Comparison of Methods for Different Time-frequency Analysis of Vibration Signal

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Abstract—**Vibration problems in rotors can be extremely frustrating and may lead to greatly reduced reliability. By utilizing the proper data collection and analysis techniques, the faults because of vibration can be discovered and predicted. The signal analysis is important in extracting fault characteristics in fault diagnosis of machinery. The traditional signal analysis can not settle for non-stationary vibration signal whose statistic properties are variant. To deal with non-stationary signal, time-frequency analysis techniques are widely used. The experiment data of oil whip vibration fault signal were analyzed by different methods, such as short time Fourier transform (STFT), Wigner-Ville distribution (WVD), Wavelet transform (WT) and Hilbert-Huang Transform (HHT). The experiment data of rubbing vibration faults signal were also analyzed by the HHT. Compared with these methods, it is demonstrated that the time-frequency resolutions of STFT and WVD were inconsistent, which were easy to cross or make signal lower. WT had distinct time-frequency distribution, but it brought redundant component. HHT time-frequency analysis can detect components of low energy, and displayed true and distinct time-frequency distribution. Therefore, HHT is a very effective tool to diagnose the faults of rotating machinery.**

Index Terms—**signal analysis, time-frequency analysis, vibration, Hilbert-Huang transform (HHT), fault diagnosis**

I. Introduction

Vibration signal analysis has been widely used in the faults detection of rotation machinery. Many methods based on vibration signal analysis have been developed. These methods include power spectrum estimation, fast Fourier transform (FFT), envelope spectrum analysis, which have been proved to be effective in fault detection. However, these methods are based on the assumption of stationary and linearly of the vibration signal. Therefore, new techniques are needed to analyze vibration for faults detection in rotating machinery. Effective detection of non-stationary signals is of great importance [1-2], as they are precursors of potential machine failures. However, their temporary nature makes the assumption of signal stationary as required by Fourier transform invalid, thus reducing its effectiveness.

To deal with non-stationary and non-linearity signals, time-frequency analysis techniques such as the Short Time Fourier Transform (STFT), Wigner-Ville distribution (WVD) and Wavelet Transform (WT) are widely used. The STFT uses sliding windows in time to capture the frequency characteristics as functions of time. Therefore, spectrum is generated at discrete time instants. An inherent drawback with STFT is the limitation between time and frequency resolutions. A finer frequency resolution can only be achieved at the expense of time resolution and vice-versa. The Wigner-Ville distribution (WVD) is a basic time-frequency representation, which is part of the Cohen class of distribution. The WVD possesses a great number of good properties and is of popular interest for non-stationary signal analysis. The difficulty of WVD is the severe cross terms as indicated by the existence of negative power for some frequency ranges. The Wavelet Transform (WT) provides a time-frequency map of the signal being analyzed[3-5]. It can achieve high frequency resolution with sharper time resolutions. A very appearing feature of the wavelet analysis is that it provides a uniform resolution for all the scales. Limited by the size of the basic wavelet function, the downside of the uniform resolution is uniformly poor resolution. The Hilbert–Huang transform (HHT) is based on the instantaneous frequencies resulting from the intrinsic mode functions of the signal being analyzed [6]; thus, it is not constrained by the uncertainty limitations with respect to the time and frequency resolutions to which other time-frequency techniques are subject. In recent years, HHT has been applied to identification of damage time instant and location in civil and mechanical structures[7-12]. Using HHT, the physical mass, damping coefficient, and stiffness matrices of a multiple degree of freedom linear system could be identified[13].

This paper investigates the utility of time-frequency methods for vibration signal analysis. The theoretical backgrounds of each time-frequency methods were introduced, and the simulation was evaluated through experimental studies performed on a test shaft. The experiment data of oil whip vibration fault signal were analyzed by different methods, such as short time Fourier transform (STFT), Wigner-Ville distribution (WVD), Wavelet transform (WT) and Hilbert-Huang Transform (HHT). The experiment data of rubbing vibration faults signal were also analyzed by the HHT. Compared with these methods, it is demonstrated that HHT is a very effective tool to analyze non-stationary and non-linearity signals.

