Research on Single and Mixed Fleet Strategy for Open Vehicle Routing Problem

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***Abstract*—Open vehicle routing problem with single and mixed fleet strategy is logistics optimization indispensable part. Hybrid genetic algorithm is used to optimize the solution. Firstly, use sequence of real numbers coding so as to simplify the problem; construct the targeted initial solution to improve the feasibility; adopt some arithmetic crossover operator to enhance whole search ability of the chromosome. Secondly, Boltzmann simulated annealing mechanism for control genetic algorithm crossover and mutation operations improve the convergence speed and search efficiency. Finally, the simulation results demonstrate the effectiveness and good quality. At the same time, it proves that mixed fleet strategy can shorten distribution distance, reduce distribution vehicle so as to reduce distribution cost and improve economic benefit.**

***Index Terms*—open vehicle routing problem, single and mixed fleet strategy, arithmetic crossover operator, boltzmann mechanism, hybrid genetic algorithm**

I. INTRODUCTION

Open vehicle routing problem is an expansion problem of the classic vehicle routing problem. The most significant difference between OVRP and VRP is that in the OVRP, vehicles do not return to the original depot after servicing the last customer on the route, or if they are required, they return by traveling the same route back. Open vehicle routing problem is a key step of logistics optimization and the indispensable part of the e-commerce activities.

At present, researches are more about the closed vehicle routing problem, as we known as the VRP, but not much research on the open vehicle routing problems. Main research methods of OVRP include accuracy algorithm, heuristic algorithm and intelligent optimization algorithm. Accuracy algorithm takes a strict arithmetical approach. Its disadvantage is, however, the inevitability of exponential explosion [1]. Letchford applies the Branch-and-Cut algorithm to solve problems as such [2]. Li purposed the heuristic algorithm with recording effect based on the scan method and insert method attempts to find solution for OVRP with practical trip length constraint [3].

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When involving large-scale practical OVRP, heuristic algorithm is usually used to find a satisfactory solution compared to other algorithms. Repoussis introduced improvised greedy algorithm to find solution for OVRP [4]. When it comes to large-scale, complex problems, intelligently optimized algorithm is much commonly applied. Brandao attempted to draft solution using the K-degree algorithm, and then solved OVRP through using tabu search algorithm [5]. Fu gained initial draft by applying random method and the farthest first heuristic algorithm, and proposed solution for capacitated OVRP [6]. Zeynep studied the solution for practical OVRP with maximum traveling time and time windows constraints [7]. Zhong Shiquan proposed the concept and principle of critical paths, and designed tabu search algorithm for OVRP with constraints of ability and distance [8]. Xiao Tianguo designed genetic algorithm for OVRP with soft time window through applying the crossover operators of self-adaptation mechanism [9]. Deng Meng introduced arithmetical model of OVRP and subsequently designed genetic algorithm that is based on natural coding [10]. Tarantilis sets a list of thresholds and selects the biggest element of the list as the threshold by the arithmetic iteration process, then continuously updating the threshold list [11].

However, compared to OVRP, there are fewer researches on single and mixed fleet strategy problems, and modeling and solving processes are far more complicated and difficult. Therefore, this article established mathematic model of OVRP and integrated designed the hybrid genetic algorithm to solve it.

II. MODEL

$$Min = \sum_{i \in S} \sum_{j \in S} \sum_{l,k \in V} X_{ijk}^{l} d_{ij} \tag{1}$$

Restraint condition,

$$\sum_{l,k \in V} \sum_{i \in S} X_{ijk}^{l} = 1, \quad j \in H \tag{2}$$

$$\sum_{i \in H} \sum_{j \in S} q_i X_{ijk}^{l} \le w_k^l, \qquad l,k \in V \tag{3}$$

$$\sum_{i \in S} X_{ijk}^{l} = Y_{ik}^{l}, \quad j \in S, \quad l,k \in V \tag{4}$$

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$$\sum_{j\in S} X_{ijk}^{l} = Y_{ik}^{l}, \quad i \in S, \quad l,k \in V \tag{5}$$

$$\sum_{l,k\in V}\sum_{i\in S} X_{ijk}^{l} d_{ij} \le D_k^l, \quad j \in H \tag{6}$$

$$X_{ijk}^{l} = 0,1 \quad i,j \in S, \quad l,k \in V \tag{7}$$

$$Y_{ik}^{l} = 0,1 \qquad i \in H, \quad l,k \in V \tag{8}$$

In the formula, $G\{g_r | r = 1,2,...,R\}$ is the distribution centre muster of a series of R. $H\{h_i | i = R+1,...R+N\}$ is the customer muster of a series of N. $S\{G\ \}\cup\{H\ \}$ is the summation of all distribution centre and customers. $V\{v_{lk} | l = 1,2,...L \quad k = 1,2,...K\}$ is l model of the muster of transportation vehicle k. q_i is the demand amount of customer i $i(i \in H)$. W_k^l is l model of the load capacity of transportation vehicle k. d_{ij} is the beeline distance from customer i to customer j. D_k^l is the maximum running distance of l model of transportation vehicle k.

