A Co-Plot- based Efficiency Measurement to Commercial Banks

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***Abstract*—For the issue of the lack in plotting analysis method in the research of DEA, this paper introduces a new graphic method—Co-Plot. It can demonstrate DMUs and variables visually in two-dimensional space, and enable the analysis of them at the same time. By applying the ratio relationship among variables, Co-Plot can be combined with DEA in a way to display the results of DEA easily and intuitively. Specific steps for graphing by Co-Plot are given, and the effectiveness of the method is proved through an illustration of commercial banks.**

***Index Terms*—data envelopment analysis; efficiency measurement; Co-Plot**

I. Introduction

Data Envelopment Analysis, also known as DEA, is based on the concept of relative efficiency. It is a mathematical planning method to evaluate relative efficiency or benefit efficiency for a group of homogeneous Decision Making Units (DMUs) with various inputs and outputs, using optimization method. Since the establishment of the first DEA model --- CCR[1] by remarkable strategist Chaines, Cooper and Rhode in 1978, DEA has attracted great attention from scholars home and abroad., of which, both theory and practice is flouring. DEA is a nonparametric strategical method, which is used to judge the efficiency though the identification of whether Desicion Making Units are located on production frontiers. Because parameters do not need to be estimated in advance, the influences of subjective factors are avoided, especially in the research of multi-input and multi-output DMUs. The enormous advantage is also reflected on caculation symplifying and error reducation.

DEA is initially applied on efficiency measurement of non-profit organizations of public services, for instance, education, health, government agency, etc. With the continuous improvement, DEA has been applied in more industries, such as finance and manufacturing sectors.[2] There has been a great deal of research carried out on the application of DEA in the efficiency measurement of commercial banks. Earlier, Mukherjee K, *et al.*[3] used DEA combined with Malmquist Index to evaluate the operation of large American commercial banks in 7 years after deregulation. Ahmet *et al.*[4] evaluated all departments of the commercial bank with credit value method.

Chinese scholars have made some beneficial exploration in the application of DEA in commercial bank efficiency measurement study as well. For example, Zhang Wei *et al.* used DEA method to examine the effectiveness of the scale of urban commercial banks, and extended the content of internal control measurement.[5] Chi Guotai *et al.* supplied a comprehensive measurement system of Chinese commercial banks based on the establishment of banking input and output indicator system.[6] Bi Gongbing *et al.* established a DEA model with two-step production system and made reasonable measurement of relative efficiency of commercial banks.[7] However, Co-Plot method is missing in the literature about DEA efficiency rating of Chinese domestic commercial banks. This paper introduces a new graphing method --- Co-Plot. Based on DEA efficiency measurement of 17 second-rank branches of one Chinese commercial bank branch, and it endeavors to demonstrate DEA result via two-dimensional plan. Different from previous research on commercial bank efficiency measurement, this paper starts from geometry to reach similar DEA result without complicated mathematical model establishment, thus demonstrates more simplicity and intuitiveness.

II. Co-Plot

A. Co-Plot and its development

The relative efficiency of each DMU can be calculated by DEA method, and sorted according to efficiency values. However, due to multivariate features, it is very difficult to make DEA result graphical in euclidean space. How to demonstrate DEA result in a simple and intuitive way has become a new research direction. The Co-Plot method introduced by this article is a new graphical method, and a special form of multidimensional scaling method. It will define each DMU as a dot in two-

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dimensional space. The position of each dot is decided by all given variables or indicators, while the variable is expressed by the ray from the center of the gravity. From the result of Co-Plot, some statements can be concluded as following. The position of similar DMUs is close to each other on the plan, which indicates that DMUs from the same group possess similar features or behaviors. The relationship between variables can be obtained through the positions of rays: being opposite in direction illustrates negative correlation, while being basically the same in direction illustrates positive correlation. With regard to variables with positive correlation, after further analysis, the ones which are of little effect to DEA results and Co-Plot graphing can be rejected.

