Accurate Modeling Method for Generalized Tool Swept Volume in 5-axis NC Machining Simulation

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Abstract—**Presented in this article is a new accurate tool swept volume generation algorithm based on a generalized cutter for five-axis NC simulation and verification. Based on the surface envelope theory and the differential geometry, using analytical method, the strict deduction process of critical curve equation and swept envelope equation for generalized cutter are given. The motion feature of the cutter in five-axis machining is analyzed and the moving frame is established; Then, the detailed solving method of the cutter's velocity is presented. Further, the accurate swept volume expression method and generation algorithm for generalized cutter are achieved. At last, the correctness and efficiency of the algorithm is verified by a fillet-end cutter's experimental result. Although this algorithm is developed for cutter swept volume, it is also suitable for generating swept volume of any rotation rigid solid.**

Index Terms—**Generalized cutter; Swept volume; Envelope theory; Five-axis NC Machining Simulation; Critical Line**

I. INTRODUCTION

The computation of swept volume plays a more and more important role in modern advanced manufacturing area, especially in NC machining simulation. The modeling and visualization of the envelope generated by the motion of an NC cutter is one of the most challenging problems in the field of NC simulation and verification. As we know, the cut operation between the cutter and block is implemented by removing the intersection part of the block and tool swept volume from the block. Thus, the modeling method of the tool swept volume is critical to the NC simulation, especially in five-axis NC machining simulation. Due to two extra freedom of tool movement in 5-axis NC machining, the computation and visualization of the tool swept volume is more difficult than that in three-axis NC simulation.

During the last decade, several approaches for modeling a swept volume generated by a 3D object have been presented. According to the theory base, they can be classified into several categories: Method based on envelope theory[1-4,6,12], sweep differential Equation(SDE)/ sweep envelope differential equation (SEDE) method[5], approach based on the Jacobian rank deficiency method[8,15] et al.. Most methods for 3D sweeping are based on the works of Wang [1], in which the theory of the envelope was formulated and conditions were defined for reconstructing the envelope generated during the motion of an object. One of the methods developed by Chung et al.[2] for representing a generalized cutter in three-axis motion is based on modeling the swept surface in a single-valued form. However, It is limited to the tool type and motion, and only can be applied in three-axis NC simulation. Also, other existing methods either aimed to specific cutter type and motion[6,7], or they are the approximate swept computation methods in 5-axis NC simulation[12,13].

In this paper, In order to acquire a universal and widespread use algorithm, we proposed a new exact tool swept volume generation algorithm in 5-axis NC simulation. It is based on the theory of the envelope, designed for a generalized cutter, and suitable for any tool movement. This paper is arranged as follows: First, a generalized cutter is established in section 2. Then, the tool movement feature in five-axis NC machining is analyzed and the moving frame is established in section 3. Next, based on the above work, the tool swept volume generation algorithm is presented, and the cutter's velocity(including translational and rotational velocity) detailed calculation method is given. Section 5 shows the test example and experiment result. At last, the discussion and conclusion are given.

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II. GENERALIZED CUTTER MODEL

Based on APT(automatically programmed tools), this paper designed a generalized cutter which is expressed by eight major parameters . It is composed of four parts: Cylinder, Upper cone, Lower cone and Corner torus. The geometric definition and mathematical description of the generalized cutter are presented below.

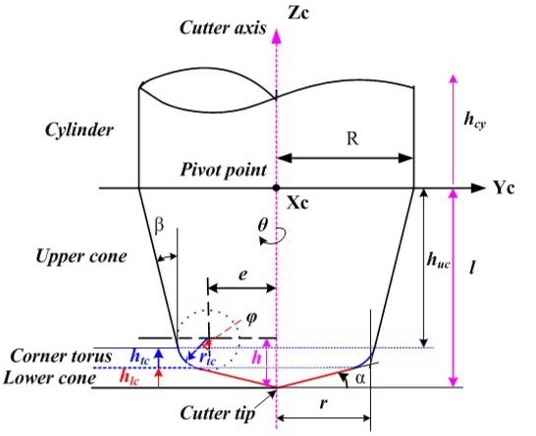

Fig. 1. Geometric definition of the generalized cutter ($X_cY_cZ_c$ is local coordinate system of the cutter, the origin is the pivot point.)

