# Multi-objective Optimization of Fuzzy Parallel Machines Scheduling Problem Using Nondominated Genetic Algorithms

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*Abstract*—A kind of unrelated parallel machines scheduling problem is discussed. The memberships of fuzzy due dates denote the grades of satisfaction with respect to completion times with jobs. Objectives of scheduling are to maximize the minimum grade of satisfaction while makespan is minimized in the meantime. Two kind of genetic algorithms are employed to search optimal solution set of the problem. Both Niche Pareto Genetic Algorithm (NPGA) and Nondominated Sorting Genetic Algorithm (NSGA-II) can find the Pareto optimal solutions. Numerical simulation illustrates that NSGA-II has better results than NPGA.

# Index Terms—parallel machines scheduling, fuzzy due-date, NPGA, NSGA-II

# I. INTRODUCTION

Scheduling problems have been researched for more than fifty years since first scheduling investigations undertaken by Jackson in 1955[1]. Parallel machines scheduling problem is a kind of important multi-machine scheduling problem. It means every machine has same work function and every job can be processed by any machine. Most of non-preemptive parallel scheduling problems are NP problems and can not be solved with determinative algorithm in polynomial time; even there are only two machines. Genetic algorithm has strong searching ability and robustness. So lots of researchers analyze parallel scheduling problem with genetic algorithm [2, 3, 4, 5].

In the conventional scheduling problem, the parameters such as job processing times, ready times, due-dates have been assumed to be deterministic. In real world, input data may be uncertain or imprecise. Ishii et al. [6] introduced the concept of fuzzy due dates to scheduling problems, fuzzy due dates scheduling problems have been investigated by many researchers [7]. Considering that input parameters of parallel scheduling problems are uncertain in real situation, we fuzzilize the due dates of jobs. For one feasible scheduling sequence, memberships of fuzzy due dates are used to denote the grades of satisfaction about due dates from the point of view of scheduling decision-makers. The objective of scheduling is not only to minimize the maximum completion times of jobs, but also to maximize the minimum grades of satisfaction of jobs' completion

times. According to analyzing the bicriteria scheduling problem, Pareto optimal solution is induced into describe the problem. First we use Niche Pareto Genetic Algorithm (NPGA) [8] to find the Pareto optimal solutions of the problem. Then we improve the searching method. Nondominated Sorting Genetic Algorithm (NSGA-II) [9] is employed and shows better result.

The outline of this paper is organized as follows. Firstly, we briefly describe the parallel problem with fuzzy due dates. Then, we introduce the conception of nondominated scheduling. After that, we give the solution of Niche Pareto Genetic Algorithm. We also give another GA based solution—Nondominated Sorting Genetic Algorithm. In order to comparing the two GA based algorithms, some numerical examples and computational results are provided to show the effectiveness of the proposed algorithms. Finally, we conclude this paper with a summary.

# II. DESCRIPTION OF PROBLEM

Give *m* machines  $M_1, M_2, ..., M_m$  and *n* jobs  $J_1, J_2, ..., J_n$ . Every machine can process one job at the same time, and arbitrary job can be process on one machine simultaneity. All machines have same function. Then each job can be process on any machine. So we call it parallel machines. The processing time of job  $J_i$  is  $p_{ij}$  on machine  $M_j$ . The *m* machines are identical machines if there are  $p_{ij} = p_j$  for all machine  $M_j$ . If the processing time  $p_{ij} = p_j/s_j$  on machine  $M_j$  for job  $J_i$ , which  $s_j$  is the speed of machine, machines are uniform machines. If  $p_{ij} = p_j/s_{ij}$ , which  $s_{ij}$  is the speed of job  $J_i$  on machines are unrelated machines.

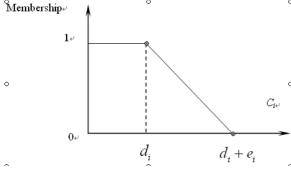
In many practical scheduling problems, due dates of jobs are uncertain or imprecise, and this kind of uncertainty or imprecision cannot be described by probability theory. For these situations where characteristics and constraints are neither deterministic nor probabilistic, the problems may often be modeled with fuzzy sets [10,11].

