A modified-Newton step primal-dual interior point algorithm for linear complementarity problems

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***Abstract*— Through some modifications on the classic-Newton direction, we obtain a new searching direction for monotone horizontal linear complementarity problem. By taking the step size along this direction as one, we set up a full-step primal-dual interior point algorithm. The complexity bound for the algorithm is derived, and the result is the best-known for linear complementarity problem.**

***Index Terms*— horizontal linear complementarity problem, interior-point algorithm, full-Newton step, complexity bound.**

I. INTRODUCTION

A monotone horizontal linear complementarity problem (LCP) is to find a pair $x, s \in R^n$ such that

$$Mx + Ns = q, \quad xs = 0, \quad x, s \geq 0, \tag{1}$$

where xs denotes the componentwise (or Hadamard) product of the vectors x and s. $q \in R^m$ and $M, N \in R^{m\times n}$, moreover M and N have the column monotonicity property, i.e., for any $u, w \in R^n$

$$Mu + Nw = 0 \Rightarrow u^T w \geq 0. \tag{2}$$

The formulation (1) includes linear and convex quadratic programming problems expressed by their optimality conditions in their usual format. Properties of this formulation are described in [1], where $R([M, N]) = n$ has been proved under the monotonicity hypothesis.

There are a variety of solution approaches for LCP which have been studied intensively. Among them, the interior-point methods (IPMs) gained much attention than other methods. Due to the close connection between LCP and linear and convex quadratic programming problems, some IPMs for linear and convex quadratic programming problems have been extended to LCP. For instance, Gonzaga et al. [2], [3] studied the largest step path following algorithm for LCP and showed that the fast convergence of the simplified largest step path following algorithm. Huang [4] proposed a high-order feasible IPM for LCP with $\mathcal{O}\left(\sqrt{n}\log\frac{\epsilon_0}{\epsilon}\right)$ iterations. Monteiro et al. [5] studied the limiting behavior of the derivatives of certain trajectories associated with the monotone LCP. Zhang [6] presented a class of infeasible IPMs for LCP and showed that the algorithm has $\mathcal{O}\left(n^2\log\frac{1}{\epsilon}\right)$ under some mild assumptions. Some other relevant references can be found in [7], [8].

Most of IPMs follow the central path and use the so-called primal-dual Newton search directions to obtain an ϵ-solution of the problem. Even for many algorithms that do not use the Newton's direction directly [9]–[12], they use the classic Newton's direction as the basis for deriving the new searching direction.

Because the importance of the Newton's direction in the designs and analyzes of IPMs, we study the Newton's direction. By using the scaled Newton direction we obtain a modified-Newton direction. Moreover we give a full-Newton step IPM for LCP, the algorithm uses the modified-Newton direction as the searching direction, which enjoys the nice property of quadratically convergent in the small neighborhood of cental path. Furthermore, we derive the complexity bound for the algorithm, and the complexity result is the best-known for LCP.

The paper is organized as follows. In Section II, the basic concepts of IPMs are given, which include the central path and the classic Newton direction. In section III, we give a scaled version of the classic Newton direction, and from which we give a modified-Newton direction. The generic algorithm is described in section IV. In section V, the properties of full-Newton step are analyzed, which include the estimation of the upper bound for duality gap and the increase of the proximity after one full-Newton step, the decrease of proximity after the parameter update is also given in this section. At the end of this section, we give a complexity result for the full-Newton step IPM. Section VI gives a simple numerical example. Section VII ends the paper with a conclusion.

Some notations used throughout the paper are as follows. $\|\cdot\|$denotes the 2-norm of a vector, $\|\cdot\|_1$ and $\|\cdot\|_\infty$ denote the 1-norm and infinity-norm, respectively. For any $x = (x_1; x_2; \ldots; x_n) \in R^n$, $x_{\min}$ denotes the smallest value of the components of x.

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II. Preliminary

We assume the following hypotheses hold: the existence of an interior feasible solution, and the existence of a strictly complementarity optimal solution.