II. STFT OF VIBRATION SIGNAL

A. Short Time Fourier Transform (STFT)

Short Time Fourier Transform (STFT) has a short data window centered at time. The window is moved to a new position and the calculation repeated.

STFT is defined for a single sample function $x(t)$,

$$STFT_x(t,\omega) = \int x(\tau)g_{t,\omega}(\tau)d\tau = \int x(\tau)g(\tau-t)e^{-j\omega\tau}d\tau$$
$$=< x(\tau), g(\tau-t)e^{j\omega\tau} >$$

$$\tag{1}$$

$$g_{t,\omega}(\tau) = g(\tau-t)e^{j\omega\tau} \tag{2}$$

Where $\| g(\tau) \|=1$, $\| g_{t,\omega}(\tau) \|=1$, and the window function $g(\tau)$ is symmetrical.

If the width of the window is represented by time duration t, its frequency bandwidth is approximately ω. A feature of the STFT is that all spectral estimates have the same bandwidth.

B. 2.2 STFT of Oil Whip Signal

1. Oil Pump Assembly
2. Oil Bearing Assembly
3. Rotor Kit Shaft with Oil Bearing Journal
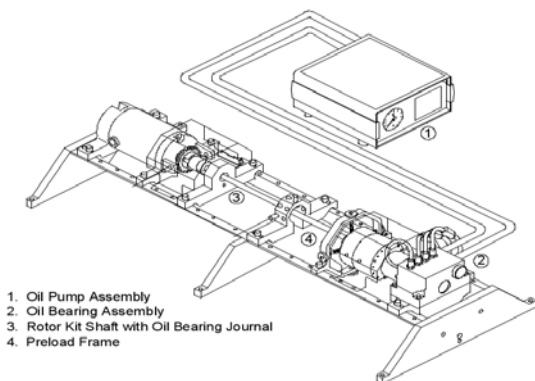4. Preload Frame

Figure 1.   Experimental test rotor system

To experimentally evaluate the effectiveness of time-frequency methods for non-stationary vibration signal, systematic tests were conducted on a rotor test system (see Figure 1). The test system consisted of an oil pump assembly, an oil bearing assembly, a rotor kit shaft with oil bearing journal and a preload frame. Tighten the set screws to firmly attach the rotor masses to the rotor. The rotor will whirl at a lower speed the closer the rotor

masses are to the fluid film bearing. The rotor kit is to go unstable place as the speed increases. The rotor will develop oil whirl to oil whip.

When the rotor running speed is near the 2th rev, the rotor will develop oil whip. Because the frequency of oil whirl approaches the high eccentricity natural resonant frequency of the rotor. During oil whip the frequency of the whip is nearly constant as rotor rpm is increased. In Figure 2 the sampling data is shown when the rotor speed is 4800rpm. The three major frequency components at 80Hz, 31Hz and 15Hz were identified, which are corresponding to fundamental frequency, whip frequency and dimidiate frequency of whip. In Figure 3 the STFT provides a time-frequency distribute. The adjacent data windows have overlapped, so the time-frequency spectrum of STFT generated redundant information.
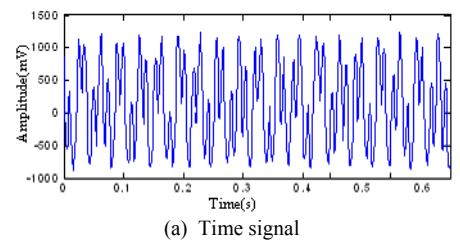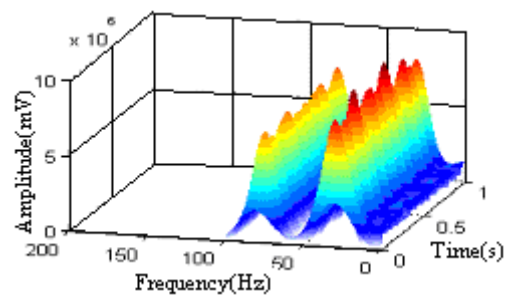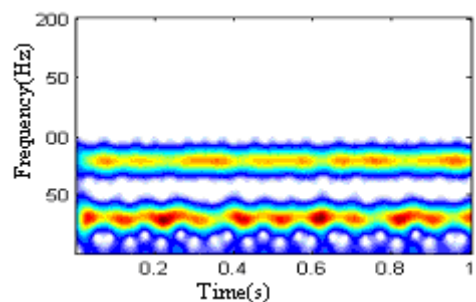