In data-analysis literature, most commonly-used graphing methods of handling large data sets are Principal Component Analysis (PCA)[8], Correspondence Analysis (CA)[9], Multidimensional Scaling (MDS)[10] , M&B method (Mareschal and Brans)[11], etc. Traditional multivariate analytical approaches, such as PCA, MDS and clustering, normally analyze variables and DMUs respectively. Although CA and M&B can analyze both simultaneously, CA is not applicable to continuous variable analysis, while the process of M&B is fairly complicated. Co-Plot, a special form of multidimensional scaling method, provides superiority beyond compare with other graphing methods of Co-Plot graphic presentation. Firstly, it overcomes the drawback of traditional multivariate analytical approach that analyze variables and DMUs respectively. It can examine both and directly perceive the relationship between them. Secondly, it can not only identify DMUs, namely outliers, with outstanding performance in two-dimensional plan, but can also find and combine or reject variables with less effect to DEA results. As a result, it performs a role of screening variables. Last but not least, its complexity will not be intensified with the increase of variables.

Co-Plot has been widely applied in many areas since its development, for instance, industrial measurement of 35 Chinese cities combined with DEA[12], application in Multicriteria Decision Making (MCDM)[13], research on social and economic differences between cities[14], performance analysis of Greek banking system[15], etc. In China, most research concerning Co-Plot concentrates on tourism[16], and have not been found in commercial banks and DEA.

Co-Plot combining with DEA is a desirable research technique, with the two complementing each other. (1) Co-Plot is applied in DEA as a geometrical graphing method. It extends the DEA approach. (2) Co-Plot is more simple and intuitive, which overcomes the abstractness of DEA mathematical model. Therefore, it is beneficial to the application and promotion of DEA. (3) Co-Plot assists the selection of index. Graphing by Co-Plot before using DEA can identify indexes with less effect to efficiency, which can even be neglected, so as to achieve the target of screening. (4) The effectiveness of Co-Plot graphing can be testified by DEA results.

B. Co-Plot Graphing[12]

Supposing there are n DMUs, the inputs and outputs of each DMU are m (regard all inputs and outputs as variables), we can use $n \times m$ order of matrix to indicate data. Before graphing, data matrix should be indicated first, followed by DMUs and variables. Co-Plot covers two maps. The first map illustrates n dots of DMUs, and based on the first one, the second map illustrates m independently-drawn rays.

The procedure of Co-Plot graphing is divided into the following four stages:

The first stage is to standardize the data matrix $X_{n\times m}$, in order to diversify the range of value-taking. Variables with different degree of dispersion and concentration can be compared fairly. The matrix $Y_{n\times m}$ is thus obtained. Y_{ij} can be obtained by:

$$Y_{ij} = (x_{ij} - \bar{x}_{\cdot j}) / S_j \qquad i = 1, \cdots n; \ j = 1, \cdots, m$$

Among this, $\bar{x}_{\cdot j}$ is the mean value of the j^th^ rank in matrix $X_{n\times m}$, and S_j is the standard deviation.

The second stage is to measure the dissimilarity of each pair of DMUs, which is to form a $n \times n$ symmetrical matrix D_{ik} from C_n^2 DMU combinations. The city-block distance is used here, which is to measure dissimilarity by the sum of absolute deviation

$$D_{ik} = \sum_{j=1}^{m} \left| \ y_{ij} - y_{kj} \ \right| \geq 0 \,, (1 \leq i, k \leq n)$$

The third stage is to use multidimensional scaling method (MDS) to illustrate the matrix D_{ik} from the previous stage. In this way, every DMU can be illustrated by a dot P_i in two-dimensional Euclidean space, which means producing n coordinates P_i = (X_{1i}, X_{2i}), ($i = 1, \cdots, n$), and this makes every rank of the standardized matrix Y = ($Y_{i1}, \cdots, Y_{im}$) illustrated as a dot (X_{1i}, X_{2i})in two-dimensional space.