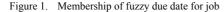
In scheduling problems, every job has due date  $d_i$ . Jobs must be completed before due dates in deterministic scheduling problems. But due dates of jobs are not crisp numbers in lots of practical scheduling. A certain delay is tolerable up to a late date  $d_i+e_i$  beyond which the order will be canceled, because the customer will resort to other

suppliers. As completion time of job  $J_i$  passes between due date  $d_i$  and late date  $d_i+e_i$ , the customer satisfaction decreases until it vanishes at the latter. The greater the delay, the lower the satisfaction. For the interval  $[d_i, d_i+e_i]$ in real number set, it is acceptable if job  $J_i$  can be finished in its interval. But the grades of satisfaction of scheduling decision-makers about completion time  $C_i$  of job  $J_i$  are different. We use fuzzy set  $\tilde{d}_i$  to describe the uncertain due date. Fuzzy set  $\tilde{d}_i$  denotes  $[d_i, d_i+e_i]$ , its membership  $f_i(x)$  is:

$$f_i(x) = \begin{cases} 1 & x \le d_i \\ (d_i + e_i - x) / e_i & d_i < x < d_i + e_i \\ 0 & d_i + e_i \le x \end{cases}$$
(1)

Membership function denotes the grade of satisfaction of completion time  $C_i$ (it is x in formula (1)) about due date. It is given by scheduling decision-makers according their scheduling experience or preference. If completion time is not beyond low bound  $d_i$ , the grade of satisfaction is 1, which is biggest. The grade of satisfaction declines with increasing of  $C_i$ , If  $C_i$  is beyond  $d_i$ . Until completion time equals or is bigger than upper bound of fuzzy due date  $d_i+e_i$ , the grade of satisfaction declines to zero. This membership is shown in Figure 1.





For above parallel scheduling problem, we consider two objective functions as following. One is maximum completion time of jobs  $C_{\text{max}}$  which is makespan of jobs:

$$C_{\max} = \max\{C_i | i=1,2,...,n\}.$$
 (2)

Another is minimum grade of satisfaction  $f_{\min}(C_i)$  of job completion time  $C_i$ :

$$f_{\min}(C_i) = \min\{f_i(C_i) | i = 1, 2, \dots, n\}.$$
 (3)

The objective of scheduling is to find optimal job sequence, which minimizes  $C_{\text{max}}$  and maximize  $f_{\min}(C_i)$  simultaneity. So the bicriteria scheduling problem can be denoted as  $R | \tilde{d}_i | (C_{\max}, f_{\min})$ .

#### **III. NO-DOMINATED SCHEDULING**

For above bicriteria scheduling problem  $R | \tilde{d}_i | (C_{\max}, f_{\min})$ , the objective is to minimize the maximum job's completion time  $C_{\max}$  and maximize the minimum grade of satisfaction  $f_{\min}(C_i)$  in the same time. For some feasible schedule  $\pi$ , let schedule vector

 $\nu^{\pi}$  consisting two elements,  $C_{\max}^{\pi}$  and  $f_{\min}^{\pi}$  in some feasible schedule  $\pi$ . We denote the solution vector as  $S^{\pi} = (C_{\max}^{\pi}, f_{\min}^{\pi})$ . Given two feasible solutions  $\pi_1$  and  $\pi_2$ , their vectors are  $S^{\pi_1} = (C_{\max}^{\pi_1}, f_{\min}^{\pi_1})$  and  $S^{\pi_2} = (C_{\max}^{\pi_2}, f_{\min}^{\pi_2})$ , respectively. We say that vector  $S^{\pi_1}$  dominates vector  $S^{\pi_2}$  if

$$C_{\max}^{\pi_1} \le C_{\max}^{\pi_2}$$
,  $f_{\min}^{\pi_1} \ge f_{\min}^{\pi_2}$  and  $S^{\pi_1} \ne S^{\pi_2}$ . (4)