(a) Time signal
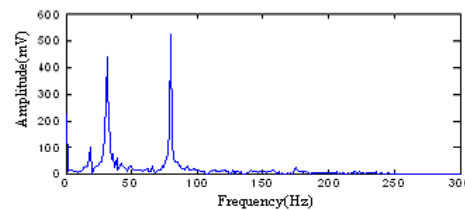
(b) Frequency spectrum

Figure 2.   FFT of oil whip vibration signal

(a) 3-D time-frequency distribution

(b) High light spectrum

Figure 3.   STFT of oil whip vibration signal

III. WVD OF VIBRATION SIGNAL

A. WVD and SPWVD

The function (3) is called the Wigner-Ville distribution (WVD). Wigner (1932) used it first in quantum mechanics and J.Ville (1948) was the first to propose using (3) for harmonic analysis.

$$WVD_x(t,f) = \int_{-\infty}^{\infty} x\left(t + \frac{\tau}{2}\right)\overline{x}\left(t - \frac{\tau}{2}\right)e^{-j2\pi\tau}d\tau \qquad (3)$$

The smooth pseudo Wigner-Ville distribution (SPWVD) selects appropriate window function. By sliding Wigner-Ville kernel function, the cross elements are restrained. The SPWVD is defined as:

$$SPWVD_x(t,f) =$$
$$\int_{-\infty}^{\infty} x\left(t - u + \frac{\tau}{2}\right)\overline{x}\left(t - u - \frac{\tau}{2}\right) \times g(\tau)w(\tau)e^{-j2\pi\tau}d\tau \qquad (4)$$

B. SPWVD of Oil Whip Signal

Figure 4 is time-frequency diagram of SPWVD. It has high time-frequency resolution and immunity of cross term interference. But high resolution cannot be obtained simultaneously in time and in frequency. The 2X frequency of vibration was weakened.
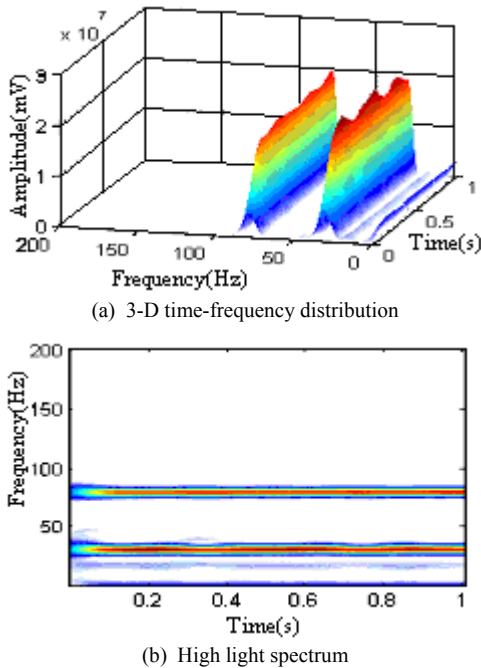

(a) 3-D time-frequency distribution

(b) High light spectrum

Figure 4.   SPWVD of oil whip vibration signal

IV. WT OF VIBRATION SIGNAL

A. Wavelet Transform (WT)

Wavelet analysis involves a fundamentally different approach. Instead of seeking to break down a signal into its harmonics, which are global functions that go on for ever, the signal is broken down into a series of local basis functions called wavelets. At the finest scale, wavelets may be very short indeed; at a coarse scale, they may be

very long. The signal $x(t)$, is plotted along the horizontal axis and the wavelet function $\psi(t)$ is plotted vertically.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi(\frac{t-b}{a}) \qquad (5)$$