In formula (1), objective function is the least value of solving the cost from distribution centre to customer. Restricted condition (3) is the restricted condition of transportation tool capacity, which satisfied that each vehicle don't exceed its load capacity running in each route. Restricted condition (4) can guarantee that vehicle reach to some customer point once time at best. Restricted condition (5) can assure that some vehicle only start from some collecting goods point. Restricted condition (6) is to satisfy that running distance in each route doesn't exceed the maximum running distance.

III. Operator Confirmation in Hybrid Genetic Algorithm

A. Sequence of Real Numbers Code

For multi-vehicle, loading is setting different cargos into different vehicle. Therefore, consider to select vehicles when coding. Adopt sequence of real numbers to code. Grade of loading cargo is $n(1,2,...,n)$. Grade of loading vehicle is $k(1,2,...,k)$. Concrete coding form is $(x_1, x_2,..., x_i,..., x_n)$. The value of each gene is $x_i = k \in (1,2,...,k)$. $x_i = k$ can be shown that i cargo is loaded in k vehicle.

B. Initial Solution Forming

Supposed hk is the total number of all customer points for vehicle k. Muster $R_k = \{y_{ik} | 0 \le i \le h_k\}$ is the customer point of vehicle k. Y_{ik} is the transportation tool of vehicle at i point. Y_{0k} is the distribution centre of starting point for vehicle k. The concrete steps are as followings.

Step1: Supposed the initial residual capacity of transportation vehicle as $w_k^1 = w_k$, $k = 0$, $h_k = 0$, $R_k = \Phi$.

Step2: The demand amount of i gene in a chromosome is q_i and $k = 1$.

Step3: If $q_i \le w_k^1$, $w_k^1 = Min\{(w_k^1 - q_i), w_k\}$. Otherwise, it shifts to step 6.

Step4: If $w_k^1 - q_i \le w_k$ and $D_{i-1} + D_i \le D_k$, $R_k = R_k \cup \{i\}$ and $h_k = h_k + 1$. Otherwise, it shifts to step 6.

Step5: If $k > K$, $k = K$. Otherwise, $k = k$.

Step6: $k = k + 1$, shift to step 3.

Step7: $i = i + 1$, shift to step 2.

Step8: Repeat from step 2 to step 7. K memorizes the total amount of all vehicles. R_k memorizes a group of feasible path.

C. Fitness function

Adopt best preserving selection algorithm, require fitness function number to non- negative, and transfer objective function into fitness function through following transformation.

$$f_m = \frac{kkz^1}{z_m} \tag{9}$$

Here, f_m is the fitness of m chromosomes, k is constant, z^1 is the corresponding distribution expense of best chromosome, and z_m is the corresponding distribution expense of actual chromosome.

D. Selecting operator

The selected probability is the following formula.

$$P_i = f_i / \sum_{i=1}^{n} f_i \tag{10}$$

According to model theorem of genetic algorithm, sample amount of model H in t generation is the following formula.

$$m(H,t) \equiv m(H,0).(1+c)^t \tag{11}$$

Here, $m(H,0)$ is the sample amount of model in initial population. When some individual number exceeds marginal value ε, the individual number should be reduced so as that it can be controlled in assigned marginal value extent. And new individual random can make up the population scale. The description is following.

Step1: Calculate the fitness value f_i of each individual for community. Suppose that the highest fitness value of actual group is f_{best}.

Step2: Calculate the total fitness value $\sum f_i$.

Step 3: t-1 generation group forming after t-1 genetic operation can have selecting operation to create group $p(t)$ according to proportion fitness.

Step4: Calculate every individual number in group $p(t)$.

Step5: Have the following operation to group $p(t)$ to create group $p^1(t)$. If some individual number exceeds the marginal value ε of t individual, delete this individual so as to control individual number in the extent ε .otherwise, copy all individuals.