A. The central path

The basic idea of the IPM is to replace the second equation in (1) by the parameterized equation $xs = \mu e$, with $\mu > 0$. Thus we consider the following system

$$Mx + Ns = q, \quad xs = \mu e, \quad x, s \geq 0. \tag{3}$$

Under the assumption, the parameterized system (3) has a unique solution for each $\mu > 0$. This solution is denoted as $(x(\mu), s(\mu))$ and are called the μ-center of LCP. The set of μ-centers (with μ running through all positive real numbers) gives a homotopy path, which is called the central path of LCP. If $\mu \to 0$, then the limit of the central path exists and since the limit points satisfy the complementarity condition $xs = 0$, the limit yields an optimal solution for LCP, see [6], [13].

B. The classic-Newton direction

In feasible IPM, we are given a positive feasible pair (x, s), and some $\mu > 0$. Our aim is to define search directions $(\triangle x, \triangle s)$ that move in the direction of the μ-center $(x(\mu), s(\mu))$. In fact, we want the new iterates $x + \triangle x$, $s + \triangle s$ to satisfy system (3) and be positive with respect to μ. After substitution this yields the following conditions on $(\triangle x, \triangle s)$

$$\begin{array}{rcl} M(x+\triangle x) + N(s+\triangle s) & = & q, \\ (x+\triangle x)(s+\triangle s) & = & \mu e, \\ x+\triangle x & > & 0, \\ s+\triangle s & > & 0. \end{array} \tag{4}$$

If we neglect for the moment the inequality constraints, then, since $Mx + Ns = q$, this system can be rewritten as follows

$$\begin{array}{rcl} M\triangle x + N\triangle s & = & 0, \\ s\triangle x + x\triangle s & = & \mu e - xs \end{array} \tag{5}$$

The unique solution of the system (5) is guaranteed by Lemma 4.1 in [13], and we obtain the so-called classic-Newton direction $\triangle x$ and $\triangle s$.

III. New search direction

To describe the ideas underlying this paper, we need to consider a scaled version of the system (5) that defines the search directions.

A. A scaled-Newton direction

Now we introduce the scaled vector v and the scaled search directions d_x and d_s according to

$$v = \sqrt{\frac{xs}{\mu}} \text{ and } d_x = \frac{v\Delta x}{x}, \ d_s = \frac{v\Delta s}{s}. \tag{6}$$

Following (6), the system (5) can be rewritten as

$$\begin{array}{rcl} \bar{M}d_x + \bar{N}d_s & = & 0, \\ d_s + d_x & = & v^{-1} - v. \end{array} \tag{7}$$

where

$$V = \text{diag}(v), \quad X = \text{diag}(x) \text{ and } S = \text{diag}(s). \tag{8}$$

$$\bar{M} = MV^{-1}X, \quad \bar{N} = NV^{-1}S. \tag{9}$$

The search directions d_x and d_s are obtained by solving (7), so $\triangle x$ and $\triangle s$ can be computed via (6).

B. A modified-Newton direction

Rearrange the second equation in (7), we obtain

$$v^2 + v(d_x + d_s) = e,$$

taking square root at both side the equation, one has

$$\left(v^2 + v(d_x + d_s)\right)^{\frac{1}{2}} = e.$$

Using Taylor series at v^2, which gives the following equation

$$v + \frac{1}{2}(d_x + d_s) = e, \tag{10}$$

rearrange the equation (10) and substitute the second equation in (7), one obtain the new Newton system

$$\begin{array}{rcl} \bar{M}d_x + \bar{N}d_s & = & 0, \\ d_s + d_x & = & 2(e - v). \end{array} \tag{11}$$

Once system (11) is solved, the $\triangle x$ and $\triangle s$ can be computed via (6) too.

It should be mentioned that the idea of equivalent algebraic transformation above was also proposed by [14] for LO case. There, the power transformation $\psi(t) = \sqrt{t}$ was focused on xs space.

Note that $d_x = d_s = 0$ if and only if $v = e$ and hence x and s satisfy $xs = \mu e$, which implies that x, s are on the μ-center $(x(\mu), s(\mu))$. Thus, we can use $\|e - v\|$ as a quantity to measure closeness to the pair of μ-centers. We therefore define

$$\sigma(x, s; \mu) = \sigma(v) = \|e - v\|, \tag{12}$$

where v is defined as (6).