The cutter geometry is shown in Fig.1, the parameters are defined as follows:

(1) ***r*** **:** the cutter radius;
(2) ***r****tc* **:** the cutter corner radius;
(3) ***l*** **:** the distance between the cutter tip and the pivot point along the cutter axis;
(4) ***e*** **:** the radial distance from the cutter axis to the cutter corner center ;
(5)***α*** **:** the angle from a radial line through the cutter tip to the cutter bottom, $0 \le \alpha < 90^{\circ}$;
(6) ***β*** **:** the taper angle between the cutter side and the cutter axis, $-90^{\circ} \le \beta \le 90^{\circ}$;
(7) ***h*** **:** the distance from the cutter tip to the corner torus center as measured along the cutter axis;
(8) ***h****cy* **:** the length of the cylinder part, denote the length of the cutter arbor.

In specific applications, if these parameters' value is given, the special cutter type is determined. We can get the common cutter, Such as ball-end cutter, flat-end cutter, fillet-end cutter and taper-end cutter et al., as shown in Fig.2.

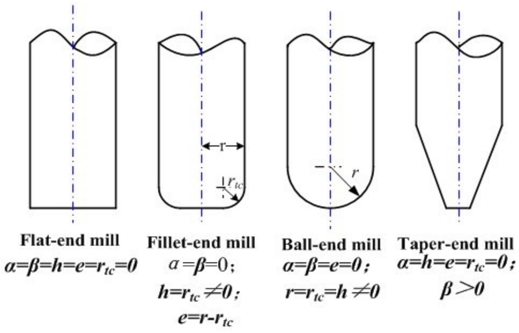

Fig.2. Common cutter type

The mathematical description of a generalized cutter Mc can be represented as follows:

$$M_c(s)=M_{cylinder}(s)\cup M_{uppercone}(s)\cup M_{torus}(s)\cup M_{lowercone}(s)$$
$$=\begin{bmatrix} R\cdot\cos\theta \\ R\cdot\sin\theta \\ k_{cylinder} \\ 1 \end{bmatrix}\cup \begin{bmatrix} (e+r_{tc}\cos\beta)\cos\theta+k_{uppercone}(l-h+r_{tc}\sin\beta)\tan\beta\cos\theta \\ (e+r_{tc}\cos\beta)\sin\theta+k_{uppercone}(l-h+r_{tc}\sin\beta)\tan\beta\sin\theta \\ -l+h-r_{tc}\sin\beta+k_{uppercone}(l-h+r_{tc}\sin\beta) \\ 1 \end{bmatrix}$$
$$\cup\begin{bmatrix} (e+r_{tc}\sin\varphi)\cos\theta \\ (e+r_{tc}\sin\varphi)\sin\theta \\ -l+h-r_{tc}\cos\varphi \\ 1 \end{bmatrix}\cup\begin{bmatrix} k_{lowercone}(e+r_{tc}\sin\alpha)\cos\theta \\ k_{lowercone}(e+r_{tc}\sin\alpha)\sin\theta \\ -l+k_{lowercone}(e+r_{tc}\sin\alpha)\tan\alpha \\ 1 \end{bmatrix} \quad (1)$$

Where, s is a parameter used to describe the cutter's geometry feature, $(\theta,k_{cylinder})$, $(k_{uppercone},\theta)$, (θ,φ) and $(k_{lowercone},\theta)$ describe the cylinder , upper cone, corner torus and lower cone respectively. $\theta\in[0,2\pi],\varphi\in[\alpha,(\frac{\pi}{2}-\beta)]$,and $h_{cy},h_{uc},h_{tc},h_{lc}$ denote the height of the four parts respectively, see in Fig.1.

III. THE TOOL MOTION IN FIVE-AXIS MACHINING

In five-axis NC machining, the tool movement include not only translation, but also rotation motion. Hence, the movement manner is heterogeneous and complicated. As a result, the difficulty of swept solving increases. To analyze the motion feature of tool in 5-axis Machining, we establish a local moving coordinates system, as shown in Fig.3.