and at least one of these two inequalities is strict. If  $\pi_1$  and  $\pi_2$  are such that

$$C_{\max}^{\pi_1} = C_{\max}^{\pi_2}, \ f_{\min}^{\pi_1} = f_{\min}^{\pi_2}.$$
 (5)

then we call them equivalent. Consequently, every equivalence class of schedules consists of all schedules with identical value of  $C_{\max}^{\pi}$  and  $f_{\min}^{\pi}$ . During the process of maximize the both criteria  $C_{\max}^{\pi}$  and  $f_{\min}^{\pi}$ , maybe there exit many nondominated schedules which have same values. We just need to find at least one feasible schedule corresponding form each equivalence class of nondominated feasible schedules.

For one feasible schedule  $\pi$ , we say it is the nondominated schedule of problem if its vector  $S^{\pi}$  is not dominated by any other vectors. So the feasible schedule  $\pi$  is the one of optimal schedules whom we are searching for.  $S^{\pi}$  is called Pareto optimal vector. For scheduling problem  $R | \tilde{d}_i | (C_{\max}, f_{\min})$ , it has lots of nondominated schedules who are not dominated by any other feasible schedules. The vectors of those nondominated schedules constitute Pareto optimal frontier of solutions vector space.

# IV. THE SOLUTION BASED ON NPGA

Niche Pareto Genetic Algorithm (NPGA). In order to find optimal schedules of problem  $R \mid d_i \mid (C_{\text{max}}, f_{\text{min}})$ , the corresponding Pareto optimal vectors should be gotten. When all Pareto optimal vectors are given, a Pareto optimal frontier can be constructed in two dimensions space. The different points in the frontier denote corresponding optimal schedules. But different optimal schedules have same Pareto optimal vectors in some case. Horn and Nafpliotis [8] pointed out that Niche Pareto Genetic Algorithm (NPGA) can find the set of Pareto optimal vectors. NPGA algorithm also can keep the diversity of Pareto optimal vectors of every generation and avoid populations to converge into several optimal vectors in population evolution process. The solving process is given as following according to the problem in our research.

**Encoding Method.** Given parallel machines scheduling problem with *n* jobs and *m* machines, n+m-1 genes denote one feasible schedule. Every chromosome is divided into m segments which are separated by m-1 machines  $M_i$  (i = 1, 2, ..., m-1). Here  $M_i$  means *i*-th machine. For example, there are six jobs and three

machines. Chromosome " $35M_11M_2462$ " denotes that job  $J_3$  is processed on machine  $M_1$  firstly and then  $J_5$  is processed on same machine secondly. In the same time, job  $J_1$  is processed on machine  $M_2$ . The left job  $J_4$ ,  $J_6$  and  $J_2$  are processed on machine  $M_3$  in sequence of  $J_4 \rightarrow J_6 \rightarrow J_2$ . So  $M_3$  doesn't show in chromosome. If there are no jobs between machines  $M_i$  and  $M_j$ , it means that  $M_j$  is idle. Here some schedules are created randomly. The initial population is created by encoding these schedules in order to get next generation population.

**Fitness function**. We choose objectives  $C_{\text{max}}$  and  $f_{\min}(C_i)$  as fitness functions for the bicriteria parallel machines scheduling problem  $R | \tilde{d}_i | (C_{\max}, f_{\min})$ .

Selection Operator. Selection operator is critical part in NPGA algorithm. In order to keep the diversity of Pareto optimal schedules in every generation of population and avoid that individual converges to one Pareto optimal schedule; NPGA uses unique tournament selection method. The selection method of NPGA algorithm is following. Firstly, two individuals are chosen randomly from current population as candidates.  $t_{dom}$ individuals also are chosen from current population as comparing set P at random. The fitness of each candidate is compared with all individuals of comparing set P. If one candidate is dominated by P and another candidate is not dominated by P, the late one is chosen to reproduction. The sharing to choose the winner if neither or both is dominated by P.