Step6: If the number of group $p^1(t)$ is less than group scale N, then randomly operate N- $p^1(t)$ new individuals. And new individual can take part in following cross and mutation operation.

Step7: Calculate the corresponding fitness value $p_i = \frac{f_i}{\sum f_i}$ of each individual in community.

Step8: Randomly create α in [0,1]. If $p_1 + ... + p_{i-1} < \alpha < p_1 + ... + p_{i-1} + p_i$, select individual i enter into the next generation community.

E. Crossover Operator

The study adopts partial Arithmetical Crossover. On one hand, partial arithmetical crossover can maintain the diversity of species evolution to guarantee convergence to the global optimum. On the other hand, it can retain the good parent genome fragments; limit appears feasible solution so as to improve the convergence rate of population.

Suppose individuals of two parents to $x^0 = (x_1, x_2, ..., x_n)$ an $y^0 = (y_1, y_2, ..., y_n)$.

After crossover, it can get the sub-individual $x^{'} = (x_1^{'}, x_2^{'}, ..., x_n^{'})$ and $y^{'} = (y_1^{'}, y_2^{'}, ..., y_n^{'})$. Select gene in parent individual as crossover part. In (0, 1), randomly generate $n-k$ random number $a_{k+1}, a_{k+2}, ..., a_n$.

After crossover, it can get individual and definite it as followings.

$$x_i^{'} = \begin{cases} x_i & i = 1,2,...,k \\ |a_i x_i + (1-a_i) y_i| & i = k+1, k+2, ..., n \end{cases} \tag{12}$$

$$y_i^{'} = \begin{cases} y_i & i = 1,2,...,k \\ |a_i y_i + (1-a_i) x_i| & i = k+1, k+2, ..., n \end{cases} \tag{13}$$

F. Mutation Operator

Mutation strategy adopts improve the non-uniform mutation. Suppose chromosome of parent individual is $x^0 = (x_1, x_2, ..., x_n)$. $t_{\max}$ is maximum evolution generation. t is current evolution generation. The factor i can mute as certain mutation probability p_m.

Chromosome of new individual is $x^1 = (x_1, x_2, ..., x_i^*, ..., x_n)$.

If $x_i^* \in [x_i^{\min}, x_i^{\max}]$, x_i^* can be decided by the following formula.

$$x_i^* = x_i + (x_i^{\max} - x_i^{\min}) \cdot (1 - r^{(1-\frac{t}{t_{\max}})^2}) \tag{14}$$

In the formula, $x_i^{\max}$ is the upper limit, $x_i^{\min}$ is the lower limit, r is random number and $r \in [0,1]$. In this study, $r = 0.5$.

If $x_i^* \notin [x_i^{\min}, x_i^{\max}]$, x_i^* can be selected by upper or lower limit.

In the beginning of evolutionary, the algorithm is adopted bigger mutation scale to maintain the population diversity. In the ending of evolutionary, when t is close to T, mutation scale is gradually narrow until to local searching.

The experiment is shown that the efficiency of partial crossover is higher than other operators.

G. Mountain climbing operation

Add mountain climbing algorithm into genetic algorithm. Improve the solution of each generation structure so as to reduce the expense of solving route and quicken algorithm constringency speed. Mountain climbing algorithm adopts 2-opt method to take point exchanging operation.

Supposed routing line before exchanging is $s=\{...,x_i, x_{i+1}, ..x_j, x_{j+1}, ..\}$, it can get the routing line $s^{'} = \{..., x_j, x_{i+1}, ...x_i, x_{j+1}, ..\}$ after exchanging location of two points. If the distance is equal to $\{d(x_{i+1}, x_j) + d(x_i, x_{j+1})\} < \{d(x_{i+1}, x_i) + d(x_j, x_{j+1})\}$, exchanging is successful and keeps exchanging result. Otherwise, exchanging attempt is failure, cancel exchanging and furbish former routing line.

To the optimized individual of each generation group through genetic operation, it can realize mountain climbing operation through searching in neighbors. The study adopts gene exchanging operator to realize climbing operation. The concrete steps are as followings.

Step1: Initial recycling time variable t=1, when the most optimal solution at present $s^* = s$ and its length is $l(s^*)$.

Step2: Randomly selecting two top points x_i, x_j in the most optimal route and $i < j$. x_i is not close to x_j.

Step3: Calculate saving distance,

$$\Delta c = \{d(x_{i+1}, x_j) + d(x_i, x_{j+1})\} < \{d(x_{i+1}, x_i) + d(x_j, x_{j+1})\} \tag{15}$$

If $\Delta c > 0$, it isn't exchanged. If $t = t + 1$, it shifts into step4.