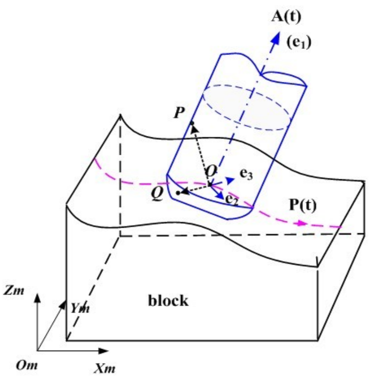

Fig.3. Moving frame and 5-axis tool motion

In the following representation, bold italics indicate a vector. The origin point is cutter location point O , take the fillet-end mill for example, Let the basis of the moving frame $e_1, e_{2,} e_3$ defined as follows:

$$e_1 = \boldsymbol{A}; \qquad e_2 = \dot{\boldsymbol{A}} \big/ |\dot{\boldsymbol{A}}|, if \ |\dot{\boldsymbol{A}}| \neq 0; \qquad e_3 = e_1 \times e_2 \ . \tag{2}$$

Where, $|\dot{\boldsymbol{A}}| \neq 0$ denote that the cutter has rotational motion(movement in 5-aixs machining); if $|\dot{\boldsymbol{A}}| = 0$, without rotational motion, Then, the moving frame is defined as follows:

$$\begin{aligned} & e_1 = \boldsymbol{A}; \\ & e_3 = \boldsymbol{A} \times \dot{\boldsymbol{P}} / |\boldsymbol{A} \times \dot{\boldsymbol{P}}|, if \ |\dot{\boldsymbol{A}}| = 0, and, \boldsymbol{A} \times \dot{\boldsymbol{P}} \neq 0; \\ & e_2 = e_3 \times e_1 \ . \end{aligned} \tag{3}$$

Where, A is the tool axis motion function A(t), which denote the instantaneous orientation of the tool axis. P is the tool center point motion function P(t), which denote the trajectory of the tool center points. $|\dot{A}| = 0$ and $A \times \dot{P} \neq 0$ it means the tool movement is only translational motion. $\dot{P}$ is the derivate of P(t), $\dot{A}$ is the derivate of A(t). Their kinematics significance are the translational velocity Vo and rotational velocity VR of the tool.

Assume the tool's two rotation axes are A and C, and the rotation angle are θ_A *and* θ_C, they can determine the tool orientation. Hence, at one moment, the tool position in 3D space can be determined by a five-tuple($x_p, y_p, z_p, \theta_A, \theta_c$), x_p, y_p, z_p denote the cutter location point (CL point)coordinate, used to describe the cutter 's translational movement. The five-tuple may also be represented as (P_i, θ_i) .The rotational matrices $R(X, \theta_A(t)), R(Z, \theta_C(t))$ are defined as follows:

$$R(X,\theta_A(t)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_A(t) & -\sin\theta_A(t) \\ 0 & \sin\theta_A(t) & \cos\theta_A(t) \end{bmatrix},$$
$$R(Z,\theta_C(t)) = \begin{bmatrix} \cos\theta_C(t) & -\sin\theta_C(t) & 0 \\ \sin\theta_C(t) & \cos\theta_C(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

According to Eq.(2),(3)and(4), applying the general rigid body motion theory, we can obtain the description of $e_1, e_{2,} e_3$ in detail:

$$e_1 = A(t) = R(Z, \theta_C(t)) \bullet R(X, \theta_A(t)) \bullet \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$
$$= \begin{pmatrix} \sin\theta_C(t) \bullet \sin\theta_A(t) \\ -\cos\theta_C(t) \bullet \sin\theta_A(t) \\ \cos\theta_A(t) \end{pmatrix}. \tag{5}$$

The concrete form of $e_{2,} e_3$ can be obtained from (2),(3)and(5).

IV. THE TOOL SWEPT VOLUME MODELING AND COMPUTATION

The swept volume(SV) is defined as the totality of all points that belong to the trace of an arbitrary moving geometric entity (operator), which moves (translates and rotates, and possibly deforms) along an arbitrary path (a curve, surface, or solid). Thus, the tool swept volume is defined as an entity that formed during the tool moving in the 3D space along an arbitrary trajectory.