In the fourth stage, based on the previous one, every variable j is expressed by the ray $\vec{X}_j$ from the center of the gravity. The principle of selecting $\vec{X}_j$ is to maximize the relativity of the actual value of variable j and its projection on the ray. Consequently, the correlation between the cosine of the angle between the rays and the corresponding variables is roughly proportional.

Using Co-Plot graphing, the issue of degree of fitting should be considered. The former two stages are to handle data matrix, while the later two are to construct graphs. Therefore, the degree of fitting is only analyzed in the later two stages. In the third stage, Guttman's SSA[17] (small space analysis) method is applied, and devaition coefficient θ is used as the measuring standard. The smaller θ is, the higher the degree of fitting is. In the fourth stage, it is required to calculate the degree of fitting of m variables respectively. Each value of degree corresponds to the maximum relevance r_j^* ($j = 1,\ldots, m$). The larger r_j^* is, the better $\hat{X}_j$ can illustrate the direction of variables, and thus the degree of fitting is higher. Normally, decisions whether variables should be kept or deleted are made according to the size of the relevance r_j^* value.

III. DEA MODEL

Suppose there are n DMUs, let X_j and Y_j respectively represent the input and output levels of the j^{th} DMU ($j = 1,\cdots,n$), and use λ_j to show the weight of DMU_j. There will be various models if input and output are used to evaluate the k^{th} DMU. Considering the ranking issue in this paper, not only the CCR model[1] is selected, but the SE-DEA[18] as well. They are as follows:

- CCR Model

$$\max \theta_k$$

$$s.t.\sum_{j=1}^{n}\lambda_j X_j \le X_k \qquad (1)$$

$$\sum_{j=1}^{n}\lambda_j Y_j \ge \theta_k Y_k$$

$$\lambda_j \ge 0;\ j = 1,\cdots,n$$

- SE-DEA Model

$$\min[\theta - \varepsilon(\hat{e}^T S^- + e^T S^+)]$$

$$s.t.\sum_{\substack{j=1\\ j\neq k}}^{n}\lambda_j X_j + S^- = \theta X_k \qquad (2)$$

$$\sum_{\substack{j=1\\ j\neq k}}^{n}\lambda_j Y_j - S^+ = Y$$

$$\lambda_j \ge 0,\ j = 1,\cdots,n$$

Among them, θ_k is the efficiency value of the k^{th} DMU. ε is a non-Archimedean infinitesimal. S^- and S^+ are slack variables of corresponding input and output. In addition, Model (1) is the output type, and Model (2) is input.

IV. APPLICATION

Banking industry is a key area of DEA application. However, the current research both home and abroad is confined to the solution of efficiency value and improvement of efficiency, and graphing analysis is rarely applied. This paper uses 17 second-rank branches of one Chinese commercial bank branch as examples, firstly adopts Model (1) and (2) to perform efficiency measurement, then uses Co-Plot graphing and analyzes the relevant results. With the DEA measurement results of commercial banks illustrated by Co-Plot, bank management will recognize the relationship between each index and efficiency more simply and intuitively, therefore the direction of efficiency improvement will be indicated.

Either DEA or Co-plot needs objective index selection, which is to select indexes with representativeness and maneuverability. This article cites the research of Bi Gongbing *et al*.[7] The selected inputting indexes include fixed assets net value, the number of employees and operating expenses, and the outputting indexes include loans and paper profits. The original data can be accessed in Bibliography[7].