Where a,b are constants, and $a > 0$. If $x(t) \in L^2(R)$, Wavelet Transform is defined as:

$$\begin{aligned} WT_x(a,b) &= \frac{1}{\sqrt{a}}\int x(t)\psi^*(\frac{t-b}{a})dt \\ &= \int x(t)\psi_{a,b}^*(t)dt = \langle x(t), \psi_{a,b}(t)\rangle \end{aligned} \qquad (6)$$

B. WT of Oil Whip Signal

The structure of a non-stationary signal can be analyzed with local features represented by closely-packed wavelets of short length. Vibration characteristics can be identified readily from a wavelet map in which the mean-square value of the vibration record is shown distributed over wavelet scale and position. Coefficient of continuous wavelet transform was plotted on time-frequency plane of oil whip signal in Figure 5. 3-D time-frequency distribution was shown in Figure 6. The three major frequency components at 80Hz, 31Hz and 15Hz were identified, which are corresponding to fundamental frequency, whip frequency and dimidiate frequency of whip. WT had distinct time-frequency distribution, but it brought redundant component decomposed in Figure 5.
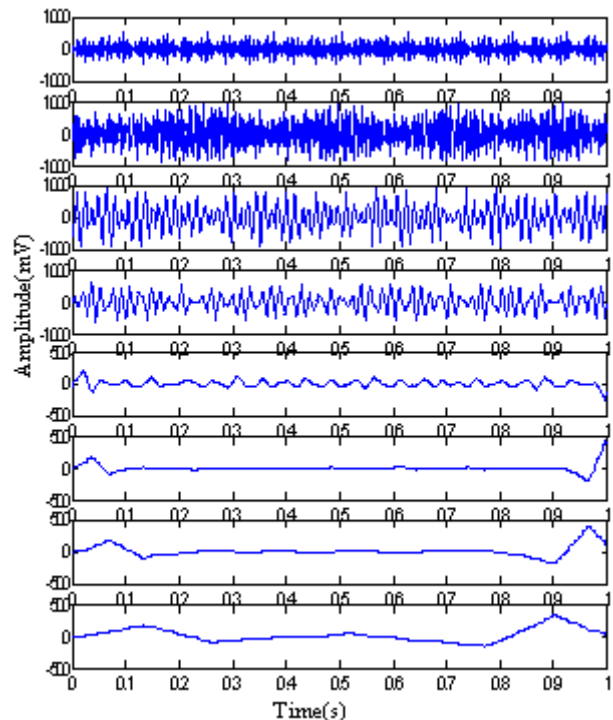

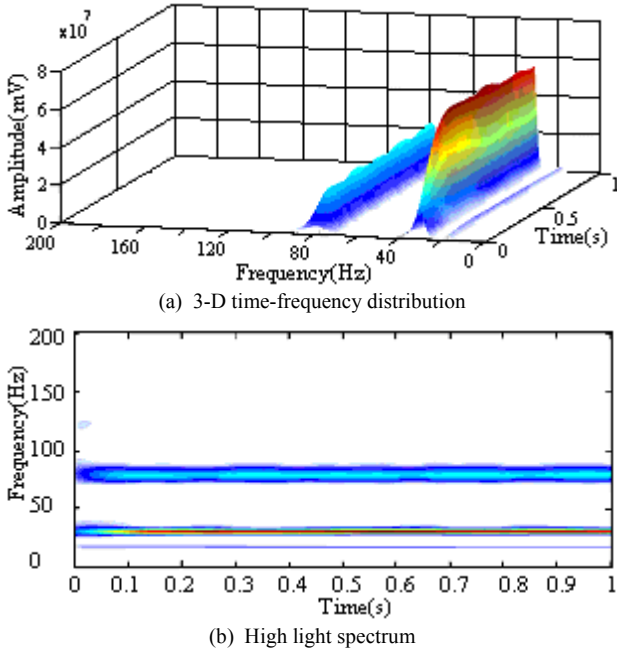Figure 5.   The decomposed components of WT

