted into the stability problem of the n-dimensional linear system(15).

Theorem 3. The coupled system with a controller (9) can be locally exponentially synchronized to $s(k)$, if it satisfies

$$\begin{bmatrix} (D(f(s(k))) + \lambda_i\Gamma)^T & I \\ I & D(f(s(k))) + \lambda_i\Gamma \end{bmatrix} < 0 ,$$

$i = 1, 2,...N$.

Proof: In this case, define a new Lyapunov function as

$$V(\eta_i(k)) = \frac{1}{2}\eta_i^T(k)\eta_i(k)$$

$i = 1, 2,..., N$.

$$V(\eta_i(k+1)) - V(\eta_i(k)) = \frac{1}{2}[\eta_i^T(k+1)\eta_i(k+1) - \eta_i^T(k)\eta_i(k)]$$

$$= \frac{1}{2}\eta_i^T(k)[(D(f(s(k))) + \lambda_i\Gamma)^T(D(f(s(k))) + \lambda_i\Gamma) - I]\eta_i(k)$$

.

Under the condition

$$\begin{bmatrix} (D(f(s(k))) + \lambda_i\Gamma)^T & I \\ I & D(f(s(k))) + \lambda_i\Gamma \end{bmatrix} < 0,$$

we have $V(\eta_i(k+1)) - V(\eta_i(k)) < 0$.

Theorem 3 is proved**.**

Theorem 4. If there exists a negtive definite matrix $P = diag(p_1, p_2,..., p_n)$, satisfying $2\lambda_i\left[\Gamma PD(f(s(k)))\right]^s - P < 0 \quad i = 1, 2,..., N,$ then the controlled system (9) is globally stable.

Proof. Select the following Lyapunov-Krasovskii of the form

$$V(\eta_i(k)) = \sum_{i=1}^{N}\eta_i(k)^T P\eta_i(k)$$

$$\Delta V = V(\eta_i(k+1)) - V(\eta_i(k))$$

$$= \sum_{i=1}^{N}(\eta_i^T(k)D^T(f(s(k))) + \lambda_i\eta_i^T(k)\Gamma)P(D(f(s(k)))\eta_i(k) + \lambda_i\Gamma\eta_i(k)) - \sum_{i=1}^{N}\eta_i^T(k)P\eta_i(k)$$

$$= \sum_{i=1}^{N}\eta_i^T(k)\left[D^T(f(s(k)))PD(f(s(k))) + \lambda_i^2\Gamma P\Gamma + \lambda_i D^T f(s(k))P\Gamma + \lambda_i\Gamma PDf(s(k)) - P\right]\eta_i(k)$$

$$= \sum_{i=1}^{N}\eta_i^T(k)\left[D^T(f(s(k)))PD(f(s(k))) + \lambda_i^2\Gamma P\Gamma + 2\lambda_i(\Gamma PDf(s(k)))^S - P\right]\eta_i(k)$$

,

where

$$\left[\Gamma PD(f(s(k)))\right]^s = \frac{1}{2}\left[\Gamma PD(f(s(k))) + (\Gamma PD(f(s(k))))^T\right]$$

Since P is a negtive denifite matrix ,we have $D^T(f(s(k)))PD(f(s(k))) < 0, \Gamma P\Gamma < 0$ and $2\lambda_i\left[\Gamma PD(f(s(k)))\right]^s - P < 0$, so we get $\Delta V = V(\eta_i(k+1)) - V(\eta_i(k)) < 0$

Then the theorem is proved.

V. SIMULATION

The control theorem's analysis above can be applied to networks with defferent size. For simplicity, we first consider a 10-nodes network, in which each node is a simple three-dimensional linear system.

$$(\dot{x}_{i1}, \dot{x}_{i2}, \dot{x}_{i3})^T = (-2x_{i1}, -3x_{i2}, -4x_{i3})^T$$

Besides we suppose that the inner-coupling marix is $\Gamma=\text{diag}(1,\ 1,\ 1)$, and the coupling matrix is

$$A=\begin{bmatrix} -4 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & -3 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & -2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -2 \end{bmatrix}$$

We want to stabilize this network onto the originally equilibrium point $s(t)=(0,0,0)^T$ with a single controller. For simpicity, we make the first node to be controlled.

When the coupling strength c=0.10, the controller $\varepsilon = 0.50$, we use the Matlab to describe the error $e_{ij}(t), i=1,2,...,10$ $j=1,2,3$. Because the equilibrium point is $s(t)=(0,0,0)^T$, the error $e_{ij}(t)$ are described on the same picture as Fig 1.