Sharing is a kind of fitness sharing function which is used to maintain individuals to distribute around Pareto optimal frontier. Sharing function sh[d] is the decreasing function about d[i, j], such that sh[0] = 1 and  $sh[d \ge \sigma_{share}]$ = 0. Here d[i, j] is the distance between *i* and *j*. Generally speaking, sharing function is denoted by triangle function:

$$sh[d] = \begin{cases} 1 - d / \sigma_{share} & d \le \sigma_{share} \\ 0 & d > \sigma_{share} \end{cases}$$
(6)

Here  $\sigma_{share}$  is niche radius, given by user. We get the niche count  $m_i$  when we add all sharing functions of individual *i* with other individuals in population:

$$m_i = \sum_{j \in Pop} sh[d[i, j]].$$
(7)

Here  $m_i$  means the crowded degree around individual *i*. In order to avoid population converge into certain individual, sharing function calls for the degradation of an individual's fitness by simply dividing the objective fitness by the niche count  $m_i$ . Then we get a new fitness  $f_i$ ' as following:

$$f_i' = f_i / m_i . \tag{8}$$

Then the fitness of individuals within one niche radius become to decrease and is lower than the fitness of individuals within other niche radius when the quantity of individuals in this niche radius is beyond certain number. So individuals with smaller fitness have less chance to keep in next generation. This guarantees the diversity of final generation of population. So in selection process of NPGA algorithm, the candidate with smaller niche count will be chosen if neither or both of two candidates are dominated by *P*.

Then choose progress can be described as following. Given S is current population, P is comparing set and consist of  $t_{dom}$  individuals.  $P_i$  is the *i*th individual.

Step one: Randomly choose two individuals as candidates from population *S*. Randomly choose  $t_{dom}$  individuals as comparing set *P*.

Step two: Let  $i \leftarrow 1$ , comparing two candidates with *i*th individual  $P_i$ . If  $P_i$  is better than some candidate, mark the candidate as dominated one. If both of candidates are marked as dominated individuals, go to step four.

Step three: Let i=i+1. If  $i \le t_{dom}$ , go to step two. Else go to next step.

Step four: If only one candidate is marked as dominated individual, remain another one which is not dominated by comparing set P. If both of individuals are dominated by comparing set P or neither one is dominated by comparing set P, use sharing mechanism.

**Crossover Operator**. Crossover operator applies partial map crossover (MPX). MPX can be viewed as a variation of two-cut-point crossover. MPX deals with illegal phenomenon according to a special repairing process. The crossover process of MPX is following. Randomly choose two individuals as parents in this generation. Then choose two genes of every parent as cut-point. The substring between two cut-point of every parent directly remains into the same position of corresponding child. The other genes of children can be gotten from another parent according certain mapping relationship.

**Mutation Operator**. Our research uses following mutation mechanism. Select one gene in every individual at random and divide the individual into two parts. Then swap these two parts. For example, pick the third gene "3" in individual (123456) as swap point. Swap the two parts and get new individual (345612).

**Stop Condition**. The algorithm stop when given stop condition is reached. Otherwise the algorithm keeps searching the Pareto optimal solutions. Here we use maximum times of propagation as stop condition.

Now the NPGA algorithm for finding the optimal schedules can be described as follows:

Step one: The initial population  $P_i$  including N individuals is generated randomly.  $P_i$  is current population. Evaluate the fitness of every individual of  $P_i$ . We can get the current best individual. Let the best individual be overall best one.

Step two: Randomly choose two individuals from current population as candidate individual. The randomly choose  $t_{dom}$  individuals to compose comparing sets *P*. Using selection operator to generate next generation individuals.

Step three: Repeat step 2 till new population is generated which is consisted of N individuals. The new population replaces the current population.

Step four. Randomly choose two individuals to get new individuals with crossover operator and mutation operator. Step five: Repeat step 4 till get N new individuals. These N new individuals consist of population  $P_{i+1}$ .