(a) 3-D time-frequency distribution

(b) High light spectrum

Figure 6.   WT of oil whip signal

V. HHT OF VIBRATION SIGNAL

A.   Hilbert-Huang Transform (HHT)

The HHT represents the signal being analyzed in the time-frequency domain by combining the empirical mode decomposition (EMD) with the Hilbert transform [6]. In contrast to the Fourier spectral analysis by which a series of sine and cosine functions of constant amplitudes are used to represent each constituent frequency components in the signal, the HHT technique is based on the instantaneous frequency calculation that results from the Hilbert transform of the signal. Generally, the Hilbert transform for any signal is defined as

$$H[x(t)] = y(t) = \frac{1}{\pi} \int \frac{x(\tau)}{t - \tau} d\tau \qquad (7)$$

Where $H(\bullet)$ denotes the Hilbert transform operation. Theoretically, any analytic signal z(t) can be expressed by the sum of its real part x(t) and imaginary part y(t), with the latter being the Hilbert transform of the real part.

$$z(t) = x(t) + jy(t) \qquad (8)$$

Equation (8) can be rewritten in a polar coordinate system as

$$z(t) = a(t)e^{j\theta(t)} \qquad (9)$$

From the instantaneous phase $\theta(t)$, the instantaneous frequency $\omega(t)$ of the signal can be derived as

$$\omega(t) = \frac{d(\theta(t))}{dt} = \frac{\dot{y}(t)x(t) - y(t)\dot{x}(t)}{x^2(t) + y^2(t)} \qquad (10)$$

Accordingly, the real part x(t) of the signal can be written in terms of the amplitude and instantaneous frequency as a time-dependent function

$$x(t) = \Re(z(t)) = \Re(a(t)e^{j \int \omega(t)dt}) \qquad (11)$$

Where the symbol $\Re(\bullet)$ denotes the real part of the signal z(t).

To effectively construct frequency spectrum of a vibration signal that contains multiple-frequency components, the signal needs to be first decomposed into mono-component functions, by means of EMD [6]. Decomposition of such a signal is based on the following observations.

1) The signal has at least two extrema, i.e., one maximum and one minimum.

2) The characteristic time scale is clearly defined by the time lapse between successive alternations of local maxima and minima of the signal.

3) If the signal has no extrema but contains inflection points, then it can be differentiated one or more times to reveal the extrema.

The EMD technique decomposes the signal into a number of Intrinsic Mode Functions (IMFs), each of which is a mono-component function. Then, the Hilbert transform is applied to calculate the instantaneous frequencies of the original signal.

After identifying all the local maxima and minima of the signal, the upper and lower envelopes are generated through curve fitting. Therefore, the cubic spline function was employed in the presented study. The mean values of the upper and lower envelopes of the signal $m_{11}(t)$ are calculated as

$$m_{11}(t) = (x_{up}(t) + x_{low}(t))/2 \qquad (12)$$

Where $x_{up}(t)$ and $x_{low}(t)$ are the upper and lower envelopes of the signal, respectively. Accordingly, the difference between the signal $x(t)$ and the envelopes of the signal $m_{11}(t)$, which is denoted as $h_{11}(t)$, is given by

$$h_{11}(t) = x(t) - m_{11}(t) \qquad (13)$$

Due to the approximation nature of the curve fitting method, $h_{11}(t)$ has to be further processed (by treating $h_{11}(t)$ as the signal itself and repeating the process continually) until it satisfies the following two conditions.

1) The number of extrema and the number of zero-crossings are either equal to each other or differ by at most one.

2) At any point, the mean value between the envelope defined by local maxima and the envelope defined by the local minima is zero.

Through the iteration process (for a total of times), the difference between the signal and the mean envelope values, is denoted as

$$h_{1k}(t) = h_{1(k-1)}(t) - m_{1k}(t) \qquad (14)$$

Where m_{1k} is the mean envelope value after the kth iteration, and $h_{1(k-1)}(t)$ is the difference between the signal and the mean envelope values at the (k-1)th iteration. The function $h_{1k}(t)$ is then defined as the first IMF component and expressed as

$$c_1(t) = h_{1k}(t) \qquad (15)$$

After separating $c_1(t)$ from the original signal $x(t)$, the residue is obtained as

$$r_1(t) = x(t) - c_1(t) \qquad (16)$$

Subsequently, the residue $r_1(t)$ can be treated as the new signal, and the above-illustrated iteration process is repeated to extract the rest of the IMFs inherent to the signal $x(t)$ as