C. Some more basic results

Let us introduce the notation

$$p_v = d_x + d_s, \ q_v = d_x - d_s,$$

then we have

$$d_x = \frac{p_v + q_v}{2}, \ d_s = \frac{p_v - q_v}{2} \text{ and } d_x d_s = \frac{p_v^2 - q_v^2}{4}. \tag{13}$$

We compare the norm of p_v and q_v by the following lemma.

Lemma 1: One has

$$\|q_v\| \leq \|p_v\|.$$

Proof:

By the monotonicity property, see (2), one has

$$M\triangle x + N\triangle s = 0 \Rightarrow \triangle x^T \triangle s \geq 0 \Leftrightarrow d_x^T d_s \geq 0. \quad (14)$$

Thus

$$\begin{aligned}\|q_v\|^2 &= e^T(d_x - d_s)^2 \\ &= e^T(d_x + d_s)^2 - 4d_x^T d_s \\ &= \|p_v\|^2 - 4d_x^T d_s \\ &\leq \|p_v\|^2,\end{aligned}$$

the result follows. ■

IV. Generic Primal-Dual IPMs for LCP

We investigate a full-Newton step algorithm using the modified-Newton direction. It is assumed that we are given a positive primal-dual pair $(x^0, s^0) > 0$ and $\mu^0 > 0$ such that (x^0, s^0) is close to the μ^0-center in the sense of the proximity measure $\sigma(x^0, s^0; \mu^0)$. In the algorithm $\triangle x$ and $\triangle s$ denote the modified-Newton step, as defined before.

Generic primal-dual IPMs for LCP

Input:
A threshold parameter $\tau > 0$;
an accuracy parameter $\varepsilon > 0$;
a fixed barrier update parameter $\theta, 0 < \theta < 1$;
a strictly feasible $(x^0; s^0)$ and $\mu^0 = (x^0)^T s^0/n$
such that $\sigma(x^0, s^0; \mu^0) \leq \tau$.
begin
$x := x^0; s := s^0; \mu := \mu^0;$
while $x^T s \geq \varepsilon$ do
begin
$x := x + \Delta x;$
$s := s + \Delta s;$
$\mu := (1 - \theta)\mu;$
end
end

The most import matter in the algorithm is how to choose the parameters that control the algorithm, i.e., the threshold parameter τ, the barrier update parameter θ so as to minimize the iteration complexity.

V. Complexity analysis

In this section, we derive the complexity bound for the IPM based on the modified-Newton direction.

A. Feasibility condition

Let $x^+ = x + \triangle x$ and $s^+ = s + \triangle s$. We want the new iterates be strictly positive, so we only have to concentrate on the sign of the vectors x^+ and s^+. We call the Newton step strictly feasible if x^+ and s^+ are positive. The main aim of this subsection is to find conditions for strict feasibility of the full-Newton step.

Lemma 2: If $\sigma(v) < 1$, then the iterates (x^+, s^+) are strictly feasible.

Proof: For each $0 \leq \alpha \leq 1$, let introduce the notation $x(\alpha) = x + \alpha\triangle x$ and $s(\alpha) = s + \alpha\triangle x$. Then we have

$$x(\alpha)s(\alpha) = xs + \alpha(s\triangle x + x\triangle s) + \alpha^2 \triangle x \triangle s,$$

by (6), we obtain

$$\frac{x(\alpha)s(\alpha)}{\mu} = v^2 + \alpha v(d_x + d_s) + \alpha^2 d_x d_s.$$

Furthermore, from (13) we get

$$\frac{x(\alpha)s(\alpha)}{\mu} = (1-\alpha)v^2 + \alpha(v^2 + vp_v) + \alpha^2 \left(\frac{p_v^2 - q_v^2}{4}\right).$$

Using the second equation of (11) we find that

$$\begin{aligned}v^2 + vp_v &= 2v - v^2 \\ &= e - (e - v)^2 \\ &= e - \frac{p_v^2}{4},\end{aligned}$$

and this relation leads to

$$\frac{x(\alpha)s(\alpha)}{\mu} = (1-\alpha)v^2 + \alpha\left(e - (1-\alpha)\frac{p_v^2}{4} - \alpha\frac{q_v^2}{4}\right). \quad (15)$$