Otherwise, execute exchanging. And the corresponding solution is $s^{'}$. And the optimal solution is $s^* = s^{'}$. If $t = 1$, it shifts into step 2.

Step4: If $l(s^*)$ isn't reduced in the last x a circulation, this algorithm is over. Otherwise, it shifts into step 2.

Step5: Repeat step 1 to step 4 till reaching certain exchanging times.

H. Simulated annealing operation

In order to avoid falling into the region optimization process, utilize Boltzmann mechanism of simulated annealing algorithm, control the crossover and mutation operation of genetic algorithm. This hybrid algorithm can make convergence of optimal solutions jump from partial optimal so as to improve the quality and searching efficiency, make up a single optimization and realize global convergence.

Suppose that the fitness value of individual in some generation is f_1. After selection, crossover and mutation of genetic algorithm, fitness value of new individual is f_2. Suppose $\Delta f = f_2 - f_1$, the current temperature is T.

According to the mutation value of individual adaptability Δf and probability value, control individuals. The concrete steps are as followings.

Step1: If $\Delta f < 0$, fitness value f_2 can be kept in the next generation. And fitness value f_1 can be removed from the overall.

Step2: If $\Delta f > 0$, calculate $\exp(-\Delta f / T)$.

Step3: When take random number in $\exp(-\Delta f / T) > [0,1]$, the individual of fitness value f_1 can be kept in the next generation, and fitness value f_2 can be removed from the overall.

Step4: When take random number in $\exp(-\Delta f / T) \le [0,1]$, the individual of fitness value f_2 can be kept in the next generation, and fitness value f_1 can be removed from the overall.

IV. Experimental Calculation and Analysis

The experiment data comes and expanded from reference [10]: assume 18 customer points, the depot assigns vehicle to satisfy customer requirements. Stochastic generate depot coordinate (50, 69) and the 18 customer coordinates, the customer requirement quantity is stochastic generate from (0, 1), shown as table1.

TABLE I.
KNOWN CONDITION OF EXAMPLES

Item	x-coordinate	y-coordinate	Distribution amount
1	70	37	0.10
2	42	86	0.50
3	30	85	0.45
4	18	59	0.14
5	19	49	0.07
6	68	89	0.29
7	30	82	0.12
8	54	64	0.30
9	15	81	0.18
10	57	44	0.95
11	96	2	0.40
12	76	90	0.26
13	14	76	0.58
14	31	84	0.97
15	70	87	0.32
16	26	75	0.39
17	99	16	0.23
18	56	61	0.35

The vehicles was classified into three types: A, B, C. Several A types with loading capacity 1, 3 of B types with types with loading capacity 1.5, 2 of C types with loading capacity 1.8. Adopt the straight line distance to measure the distance between logistics centre and those customers.

A. Solution of Single Fleet for OVRP

After many trails, adopt following parameters: group scale $NN = 50$, maximum iterations are $t_{\max} = 500$. Crossover operator is $p_c = 0.87$. Mutation operator is $p_m = 0.01$ .Initial temperature is T_0 =200. Temperature coefficient is $\delta = 0.88$. Randomly get the solutions for 10 times.

The average value of total distance is 327.99 km and the average using vehicles are eight. The calculation result of algorithm is relatively steady. The total distance of Sub-optimal solution is only better 0.31 percent than the best. From the calculating efficiency, there are three times to reach the best solution of ten times, which means that efficiency is much higher.

TABLE II.
CALCULATION RESULTS BY HYBRID GENETIC ALGORITHM

Calculation order	Solution by hybrid genetic algorithm	
	Total distance	Vehicle amount
1	322.6	8
2	328.0	8
3	328.9	8
4	321.6	8
5	322.6	8
6	328.0	8
7	328.9	8
8	321.6	8
9	356.1	8
10	321.6	8
Average value	327.99	8
Standard deviation	3.287	0

Here, the total distance of best solution is 321.6 km and concrete running path can be seen in table 3 and figure 1.