Step six: Evaluate the fitness of every individual of  $P_{i+1}$ . We can get the best individual and worst individual. Replace the worst individual of  $P_{i+1}$  with overall best individual. If the best individual of  $P_{i+1}$  is better than overall best individual, replace the overall best individual with the best individual of  $P_{i+1}$ .

Step seven: If stop condition is not satisfied, Let  $P_{i+1}$  be current population  $P_i$ . Else, go to end.

Figure 2 gives the procedure of NPGA algorithm.

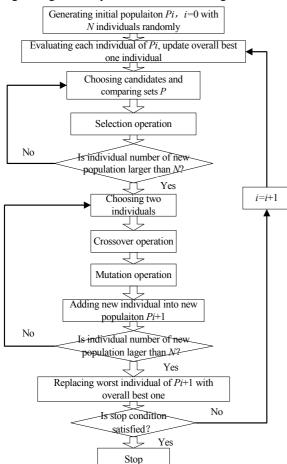


Figure 2. Membership of fuzzy due date for job

#### V. THE SOLUTION BASED ON NSGA-II

**Nondominated Sorting Genetic Algorithm (NSGA-II)**. NSGA-II algorithm is the modified method of Nondominated Sorting Genetic Algorithm (NSGA) by Deb et al [8]. NSGA algorithm, which is given by Srinivas and Deb, has following shortages:

--First, high computing complexity of NSGA.

--Second, lack of elitism.

--Third, need for specifying the share parameter  $\sigma_{\scriptscriptstyle share}$  .

NSGA-II algorithm improves the NSGA algorithm and solves above three shortages successfully. The details of NSGA-II algorithm are following.

**Encoding Method.** Given parallel machines scheduling problem with n jobs and m machines, n+m-1 genes denote one feasible schedule. Every chromosome is

divided into m segments which are separated by m-1 machines  $M_i$  (i = 1, 2, ..., m-1). Here  $M_i$  means i-th machine. For example, there are six jobs and three machines. Chromosome " $35M_11M_2462$ " denotes that job  $J_3$  is processed on machine  $M_1$  firstly and then  $J_5$  is processed on same machine secondly. In the same time, job  $J_1$  is processed on machine  $M_2$ . The left job  $J_4$ ,  $J_6$  and  $J_2$  are processed on machine  $M_3$  in sequence of  $J_i \rightarrow J_6 \rightarrow J_2$ . So  $M_3$  doesn't show in chromosome. If there are no jobs between machines  $M_i$  and  $M_j$ , it means that  $M_j$  is idle. Here some schedules are created randomly. The initial population is created by encoding these schedules in order to get next generation population.

**Fitness function**. We choose objectives  $C_{\max}$  and  $f_{\min}(C_i)$  as fitness functions for the bicriteria parallel machines scheduling problem  $R \mid \tilde{d}_i \mid (C_{\max}, f_{\min})$ .

**Nondominated Schedules**. All individuals of current population are sorted according to corresponding fitness functions. Firstly, every Pareto optimal individual is selected in current population. The rank number of these Pareto optimal individuals is one. Then delete individuals whose rank number are one and search Pareto optimal individuals in left population whose rank number are assigned as two. Delete individuals with rank number two and continue to search Pareto optimal individuals in left population. Current Pareto optimal individuals are labeled with rank number three. Repeat above process until every individual has it own rank number. That means individuals have been sorted by its fitness function.

Selection Operator. For NSGA-II algorithm, selection process is very important. In order to keep the diversity of Pareto optimal schedules in every generation of population and avoid that individual converges to one Pareto optimal schedule; NSGA-II algorithm uses unique crowded-comparison approach instead of sharing function of NSGA. The selection strategy of NSGA-II is showed as following. First, NSGA-II algorithm defines density estimation. In order to get density estimation surrounding one individual in the population, NSGA-II algorithm calculates the average distance of two most closed individuals on either side of this individual along each objective. In every generation of population, NSGA-II need choose appropriate individuals from current Pareto optimal individuals into next generation. It means that it depends on density estimation of current Pareto optimal individuals which individuals can be remained in next generation. The total density estimation of individual can be gotten by accumulation all density estimation of its different objective functions. Specifically, current Pareto optimal individuals are ranked from smallest one to biggest one according to the value of this objective function for certain objective function. For biggest and smallest individuals on the end, the density estimation of their objective function is assigned an infinite value. For the other individuals between biggest one and smallest one, their density estimation is absolute normalized difference of the objective function values of two adjacent individuals. Then the total density estimation of one individual can be figured out by accumulation the density estimation of all different functions of this individual.