$$r_2(t) = r_1(t) - c_2(t), \cdots, r_n(t) = r_{n-1}(t) - c_n(t) \qquad (17)$$

The signal decomposition process is terminated when $r_n(t)$ becomes a monotonic function, from which no further IMFs can be extracted. By substituting (17) into (16), the signal $x(t)$ is decomposed into a number of intrinsic mode functions that are the constituent components of the signal. As a result, the signal $x(t)$ can be expressed as

$$x(t) = \sum_{i=1}^{n} c_i(t) + r_n(t) \qquad (18)$$

Where $c_i(t)$ represents the ith intrinsic mode function, and $r_n(t)$ is the residue of the signal decomposition. Equation (18) provides a complete description of the empirical mode decomposition process [6], which can be evaluated by checking the amplitude error between the reconstructed and the original signal.

Based on (7)–(10) and (18), (11) can be modified as

$$x(t) = \Re\left(\sum_{i=1}^{n} a_i(t)e^{j\int \omega_i(t)dt}\right) \qquad (19)$$

In which $a_i(t) = \sqrt{c_i(t)^2 + H[c_i(t)]^2}$ and $\omega_i(t) = d(\tan^{-1}(H[c_i(t)]/c_i(t))/dt$. The term $r_n(t)$ in (12) is not included in (13) as it is a monotonic function and does not contribute to the frequency content of the signal.

Comparing (20) with the Fourier-based representation of a signal $x(t)$ given by

$$x(t) = \Re\left(\sum_{i=1}^{\infty} A_i e^{j\Omega_i t}\right) \qquad (20)$$

Where both A_i and Ω_i are constants, it becomes evident that the EMD process enables flexible representation of a dynamic signal by revealing its time-dependent amplitude and the characteristic frequency components at various time instances. The signal is thus represented by a time-frequency distribution. The underlying HHT of the signal is mathematically defined as

$$HHT(t, \omega) = \sum_{i=1}^{n} HHT_i(t, \omega) \equiv \sum_{i=1}^{n} a_i(t, \omega_i) \qquad (21)$$

$HHT_i(t, \omega)$ represents the time-frequency distribution obtained from the ith IMF of the signal. The symbol denotes "by definition," and combines the amplitude and instantaneous frequency of the signal together.

B.   HHT of oil whip signal

The first 8 mode of HHT was displayed In Figure 7. Compared with decomposed based on WT in Figure 5, the mode of HHT had little residual. Time-frequency distribution was shown in Figure 8. The three major frequency components at 80Hz, 31Hz and 15Hz were identified, which are more distinct. In comparison, the

Hilbert–Huang transform (HHT) is based on the instantaneous frequencies resulting from the intrinsic mode functions of the signal being analyzed; thus, it is not constrained by the uncertainty limitations with respect to the time and frequency resolutions to which other time-frequency techniques are subject. The test results that HHT provides a more effective time-frequency distribute.
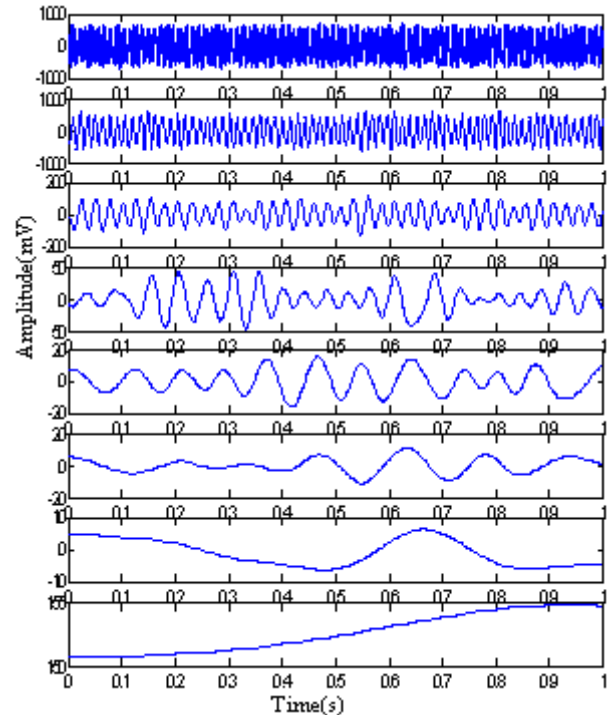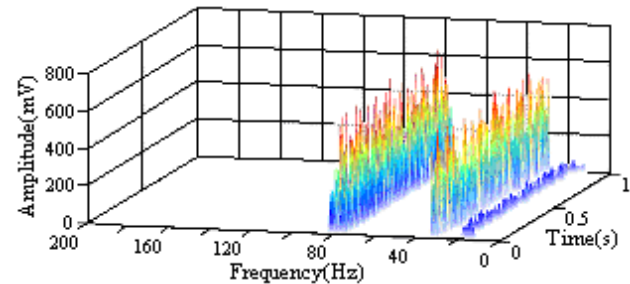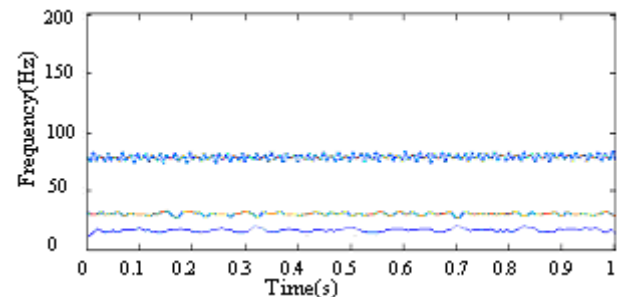