According to the theory of envelope for surface family, in NC machining, during the cutting process, the cutter at different time can constitute a single parameter surface family, called tool swept envelope, which is the boundary surfaces of tool swept volume. It is composed of the following three parts:(1)the boundary surface of the tool in the initial point;(2)boundary surface in final point;(3)the middle envelope that formed during the tool moving from initial to final point. Their closed-form constitutes the swept volume. Once the tool's shape is determined, the first two parts are easy to acquire. Thus, how to obtain the middle envelope is the key issue.

A. Algorithm description

Step 1: Read in NC data, Obtain the tool information;

Step 2: Establish the tool model according to the tool information;

Step 3:Caculate the critical line, obtain the middle swept envelope. According to the specific tool and its motion information, establish the critical line equation, and solve the equation, acquire the swept profile and the middle envelope.

Step 4: Generate the swept volume. Compute the boundary surface of tool in start and end point, closed the three parts to acquire the complete swept volume, give its 3D model.

B. Swept envelope

According to the envelope theory, the swept volume boundary surface is created through the swept profile(a.k.a critical line) set, which means that the envelope surface and the intermediate generator solid M(s,t) tangentially touch along the swept profile at a given time t . So if a point P lies on the swept profile(it means that P is not only on the cutter surface, but also on the swept envelope surface), the velocity vector of point P at time t must be in the tangent plane of the SV boundary surface at this point. That is to say,

$$f(s,t) = \boldsymbol{N(P)} \cdot \boldsymbol{V(P)} = \boldsymbol{N}(s,t) \cdot \boldsymbol{V}(s,t) = 0 \ . \qquad (6)$$

Here, V(P) is the instantaneous velocity vector and N(P) stands for the normal vector of the SV envelope surface and the boundary surface of M(s,t) at point P.

From Equation.(1) , the surface normal of the cutter M(s,t) can be defined as follows:

(1) The surface normal of the cylinder part $\overrightarrow{N}_{cylinder}$ is defined as:

$$\overrightarrow{N}_{cylinder} = R\begin{bmatrix}\cos\theta \\ \sin\theta \\ 0\end{bmatrix} \qquad (7)$$

(2) The surface normal of the upper cone $\overrightarrow{N}_{uppercone}$ is defined as:

$$\overrightarrow{N}_{uppercone} = \frac{\partial M_{uppercone}}{\partial k_{uppercone}} \times \frac{\partial M_{uppercone}}{\partial\theta} =$$

$$(e + r_{tc}\cos\beta + l - h + r_{tc}\sin\beta)\cdot(l - h + r_{tc}\sin\beta)\begin{bmatrix}\cos\theta \\ \sin\theta \\ -\tan\beta\end{bmatrix} \qquad (8)$$

(3) The surface normal of the corner torus $\overrightarrow{N}_{cornertorus}$ is defined as:

$$\overrightarrow{N}_{cornertorus} = \frac{\partial M_{cornertorus}}{\partial\theta} \times \frac{\partial M_{cornerorus}}{\partial\varphi} = (e + r_{tc}\sin\varphi) r_{tc}\begin{bmatrix}\sin\varphi\cos\theta \\ \sin\varphi\sin\theta \\ -\cos\varphi\end{bmatrix} \qquad (9)$$

(4) The surface normal of the lower cone $\overrightarrow{N}_{lowercone}$ is defined as:

$$\overrightarrow{N}_{lowercone} = \frac{\partial M_{lowercone}}{\partial k_{lowercone}} \times \frac{\partial M_{lowercone}}{\partial\theta} = k_{lowercone}(e + r_{tc}\sin\alpha)^2\begin{bmatrix}\cos\theta\tan\alpha \\ \sin\theta\tan\alpha \\ -1\end{bmatrix} \qquad (10)$$

In order to present the algorithm clearly, we will take the fillet end mill as an example in the following description. It is also suitable for other tool type.