Evidently, the inequality $x(\alpha)s(\alpha) > 0$ is satisfied if

$$\left\|(1-\alpha)\frac{p_v^2}{4} + \alpha\frac{q_v^2}{4}\right\|_\infty < 1.$$

Since

$$\begin{aligned}\left\|(1-\alpha)\frac{p_v^2}{4} + \alpha\frac{q_v^2}{4}\right\|_\infty &\leq (1-\alpha)\frac{\|p_v^2\|_\infty}{4} + \alpha\frac{\|q_v^2\|_\infty}{4} \\ &\leq (1-\alpha)\frac{\|p_v^2\|}{4} + \alpha\frac{\|q_v^2\|}{4} \quad (16)\end{aligned}$$

By Lemma 1, one has

$$\begin{aligned}(1-\alpha)\frac{\|p_v^2\|}{4} + \alpha\frac{\|q_v^2\|}{4} &\leq \frac{\|p_v^2\|}{4} \\ &\leq \frac{\|p_v\|^2}{4} \\ &= \sigma(v)^2 \\ &\leq 1 \quad (17)\end{aligned}$$

Hence, for each $0 \leq \alpha \leq 1$ we have $x(\alpha)s(\alpha) > 0$. Consequently, the sign is the continuous function of α, $x(\alpha)$ and $s(\alpha)$ remains the same for every $0 \leq \alpha \leq 1$. Hence $x(0) = x > 0$ and $s(0) = s > 0$ yields $x(1) = x^+ > 0$ and $s(1) = s^+ > 0$. This completes the proof. ■

B. Duality gap

In general the new iterates x^+ and s^+ do not coincide with μ-centers. But we have the surprising property that the duality gap less than the value at the μ-centers, where the duality gap equals $n\mu$.

Lemma 3: Let $x^+ = x + \triangle x$ and $s^+ = s + \triangle s$. Then we have

$$(x^+)^T s^+ \leq \mu n.$$

Proof: Observe that making the substitution $\alpha = 1$ in (15) that equation becomes

$$\frac{x^+ s^+}{\mu} = e - \frac{q_v^2}{4},$$

and using this equation we get

$$\begin{aligned}(x^+)^T s^+ &= e^T(x^+s^+)\\ &= \mu\left(e^Te - \tfrac{e^Tq_v^2}{4}\right)\\ &= \mu\left(n - \tfrac{\|q_v\|^2}{4}\right)\\ &\le \mu n.\end{aligned}$$

This implies the lemma. ■

C. Quadratic convergence

we first estimate the increase of the proximity after one full-Newton step.

Theorem 4: Let $\sigma^+ := \sigma(x^+, s^+; \mu)$ and $v^+ := \sqrt{\dfrac{x^+s^+}{\mu}}$. If $\sigma(v) \le 1$. Then

$$\sigma^+ \le \frac{\sigma^2}{1+\sqrt{1-\sigma^2}}.$$

Hence, the full-Newton step is quadratically convergent.

Proof: We deduce from Lemma 2 that the full-Newton step is strictly feasible, thus $x^+ > 0$ and $s^+ > 0$. Observe that making the substitution $\alpha = 1$ in (15) that equation becomes

$$\left(v^+\right)^2 = e - \frac{q_v^2}{4}. \tag{18}$$

Thus

$$\begin{aligned}v^+_{\min} &= \sqrt{1 - \frac{\|q_v^2\|_\infty}{4}}\\ &\ge \sqrt{1 - \frac{\|q_v\|^2}{4}}\\ &\ge \sqrt{1 - \frac{\|p_v\|^2}{4}}\\ &= \sqrt{1-\sigma^2},\end{aligned} \tag{19}$$