Figure 7.   The mode of HHT for oil whip signal

(a) 3-D time-frequency distribution

(b) High light spectrum

Figure 8.   HHT of oil whip vibration signal

C. HHT of early local rubbing signal

To experimentally evaluate the effectiveness of HHT for non-stationary vibration signal analysis, systematic tests were conducted on a rotor test system. In the following the HHT is applied in early local rubbing, circumferential rubbing. By comparing the HHT with original method, the obvious advantage and validity can be clearly expressed.
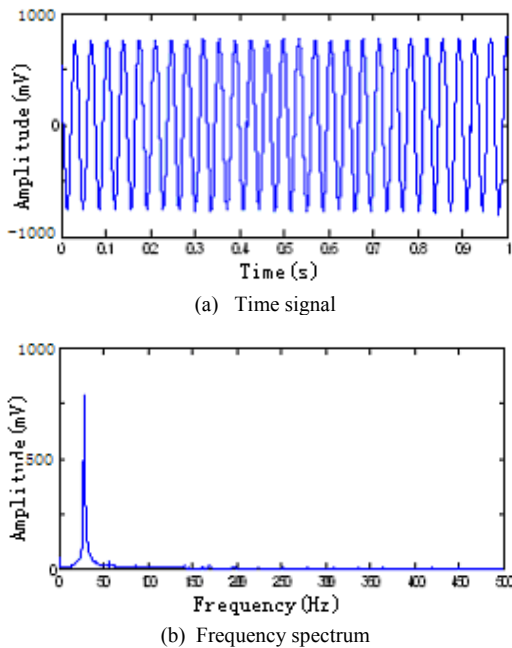

(a) Time signal

(b) Frequency spectrum

Figure 9. Time and spectral diagram of early local rubbing signals
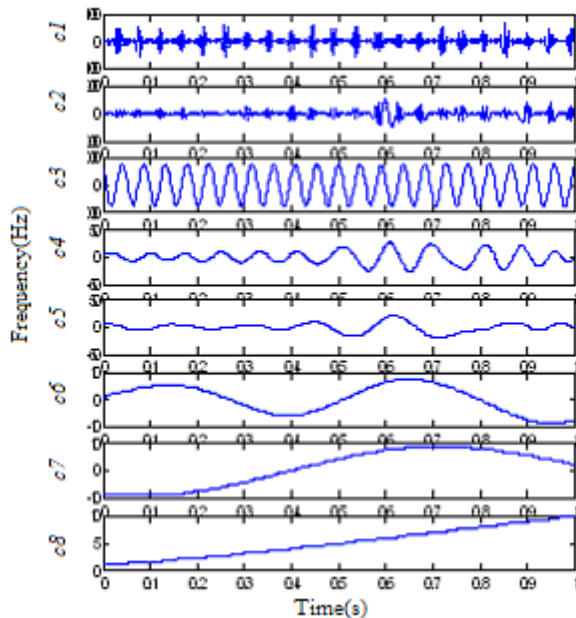