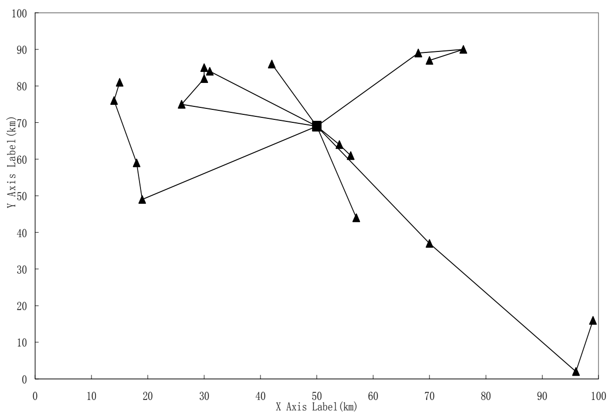

Figure 1. Optimal routes on solving of Single Fleet for OVRP

TABLE III.
OPTIMAL RESULTS BY HGA

No.	Running route	Loading / capacity / Vehicle type	Distance
1	0-1-11-17	0.73/1.0/A	95.6
2	0-5-4-13-9	0.97/1.0/A	69.5
3	0-8-18	0.65/1.0/A	10.0
4	0-16-7-3	0.96/1.0/A	35.8
5	0-6-12-15	0.87/1.0/A	41.7
6	0-14	0.97/1.0/A	24.2
7	0-2	0.50/1.0/A	18.8
8	0-10	0.95/1.0/A	26.0
Total vehicles		8	
Total distances		321.6 km	

B. Solution of Mixed Fleet for OVRP

Adopt the same hybrid genetic algorithm to solve the problem, and calculating results can be seen table 4.

The average value of total distance is 289.34 km and the average using vehicles are five. The calculation result of algorithm is relatively steady. The total distance of Sub-optimal solution is only better 2.87 percent than the best. From the calculating efficiency, there are three times to reach the best solution of ten times, which means that efficiency is much higher.

TABLE IV.
OPTIMAL RESULTS BY HGA

Calculation order	Solution by hybrid genetic algorithm	
	Total distance	Vehicle amount
1	294.4	5
2	282.4	5
3	290.5	5
4	294.6	5
5	282.4	5
6	290.5	5
7	290.8	5
8	294.6	5
9	282.4	5
10	290.8	5
Average value	289.34	5
Standard deviation	1.607	0

Here, the total distance of best solution is 282.4 km and concrete running path can be seen in table 5 and figure 2.

TABLE V.
OPTIMAL RESULTS BY HGA

No.	Running route	Loading / capacity / Vehicle type	Distance
1	0-7-3-14	1.54/1.8/C	28.3
2	0-10-1-11-17	1.68/1.8/C	98.7
3	0-2-6-12-15	1.37/1.5/B	59.8
4	0-5-4-13-9-16	1.36/1.5/B	82.0
5	0-18-8	0.65/1.0/A	13.6
Total vehicles		5	
Total distances		282.4km	

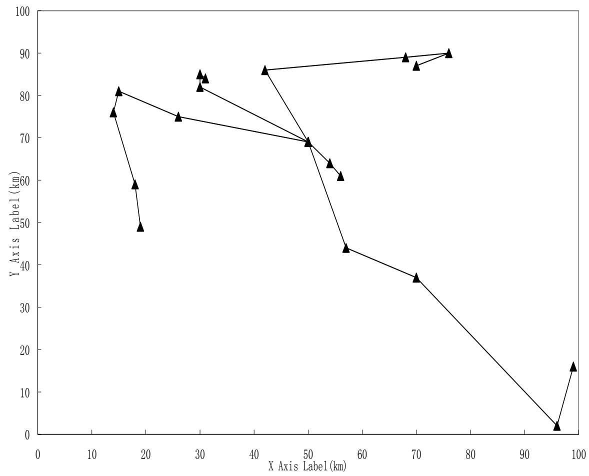

Figure 2. Optimal routes on solving of Mixed Fleet for OVRP

C. Comparison on Single/Mixed Fleet

It can be shown from optimal solution of two operating strategies that the distance adopting MF operation strategy is less 12.19 percent than SF, and the vehicle is reduced by 37.5 percent.

TABLE VI.
COMPARISON ON SINGLE AND MIXED FLEET STRATEGY

Operating Strategy	Average distance	Optimal distance	Optimal fleet
Single fleet	327.99	321.6	8
Mixed fleet	289.34	282.4	5
Saving degree on MF than SF (%)	11.78	12.19	37.5

V. CONCLUSIONS

In all, hybrid genetic algorithm proposed has much highly searching ability, much quicker convergence speed, stronger overcoming getting into partial optimal ability. Therefore, it is more practical significance and value so as to reduce operating cost and improve economic benefit.

Adopting mixed fleet operating strategy can reduce circuitous and crossover route in big extents so as to shorten distance and reduce cost.

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