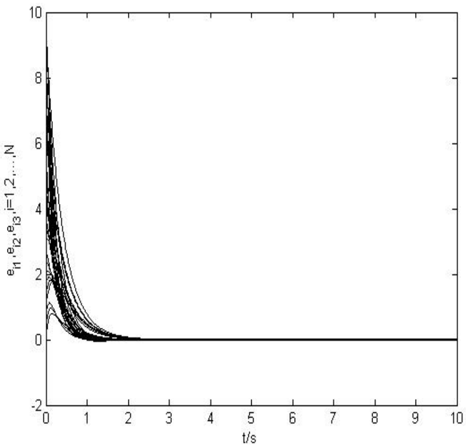

Fig1. Synchronization errors for the network with $c=0.10, \varepsilon=0.50$

From the picture we can see that a single controller can pin the complex network.

Ⅵ.CONCLUSION

In this paper, we prove that a single controller can pin a coupled complex network to a homogenous solution, which is investigated for both continuous-time and discrete-time cases. Sufficient conditions are presented to guarantee the convergence of the pinning process locally and globally. Simulations also verify our theoretical results.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support from the Humanities and Social Sciences Foundation of Ministry of Education (HSSF) (grants Nos. 10YJC790062), the Higher School Philosophy & Social Sciences Foundation of Jiangsu Province Education Department under grants Nos. 2010SJB790014, the Nanjing University of Finance and Economics Foundation of China (A2010007), as well as the Young Teacher Foundation of Soochow University under Gant No.BV11000611.

REFERENCES

[1] D.J. Watts and S.H. Strogatz 1998 *Nature*.393 440

[2] S.H. Strogatz 2001 *Nature*. 410 268

[3] A.L. Barabsai, R. Albert and H. Jeong 1999 *Physica A.* 272 173

[4]Wang X F and Chen G 2002 *IEEE Trans.* 49 54

[5] Hu G, Xiao J, Gao J, Li X, Yao Y and Hu B 2000 *Phys. Rev.E.*62 (3) 3043

[6] Li X and Chen G 2004 *IEEE Trans. Circuits. Syst.-I.* 51 (10) 2074

[7] Wang X F 2002 *Int. J. Bifurcat. Chaos.* 12 (5) 885

[8] Wang X F and Chen G 2003 *IEEE Circuits Syst. Mag.* 3(1) 6

[9] R.B. Bapart 1997 Cambridge University, Cambridge, UK

[10] L. M. Pecora and T. L, Carroll 1998 *Phys. Rev. Lett.* 80 2109

[11] L. M. Pecora, T. Carroll, G. Johnson, D. Mar, and K. S.Fink 2000 *Int. J.Bifurc. Chaos.* 10 273

[12] Wang X F and Chen G 2002 *IEEE Trans. Circuits Syst.* I 49 54

[13] M. Barahona and L. M. Pecora 2002 *Phys. Rev. Lett.* 89 1

[14] T. Nishikawa, A. E. Motter, Y. C. Lai, and F. C. Hoppensteadt 2003 *Phys. Rev.Lett.* 91 1

[15] Wu C W and Chua L O 1995 *IEEE Trans. Circuits Syst.* 42 430

[16] Lu W L and Chen T P 2004 *IEEE Trans. Circuits Syst. I.* 51 2491

[17]Y.M. Moon, P. Park, W.H. Kwon and Y.S. Lee 2001 *Int. J. Control.* 74 (14) 1447

[18] Wu C W 2005 *Nonlinearity.* 18 1057

[19] Wu C W, Yang T, and Chua L O 1996 *Int. J. Bifurc. Chaos.* 6 455

[20] Hong Y G, Qin H S, and Chen G R 2001 *Int. J. Bifur. Chaos*. 11 1149

[21]Lian K Y, Liu P, Chiang T S, and Chiu C S 2002 IEEE *Trans. Circuits Syst. I.* 49 17

[22] H. G. Z. Qu 1994 *Phys. Rev. Lett.*72 68

[23] N. Parekh, S. Parthasarathy, and S. Sinha 1998 *Phys. Rev. Lett.*81 1401

[24] R. O. Grigoriev, M. C. Cross, and H. G. Schuster 1997 *Phys. Rev. Lett.*79 2795

[25] Wang X F and Chen G R 2002 *Phys. A.* 310 521

[26] Li X, Wang X F and Chen G R 2004 *IEEE Transactions on Circuits and Systems-I.*51 2074

[27] A. Greilich, M. Markus, and E. Goles 2005 *Eur. Phys. J. D.* 33 279

[28] Chen T P,Liu X W and Lu W L 2007 *IEEE Transactions on Circuits and Systems-I.*54 1317.