Figure 10. Time of early local rubbing fault signals by EMD

The vibration signal of early local rubbing fault is measured and shown in Figure 9(a). Its spectrum in Figure 9(b) reveals the fundamental frequency at 31Hz. The fundamental frequency was obvious, but the twice rotational frequency (2X) is negligible and the

characteristic of early local rubbing can not be displayed in FFT spectral diagram. Figure 10 displays the empirical mode decomposition in eight IMFs. Using EMD method, the original vibration signals of rubbing faults can be decomposed into intrinsic modes. The decomposition identifies eight modes: c1-c8, which represents the different frequency components exited by the inner race defects, c8 is the residue, respectively. Mode c1 contains the highest signal frequencies, mode c2 the next higher frequency band and so on. Mode c3 contains the fundamental frequency, and modes c4-c7 contains differential frequencies, whose amplitudes are very small. But they can be characteristics of early local rubbing.

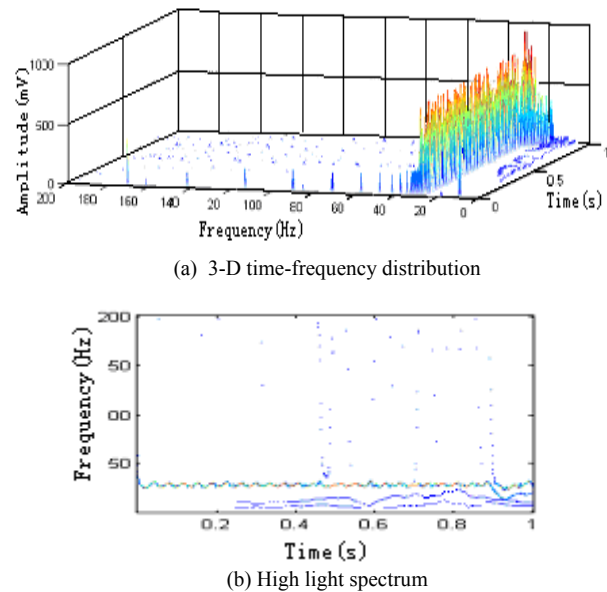(a) 3-D time-frequency distribution

(b) High light spectrum

Figure 11. HT of early local rubbing fault signals

In Figure 11 the HHT result of the early local rubbing fault vibration signal is illustrated. The energy distributing of vibrations at normal and compression-decreasing conditions is given in Figure 3, and their frequencies are computed using (15), respectively. As shower in two figures, the energy of core vibration mainly focuses on the fundamental frequency, a little exists round about 1/2X, 1/3X and high frequencies 2X, 3X and so on. But differential frequencies can be considered as the characteristics of early local rubbing. Therefore, HHT is a very effective tool to diagnose the early faults of rotor system. And it provides new means for state detection and fault diagnosis of rotating machine.

V. CONCLUSION

The experiment data of oil whip vibration fault signal were analyzed by different methods, such as short time Fourier transform (STFT), Wigner-Ville distribution (WVD), Wavelet transform (WT) and Hilbert-Huang Transform (HHT). Compared with these methods, it is demonstrated that the time-frequency resolutions of STFT and WVD were inconsistent, which were easy to cross or make signal lower. WT had distinct time-frequency distribution, but it brought redundant component. HHT time-frequency analysis can detect

components of low energy, and displayed true and distinct time-frequency distribution. Therefore, as a time-frequency signal decomposition technique, the Hilbert-Huang transform provides an effective tool for analyzing vibration signal. Research is being continued to systematically investigate the suitability and constraints of the HHT for non-stationary signal analysis, using vibration signals from different faults. Furthermore, integration of the EMD process with enveloping spectrum analysis is also being investigated, with the goal of realizing an adaptive signal demodulation approach to account for varying machine conditions in real-world applications.

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