The DEA efficiency value obtained from Model (1) and (2) is shown in Table 1:

TABLE I
RESULT OF EFFICIENCY CALCULATION

决策单元	CCR 效率	超效率	决策单元	CCR 效率	超效率
1	1.0000	2.0928	**10**	0.2146	0.2146
2	0.4510	0.4510	**11**	0.2976	0.2976
3	0.5676	0.5675	**12**	0.3442	0.3442
4	0.4059	0.4059	**13**	0.1373	0.1373
5	0.9090	0.9090	**14**	0.3827	0.3827
6	0.9558	0.9558	**15**	0.2561	0.2561
7	0.6858	0.6858	**16**	0.2660	0.2660
8	0.3713	0.3713	**17**	0.1588	0.1588
9	0.2524	0.2524			

TABLE II
RATIO DATA

	r_{11}	r_{21}	r_{31}	r_{12}	r_{22}	r_{32}
1	120.1746	0.0120	100.0372	3.6948	0.0004	3.0756
2	32.9381	31.8869	40.9477	1.1158	1.0802	1.3871
3	47.5501	53.3519	56.7761	1.0658	1.1958	1.2726
4	29.6707	31.4412	40.6070	0.6219	0.6591	0.8512
5	73.2769	66.5203	90.9296	1.7654	1.6027	2.1907
6	53.1882	80.0437	95.6202	0.7536	1.1342	1.3549
7	38.2165	42.7021	68.6025	0.8510	0.9508	1.5276
8	16.6637	28.8938	37.1395	0.3934	0.6822	0.8769
9	19.3570	19.4880	25.2507	0.2888	0.2907	0.3767
10	7.3996	14.4756	20.1911	0.2419	0.4732	0.6600
11	17.0455	20.8993	29.7748	0.4451	0.5457	0.7775
12	21.4288	31.9745	34.4292	0.5852	0.8731	0.9402
13	7.0398	9.9924	13.7398	0.2007	0.2849	0.3918
14	24.5370	30.7885	38.2800	0.6526	0.8176	1.0166
15	17.7826	18.5724	25.6225	0.2759	0.2882	0.3976
16	13.8287	21.8317	26.6056	0.2998	0.4733	0.5768
17	7.4096	9.8930	15.8883	0.0167	0.0223	0.0358

DEA and Co-Plot is connected by ratio relationship, which corresponds to the weight in DEA. Consequently, before operating Co-Plot, this paper defines six ratios r_{ij} and regards each one as a variable:

$$r_{ij} = \frac{y_j}{x_i}, \quad \forall i = 1,2,3 \quad j = 1,2$$

x_i and y_j represent bank input and output. It can be seen from the expression of r_{ij}, the larger the ratio value is, the DMU is more useful with respect to this ratio; in the Co-plot graph, which means the projection of the dot representing DMU is larger on the ray representing this ratio. The value r_{ij} calculated by original data from the 17 commercial banks is shown in Table 2. Using data in Table 2 to perform Co-Plot graphing, a plane figure can be obtained as shown in Figure 1.