Furthermore, (18) and (19) lead to

$$\begin{aligned}\sigma(v^+) &= \left\|\frac{e-(v^+)^2}{e+v^+}\right\|\\ &\le \frac{1}{1+v^+_{\min}}\|e-(v^+)^2\|\\ &\le \frac{1}{1+\sqrt{1-\sigma^2}}\left\|\frac{q_v^2}{4}\right\|\\ &\le \frac{\sigma^2}{1+\sqrt{1-\sigma^2}},\end{aligned}$$

the last inequality follows from the fact that

$$\|q_v^2\| \le \|q_v\|^2 \le \|p_v\|^2.$$

Consequently, we have $\sigma(v^+) < \sigma^2$, and this implies the lemma. ■

Theorem 4 implies that after a full-step the proximity to the μ-center is small than the square of the proximity before the full-step. In other words, full-step is quadratically convergent. Moreover, the theorem defines a neighborhood of the μ-center where the quadratic convergence occurs, namely $\sigma(v) \le 1$. This result is extremely important. It implies that when the present iterate is close to the μ-center, then only a small number of full-steps brings us very close to the μ-center.

D. Proximity changes after one iteration

After the full-Newton step, a μ-update will arise the changes of μ-center. We assume that μ is reduced by the factor $(1-\theta)$ in each iteration.

Lemma 5: Let $\sigma = \sigma(x, s; \mu) < 1$ and $\mu^+ = (1-\theta)\mu$, where $0 < \theta < 1$. We have

$$\sigma(x^+, s^+; \mu^+) \le \frac{\theta\sqrt{n}+\sigma^2}{1-\theta+\sqrt{(1-\theta)(1-\sigma^2)}}$$

Proof: From (18) and (19) we deduce

$$\begin{aligned}&\sigma(x^+, s^+; \mu^+)\\ =\;& \left\|e - \sqrt{\frac{x^+s^+}{\mu^+}}\right\|\\ =\;& \frac{1}{\sqrt{1-\theta}}\left\|\sqrt{1-\theta}e - v^+\right\|\\ =\;& \frac{1}{\sqrt{1-\theta}}\left\|\frac{(1-\theta)e-(v^+)^2}{\sqrt{1-\theta}e+v^+}\right\|\\ \le\;& \frac{1}{\sqrt{1-\theta}(\sqrt{1-\theta}+\min(v^+))}\left\|-\theta e + \frac{q_v^2}{4}\right\|\\ \le\;& \frac{1}{1-\theta+\sqrt{(1-\theta)(1-\sigma^2)}}\left(\theta\sqrt{n}+\frac{\|q_v^2\|}{4}\right)\\ \le\;& \frac{\theta\sqrt{n}+\sigma^2}{1-\theta+\sqrt{(1-\theta)(1-\sigma^2)}},\end{aligned}$$

which completes the proof. ■

E. Fixing the parameter

We want to find an update parameter θ and a threshold parameter τ. Thus, after each iteration of the algorithm, the property

$$\sigma(x, s; \mu) \le \tau$$

is maintained, and hence the algorithm is well defined.

By Lemma 5, it suffices if

$$\frac{\theta\sqrt{n}+\sigma^2}{1-\theta+\sqrt{(1-\theta)(1-\sigma^2)}} \le \tau. \tag{20}$$

The left side of the inequality (20) is monotonically increasing according to σ, it certainly suffices if

$$\frac{\theta\sqrt{n}+\tau^2}{1-\theta+\sqrt{(1-\theta)(1-\tau^2)}} \le \tau. \tag{21}$$

At this state, if we set

$$\tau = \frac{1}{2} \tag{22}$$

and assume that $n \ge 4$, it suffices if

$$\theta = 1/2\sqrt{n} \tag{23}$$

that the inequality (21) certainly establish. Thus the full-Newton step interior-point algorithm well defined for LCP.

F. Complexity bound

In the previous subsections we have found that if at the start of an iteration the iterates satisfy $\sigma(v) \leq \tau$, then after a full step and an μ-update, the iterates satisfy $\sigma\left(x^+, s^+; \mu^+\right) \leq \tau$, where τ and θ as defined in (23) and (22).

Lemma 6: If the barrier parameter μ has the initial value μ^0 and is repeatedly multiplied by $1-\theta$, with $0<\theta<1$, then after at most

$$\left\lceil \frac{1}{\theta} \log \frac{n\mu^0}{\varepsilon} \right\rceil$$

iterations we have $x^T s \leq \varepsilon$.