The family of cylindrical surfaces of the tool in reference coordinates can be described as:

$$S_{cylinder}(\theta,k,t) = P(t) + k\cdot e_1 + R\cos\theta\cdot e_2 + R\sin\theta\cdot e_3 \ . \qquad (11)$$

Where, $0 \le k \le h_{cy}$, $R = e + r_{tc}, \theta \in [0, 2\pi]$,Let P be an arbitrary point on the cylindrical surface, see in fig.3. in the reference coordinate system, the normal N(P) is denoted as follow:

$$\boldsymbol{N(P)} = \cos\theta\cdot e_2 + \sin\theta\cdot e_3 \qquad (12)$$

According to the characteristic of the rigid body rotation, the rotational velocity of the tool axis is:

$$\dot{\boldsymbol{A}} = \omega \times \boldsymbol{A} \ . \qquad (13)$$

ω is the angular velocity of the axis, A is the unit vector of the tool axis, so the rotational velocity of a point on the tool surface can be given as: $\boldsymbol{V_R} = \omega \times \overrightarrow{OP}$,

Here, $\overrightarrow{OP}$ is the vector from point O to P, as shown in fig.3. Hence,

$$\begin{aligned}\boldsymbol{V(P)} &= \dot{P}(t) + \dot{A}(t) = \boldsymbol{V_o} + \boldsymbol{V_R} \\ &= \boldsymbol{V_o} + \omega\times\overrightarrow{OP} \\ &= \boldsymbol{V_o} + |\omega\times e_1|\bullet k\bullet e_2 + \mathrm{R}\cos\theta\bullet(\omega\times \mathrm{e}_2) \\ &\quad + \mathrm{R}\sin\theta\bullet(\omega\times \mathrm{e}_3) \ .\end{aligned} \qquad (14)$$

For typical five-axis milling machine(AC type), the instantaneous angular velocity $\omega(t)$ is given as:

$$\omega(t)=\dot{\theta}_C(t)+R(Z,\theta_C(t))\cdot\dot{\theta}_A(t)=\begin{pmatrix}\cos\theta_C(t)\cdot\dot{\theta}_A(t)\\ \sin\theta_C(t)\cdot\dot{\theta}_A(t)\\ \dot{\theta}_C(t)\end{pmatrix}. \quad (15)$$

According to (6),(12)and(14), we can obtain the critical line equation:

$$f(\theta,k,t)=N(P)\cdot V(P)$$
$$=\boldsymbol{V_o}\cdot\cos\theta\cdot e_2+k\cos\theta\cdot|\omega\times e_1|+\boldsymbol{V_o}\cdot\sin\theta\cdot e_3=0. \quad (16)$$

Therefore,

$$\theta(k,t)=\begin{cases}\tan^{-1}\left(-\dfrac{\boldsymbol{V_o}\cdot e_2+k\cdot|\omega\times e_1|}{\boldsymbol{V_o}\cdot e_3}\right), & if\ \boldsymbol{V_o}\cdot e_3\neq 0\\ \dfrac{3\pi}{2}, & if\ \boldsymbol{V_o}\cdot e_3=0\end{cases}. \quad (17)$$

Take (17) into (11), the swept envelope of cylinder part can be obtained.

Using the similar analysis process, we can obtain other parts' swept envelope of tool. Let Q be an arbitrary point on the torus surface, see in fig.3. The family of toroidal surfaces can be represented as:

$$S_{torus}(\theta,\varphi,t)=P(t)+(e+r_{tc}\sin\varphi)\cos\theta\cdot e_2$$
$$+\ (e+r_{tc}\sin\varphi)\sin\theta\cdot e_3-r_{tc}\cos\varphi\cdot e_1. \quad (18)$$

Then,

$$N(Q)=-\cos\varphi\cdot e_1+\sin\varphi\cdot\cos\theta\cdot e_2+\sin\varphi\cdot\sin\theta\cdot e_3. \quad (19)$$

$$V(Q)=\boldsymbol{V_o}+\boldsymbol{V_R}=\boldsymbol{V_o}+\omega\times\overrightarrow{OQ}$$
$$=\boldsymbol{V_o}-r_{tc}\cdot\cos\varphi\cdot(\omega\times e_1)$$
$$+(e+r_{tc}\cdot\sin\varphi)\cdot\cos\theta\cdot(\omega\times e_2) \quad (20)$$
$$+(e+r_{tc}\cdot\sin\varphi)\cdot\sin\theta\cdot(\omega\times e_3).$$