NSGA-II algorithm is not like Niche Pareto Genetic Algorithm (NPGA) [8] and doesn't require estimating the value of sharing parameter  $\sigma_{share}$ . The latter two algorithms use sharing function to select individuals which can be remained in next generation during selection process. But the number of optimal individuals should be known in advanced if we estimate the value of sharing parameter. For some situation that the distribution of optimal individual can not be known in advanced, it becomes difficult to select sharing parameter. Otherwise the value of sharing parameter has great influence to final result.

**Crossover Operator.** Crossover operator applies partial map crossover (MPX). MPX can be viewed as a variation of two-cut-point crossover. MPX deals with illegal phenomenon according to a special repairing process. The crossover process of MPX is following. Randomly choose two individuals as parents in this generation. Then choose two genes of every parent as cut-point. The substring between two cut-point of every parent directly remains into the same position of corresponding child. The other genes of children can be gotten from another parent according certain mapping relationship.

**Mutation Operator.** Our research uses following mutation mechanism. Select one gene in every individual at random and divide the individual into two parts. Then swap these two parts. For example, pick the third gene "3" in individual (123456) as swap point. Swap the two parts and get new individual (345612).

**Elitist selection strategy**. Deb et al point out that elitist selection strategy can dramatically speed up converging speed and performance and keep all current Pareto optimal individuals remain in next generation of population. It can enhance performance of algorithm and speed up that population converges into Pareto optimal solutions.

**Stop Condition**. The algorithm stop when given stop condition is reached. Otherwise the algorithm keeps searching the Pareto optimal solutions. Here we use maximum times of propagation as stop condition.

Then NSGA-II algorithm can be described in figure 3 as following.

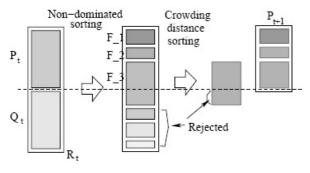


Figure 3. The solution of NSGA-II

Step one: Create initial population  $P_t$ , t=1 with N individuals at random, which  $P_t$  is current population.

Step two: Randomly choose two individual as parents and use crossover and mutation operator to get two new individual. Repeat above selection, crossover and mutation process till N new individuals are created. These N new individuals compose population  $Q_t$ .

Step three: Combine the population  $P_t$  and  $Q_t$  to get bigger population  $R_t$ . Sort all individuals in  $R_t$  according density estimation and get biggest rank number F 1.

Step four: Choose all individuals whose rank number is one to remain in next generation of population  $P_{t+1}$ . If there are no *N* individuals, continue to choose the individuals with next rank number. When it goes to rank number  $F_n$ , go to step six if the size of population  $P_{t+1}$ exactly reaches *N*. And go to step five if individuals are more than *N*.

step five: Calculate the density estimation of all individual with rank number  $F_n$ . Put right individuals into population  $P_{t+1}$  according to descending order of density estimation of individuals till the number of individuals in population  $P_{t+1}$  is exactly N.

Step six: Set  $P_{t+1}$  as current population  $P_t$  and go to step two if stop condition doesn't been reached.