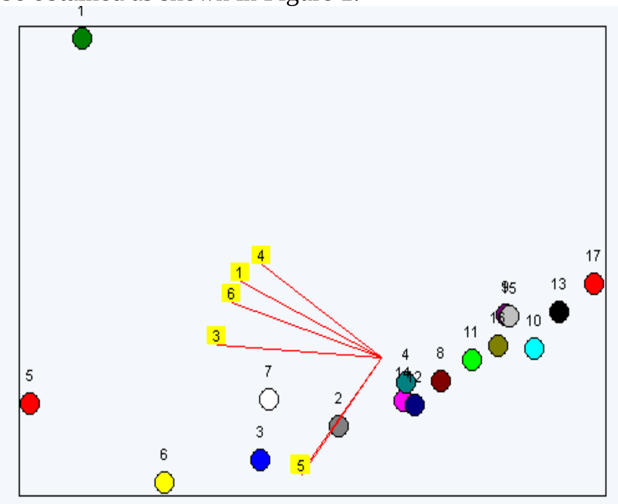

Figure 1

In Figure 1, 17 DMUs are presented by circles; 6 ratios are presented by rays. Among them, 1, 2, 3 respectively represent r_{11}, r_{21}, r_{31}; 4, 5, 6 represent r_{12}, r_{22}, r_{32}. According to the theory of Co-Plot, super-efficient DMUs are normally located in the peripheral space of the plan, while the DEA effective DMUs are located slightly within, instead of the central position. It can be indicated from Figure 1, the projection of Bank 1 is the largest on four ratios (r_{11}, r_{31}, r_{12}, r_{32}), and in the middle position on the other two ratios. Overall, it is super-efficient or at least effecient DMU. In the left 16 banks, the projection of Bank 5 and Bank 6 is smaller than that of Bank 1, but quite large on r_{21} and r_{22}. Therefore, their efficiency value is high, while lower than that of Bank 1. Similarly, the efficiency value of Bank 2, 3 and 7 is even lower. The projection of the other 11 commercial banks on the 6 ratios is very small, as a result, the efficiency value is extremely low. Apparently, the above analysis is similar to DEA calculation result, which can be testified by comparing with Table 1.

Examining the Co-Plot graphs of the 17 commercial banks, it is not difficult to find that ratio r_{21} and r_{22} (Ray 2 and 5) basically overlap, which indicates the number of employees is consistent with the effects on the two outputs. Also, the values of most banks on the two ratios are similar, which means the impact of the number employees are not obvious on banking efficiency, while the fixed assets net value affect the efficiency value much more. This conclusion is exactly the same with that of Duan Yongrui's research, which argues the primary cause of low efficiency and little benefit of Chinese commercial banks is not due to staff redundancy, therefore layoffs will not boost bank profits, however the the space for lowering operating expenses is large.[2]

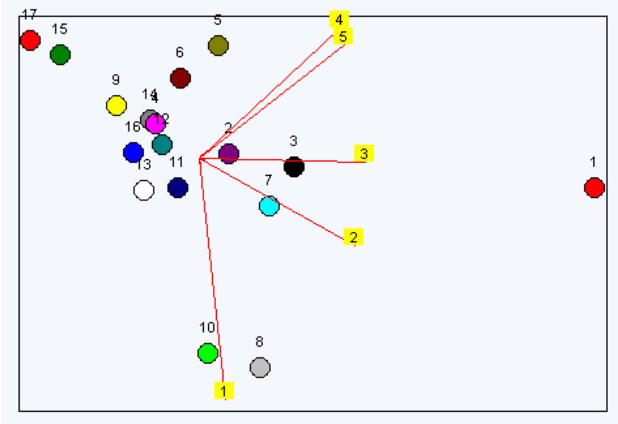

Figure 2

In order to further explore the features of the original data, this paper will not adopt ratio analysis. Instead, original data will be put directly into Co-Plot and the result in Figure 2 will be obtained, among which, respectively, 1, 2 and 3 represent three types of input, 4 and 5 represent two types of output. Via calculation, we have achieved coefficient variation θ=0.041 (fairly low), and the correlation coefficients of five variables are 0.98, 0.98, 0.98, 0.98 and 0.97 (all quite high). So the overall degree of fitting is very good. Form Figure2, it can be found out directly, the projection of Bank 1 is the largest on two outputs, located on the periphery of the graph, and is super-efficient. This is in accord with the sorting result calculated by Anderson and Pertersen's Super-efficiency Model. (as seen in Table 1)

V. Conclusion

Co-Plot is a fairly new graphic method, and a special form of multidimensional scaling method. It expresses each DMU by a dot in two-dimensional space and every variable by a ray from the center of the gravity. The result can be demonstrated by plane figure. When using ratio data collection between variables to draw graphs, Co-plot can show DEA result in a two-dimensional space because of the association with the concept of efficiency from DEA. Compared with other graphic tools, the outstanding advantage of Co-Plot is that it can consider DMU and the variable at the same time.

This paper adopts 17 second-rank branches of one Chinese commercial bank branch as examples, demonstrates the Co-Plot graph respectively of ratio data and original data, and makes relevant analysis regarding DEA efficiency value and its sort. It shows the application of Co-Plot in DEA through the examples of the 17 second-rank commercial banks, and simultaneously offers beneficial experience for commercial bank efficiency improvement.

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