Proof: At the initial point, one has $(x^0)^T s^0 = n\mu^0$, after one iterate, by Lemma 3, the duality gap

$$(x^1)^T s^1 \leq (1-\theta) n\mu^0,$$

thus, after k iterates, the duality gap satisfies

$$(x^k)^T s^k \leq (1-\theta)^k n\mu^0.$$

So, it suffices if

$$(1-\theta)^k n\mu^0 \leq \varepsilon,$$

taking logarithm gives

$$k \log(1-\theta) + \log n + \log \mu^0 \leq \log \varepsilon. \tag{24}$$

Since

$$\log(1-\theta) \leq -\theta.$$

It certainly suffices if

$$-k\theta + \log n + \log \mu^0 \leq \log \varepsilon,$$

this gives

$$k \geq \frac{1}{\theta} \log \frac{n\mu^0}{\varepsilon},$$

this completes the proof. ∎

The following theorem holds trivially.

Theorem 7: Setting $\tau = 1/2$ and $\theta = 1/2\sqrt{n}$, the initial duality gap is $(x^0)^T s^0 = n\mu^0$, the modified-full-Newton step primal-dual IPMs for LCP has the complexity bound

$$\mathcal{O}\left(2\sqrt{n} \log \frac{n\mu^0}{\varepsilon}\right).$$

Proof: Substitution (23) in Lemma 6, the result follows. ∎

VI. A SIMPLE NUMERICAL EXPERIMENT

In general, though there exists $(x^0; s^0) > 0$ for the LCP problem is strictly feasible, we don't know the value of $(x^0; s^0)$. Thus we should modify the system (5) as follows

$$\begin{aligned} M\triangle x + N\triangle s &= q - Mx - Ns, \\ s\triangle x + x\triangle s &= 2\left(\mu x s^{\frac{1}{2}} - xs\right). \end{aligned} \tag{25}$$

We consider the following example:

$$M = \begin{pmatrix} 0.0368 & 0.0188 & 0.0920 & 0.0211 & 0.0332 & 0.0162 \\ 0.0188 & 0.0393 & 0.0634 & 0.0176 & 0.0300 & 0.0248 \\ 0.0920 & 0.0634 & 0.4293 & 0.0617 & 0.1355 & 0.1124 \\ 0.0211 & 0.0176 & 0.0617 & 0.0203 & 0.0239 & 0.0107 \\ 0.0332 & 0.0300 & 0.1355 & 0.0239 & 0.0513 & 0.0480 \\ 0.0162 & 0.1248 & 0.0124 & 0.0107 & 0.0480 & 0.0824 \end{pmatrix},$$

$$q = (0.1630, -0.2820, 0.4500, -0.3560, 0.2420, -0.2489)^T$$

and $N = -E$.

Without loss of generality, we choose $x^0 = s^0 = e$ as the initial point. Setting $\epsilon = 10^{-8}$, $\tau = \frac{1}{2}$ and $\theta = \frac{1}{2\sqrt{n}}$.

After 90 iterates, an optimal solution of the example is given by

$$x^* = (0.4169 \;\; 0.0000 \;\; 0.0000 \;\; 0.0000 \;\; 4.4476 \;\; 0.0000)^T$$

and

$$s^* = (0.0000 \;\; 0.4233 \;\; 0.1910 \;\; 0.4711 \;\; 0.0000 \;\; 0.4691)^T.$$

VII. CONCLUSION

In this paper, we gave a full-Newton step IPM for LCP, the full-Newton step has the quadratically convergent property in the small neighborhood of central path. The complexity bound is the best-known results for LCP.

Although the full-Newton step IPM based on the new search direction admits the best-known iteration bound, however, from a practical perspective it may be not so efficient. The reason may come from the finite barrier property of the equation $d_x + d_s = 2(e-v)$, i.e., at the boundary of the feasible solution set, where the elements of v equal to zero, one obtains that $d_x + d_s = 2e$, which implies that it difficult to design the large-update algorithm based on this searching direction.

Our further research may focus on designing the infeasible algorithm based on this modified-Newton direction.

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