# VI. SIMULATION EXAMPLE

In order to validate the effectiveness of NSGA-II algorithm, we use Visual C++6.0 to realize the algorithm

n×	NPGA				NSGA-II			
m	A	<i>T</i> (s)	$C_{\max}$	$f_{\min}$	A	<i>T</i> (s)	$C_{\max}$	$f_{\min}$
30	4	11	147.00 148.00	0.415 0.662	2	12	147.00	0.415
×3			149.00 153.00	0.670 0.729			150.00	0.701
40	2	12	118.00	0.852	2	12	120.00	0.855
×4			122.00	0.860			125.00	0.833
50	3	13	100.00	0.833	3	13	99.00	0.814
×5			106.00 110.00	0.849 0.861			105.00 109.00	0.854 0.858
	2	14	192.00	0.370		14	181.00	0.271
60					5		185.00	0.322
×6			195.00	0.648			190.00 194.00	0.357 0.579
							197.00	0.684

TABLE I. Results of four problems with NAGA-II and NPGA

n×m means the size of problem. A is quantity of optimal individuals. T is running time of computer for the responding problem, whose unit is second.

and make several example to simulate it on computer with PIII CPU. The size of problems  $n \times m$  (n jobs and mmachines) is 30×3, 40×4, 50×5 and 60×6, respectively. The processing times of jobs are integer numbers of interval [0, 50] gotten at random. Randomly create one scheduling sequence S and calculate the completion times  $C_i(S)$  of each job  $J_i$ . The low bound of fuzzy due date  $\tilde{d}_i$ of job  $J_i$  is denoted by  $d_i = C_i(S)$ -x, which x is the random number in the interval [1, 100]. The span  $e_i$  of fuzzy due date  $\tilde{d}_i$  also is the random number of close interval [1, 100]. Above-mentioned four problems are NP-hard problems even in the case of single objective function. Here we also use Niched Pareto Genetic Algorithm (NPGA) of Horn and Nafpliotis to solve these four instances. We compare the resolution and running time of NPGA and NSGA-II. The two algorithms consider maximum completion time and fuzzy due date of jobs simultaneity when the size of population is 100 individuals. We set that crossover probability is 0.8 and mutation probability is 0.1. In NPGA algorithm, the quantity of individuals in comparing set is ten. And the shape of niche of sharing function is 0.5. Then results are showed as following table 1.

# VII. SUMMARY

Our research discusses a kind of parallel machine scheduling problem with fuzzy due dates. Fuzzy due dates mean those due dates of jobs are not crisp numbers and can vary in certain interval. It is acceptable if the completion times of jobs are in this interval. But for scheduling decision-makers, the grades of satisfaction of jobs completion times are different. So it makes jobs scheduling more flexible and realistic. Here the purpose of scheduling is to minimize the maximum completion times of jobs and maximize the minimum grades of satisfaction of fuzzy due dates. For the bicriteria scheduling problem, the concept of nondominated schedules is introduced. The solution vector  $(C_{\text{max}}, f_{\text{min}})$  of nondominated schedule doesn't dominated by the solution vector of any other schedules. So these nondominated solution vectors are Pareto optimal solutions. All Pareto optimal solutions construct the set of Pareto optimal solutions.

NPGA algorithm can solve scheduling problem in our research. But NPGA algorithm requires that scheduling decision-makers give shape of niche or estimate shape of niche. Scheduling decision-makers also should give individual quantity of comparing set in tournament selection. These two parameters have some influence to NPGA algorithm. But there is not deterministic mathematical method or formula to compute these two parameters. It depends on experience to determine them. It affects the usage of NPGA algorithm in practical scheduling application. So we apply NSGA-II algorithm to search the Pareto optimal solutions of parallel machines scheduling problem with fuzzy due dates. NSGA-II algorithm uses crowed-comparison approach to keep appropriate individuals into next generation. It is different from sharing function of NPGA. Crowedcomparison approach doesn't require estimating the shape of niche. It selects individuals by computing ranks of nondominated sets of individuals and crowed distances of individuals. This method makes selection operator more clear and convenient. Moreover, crowedcomparison approach needn't choose comparing set and avoid estimate size of comparing set. All of above advantages make NSGA-II algorithm is easier to use and less computation than NPGA. The original version of NSGA-II algorithm given by Deb requires storing dominated individual set of every individual and it needs

 $O(n^2)$  space complexity. Our research modified original version and give a kind of computing method with smaller space complexity. The new method makes space complexity reduce to O(n).

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