According to Eq.(6),(19)and(20), we can obtain the follow formula:

$$f(\theta,\varphi,t)=\sin\varphi[\cos\theta(e_2\cdot\boldsymbol{V_o})+\sin\theta(e_3\cdot\boldsymbol{V_o})]$$
$$+\cos\varphi[(-e_1\cdot\boldsymbol{V_o})+e\cdot\cos\theta\cdot(e_3\cdot\omega)]=0. \quad (21)$$

Therefore,

$$\varphi(\theta,t)=\tan^{-1}\left[\frac{(\boldsymbol{V_o}\cdot e_1)-e\cdot\cos\theta(\omega\cdot e_3)}{(\boldsymbol{V_o}\cdot e_1)\cos\theta+(\boldsymbol{V_o}\cdot e_3)\sin\theta}\right]. \quad (22)$$

According to the definition of the fillet end mill, $\varphi\in[0,\frac{\pi}{2}]$, assume when $\varphi=\frac{\pi}{2}$, $\theta=\theta_t$. Then, there exist explicit solutions of $\varphi(\theta,t)$ in the field $\theta\in[\theta_t,\theta_t+\pi]$. From Eq.(21), we can get:

$$\theta_t(\varphi=\frac{\pi}{2})=\tan^{-1}\left[-\frac{V_o\cdot e_2}{V_o\cdot e_3}\right], if\ V_o\cdot e_3\neq 0;$$
$$or\quad \theta_t=3\pi/2,\ if\ V_o\cdot e_3=0. \quad (23)$$

C. Translational and rotational velocity

From the above description, to get the individual point velocity on the cutter, three varying $\boldsymbol{V_o},\dot{\theta}_A(t)\ and\ \dot{\theta}_C(t)$ were introduced. Normally, this information is not included in the NC-data, need to solving. For linear interpolation tool movements, the intermediate motion of the cutter between two NC-point can be represented as follows:

$$\begin{bmatrix}P(t)\\ \theta(t)\end{bmatrix}=\begin{bmatrix}P_i\\ \theta_i\end{bmatrix}+\frac{t}{\Delta t}\left(\begin{bmatrix}P_{i+1}\\ \theta_{i+1}\end{bmatrix}-\begin{bmatrix}P_i\\ \theta_i\end{bmatrix}\right), 0<t\le\Delta t. \quad (24)$$

Here, $\Delta t=\left\|\begin{bmatrix}P_{i+1}\\ \theta_{i+1}\end{bmatrix}-\begin{bmatrix}P_i\\ \theta_i\end{bmatrix}\right\|\Big/v_f$. (25)

v_f is the feedrate from the current NC-point to the next NC-point, which is defined in the NC-data. Therefore, we can get the translational velocity of the CL point:

$$\boldsymbol{V_o}=\frac{P_{i+1}-P_i}{\Delta t}=\begin{bmatrix}\frac{\Delta x_p}{\Delta t} & \frac{\Delta y_p}{\Delta t} & \frac{\Delta z_p}{\Delta t}\end{bmatrix}^T. \quad (26)$$

and the velocity of orientation angles:

$$\dot{\theta}(t)=\begin{bmatrix}\dot{\theta}_A(t)\\ \dot{\theta}_B(t)\\ \dot{\theta}_C(t)\end{bmatrix}=\frac{\theta_{i+1}-\theta_i}{\Delta t} \quad (27)$$

D. Generate the integrated swept volume

To acquire the completed tool swept volume, we define (6) as the tangency function[1], according to the tangency function, the points on the cutter surface can be classified as three parts: egress points, grazing points and ingress points, see in Fig.4. Thus , at a specific time t, the cutter surface can be denoted as a point set as follows:

$$M_M(s,t) = M_M^+(s,t) \cup M_M^0(s,t) \cup M_M^-(s,t) \tag{28}$$

Where,

(1) $M_M^+(s,t)$ denotes the egress points that satisfy $f(s,t) > 0$;

(2) $M_M^0(s,t)$ denotes the grazing points that satisfy $f(s,t) = 0$ just the points lie on the critical line(swept profile);

(3) $M_M^-(s,t)$ are the ingress points with $f(s,t) < 0$.

The integrated tool swept volume envelope $E(s,t)$ can be defined as a set which is composed of three parts: the tool's ingress part at initial point of the motion path, the middle envelope during the tool's cutting process and the tool's egress part at the final point. Here, tinitial and tfinial are the time parameter of the tool.

$$E(s,t) = \left\{ \begin{matrix} M_M^-(s, t_{initial}) \cup \\ M_M^0(s, t_{initial} \le t \le t_{final}) \cup \\ M_M^+(s, t_{final}) \end{matrix} \right\} \tag{29}$$

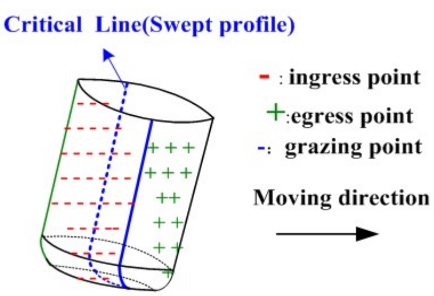

Fig.4. Egress ,ingress and grazing point on the tool surface

V. EXPERIMENT AND DISCUSSION

The designed method has been implemented on 2.20 GHz personal computer ,by using VC++ programming language and ACIS 3D modeling engine tools. Two experiments are given as follow:

A. generate the 5-axis tool swept volume for a specific motion.

A fillet end cutter with the following parameters is used to demonstrate the swept volume: $\boldsymbol{R}$=0.6cm; $\boldsymbol{h_{cy}}$=2.5cm; $\boldsymbol{e}$=0.4cm; $\boldsymbol{r_{tc}}$=0.2cm.

The moving trajectory of the fillet-end cutter is defined as:

Initial point configuration:

$(P(t), \theta(t)) = ((0,0,0),(75^\circ, 80^\circ, 15^\circ))$;

Finial point configuration:

$(P(t), \theta(t)) = ((30,5,0),(45^\circ, 80^\circ, 86^\circ))$

Fig.5. demonstrating the tool swept generation and modeling process. (a) the tool's initial and final position;(b) the different tool posture at varies time during tool moving ;(c) the generated swept volume using presented method.

Fig.5.(a)

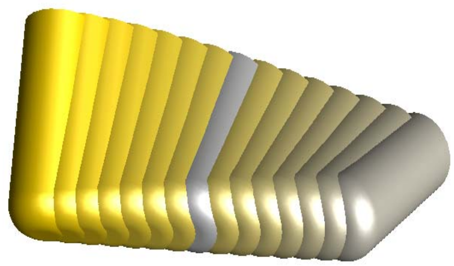

Fig.5.(b)

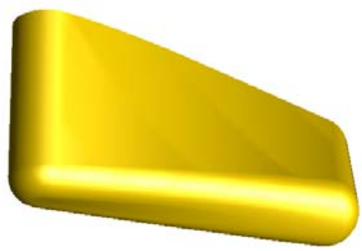

Fig.5.(c)

Fig.5. fillet end tool swept volume

B. Five-axis machining simulation result of a impeller part

Fig.6. shows the five-axis machining result of a impeller part, using the proposed swept generation algorithm.

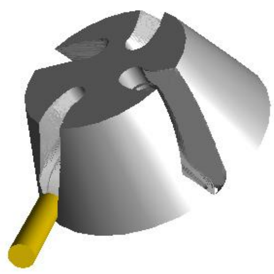

Fig.6. Simulated machining part

The experiment results demonstrate that the tool swept volume generated from the proposed algorithm is smooth and continuous, has high sense of reality and a good visual effect. Hence, the proposed algorithm is quite suitable for the five-axis machining simulation which requires high accuracy and high reality and NC verification.

VI. CONCLUSION

This paper develops a new exact tool swept volume generation algorithm for 5-axis NC simulation. It is based on the generalized cutter which is modeled first, and suitable for any tool movement. Thus, it is universal and may be widely used in NC simulation and verification. Two experimental results shows the feasibility and effectiveness of the proposed method. Compared with the existing tool swept volume approximate methods, the developed method can acquire better reality and visual effect. The main contributions of this paper are as follows:

(1) Establish a generalized cutter model, give its geometric definition and mathematical description;
(2) Demonstrate the calculation method of critical line by strict and detailed mathematical deduction;
(3) Analyze the 5-axis machine tool motion, including translation and rotation movements, give the solving method of translation and rotation velocity.
(4) The proposed method is not only used in NC simulation, but also can be applied to other rigid rotation movement.

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