A New Fuzzy Risk Analysis Method based on Generalized Fuzzy Numbers

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***Abstract*—Risk analysis plays an important role in many application systems. The current researches prefer to use fuzzy set theory for risk analysis. In this paper, we present a new fuzzy risk analysis method based on generalized fuzzy numbers. Firstly, we define new arithmetic operations between generalized fuzzy numbers. Then, we propose a new method to measure the degree of similarity between generalized fuzzy numbers. Finally, we apply the new arithmetic operations between generalized fuzzy numbers and proposed similarity measure to develop a new method to deal with fuzzy risk analysis problems. The greatest advantage of the new method is that it has less computational complexity. When dealing with the risk analysis problems, the predominance of new method has been showed: easier and more useful.**

***Index Terms*—risk analysis, generalized fuzzy numbers, fuzzy arithmetic, similarity measures**

I. INTRODUCTION

Today, in industry, government and research institutions, more and more experts and managers spend a lot of time and effort improving their understanding of risk analysis to make correct decision [1]. Fuzzy arithmetic and domain experts' knowledge are widely used in risk analysis of complex systems. When information about risk is captured in natural language, the words are termed linguistic variables. Thus, instead of numerical values, a more realistic approach may be to use linguistic assessments, which can be analyzed using fuzzy set theory [2]. Reference [3-5] show that some researches have been done in construction of risk assessment . Since Kangari and Riggs point out linking fuzzy set theory with risk analysis is one promising area in 1989[4], some methods have been presented to calculate the degree of similarity between fuzzy numbers:

The method proposed by Kangari and Riggs in reference [4] is not efficient due to the amount of computation time required for performing the complicated fuzzy number arithmetic operations and time for performing linguistic approximation. Chen (1996) [6] has presented a more efficient fuzzy risk analysis method. In reference [7], Chen and Chen (2003) pointed out the drawbacks in the similarity measure used in reference [6] and proposed a new method to determine the degree of similarity between generalized fuzzy numbers and developed a new method to deal with the fuzzy risk analysis problem. However, Chen and Chen (2003)'s method still has two shortcomings: It cannot correctly determine the degree of similarity between generalized fuzzy numbers in some situations [2]. In addition, the center-of-gravity-based similarity measure in it cannot determine the degree of similarity when the generalized fuzzy numbers are not in [0, 1]. Thus, Liu (2009) [2]

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present a new similarity measure of generalized fuzzy numbers to solve the problems.

In this paper, we present a new fuzzy risk analysis method based on generalized fuzzy numbers. It combines the concepts of new definition of arithmetic operations and new similarity measurement between generalized fuzzy numbers. The greatest advantage of the new method is that it has less computational complexity. When dealing with the risk analysis problems, the predominance of new method has been showed: easier and more useful.

The rest of this paper is organized as follows. In Section II, we briefly review the definitions of generalized fuzzy numbers. In Section III, we present a new fuzzy risk analysis method based on generalized fuzzy numbers. In Section IV, we use the proposed similarity measure of generalized fuzzy numbers to deal with the fuzzy risk analysis problems. The conclusions are discussed in Section V.

II. GERNERALIZED FUZZY NUMBERS

A. Basic Concept of Generalized Fuzzy Numbers

Let X be a universe of discourse, $\tilde{A}$ is a fuzzy subset of X if for all $x \in X$, there is a number $u_{\tilde{A}}(x) \in [0,1]$ assigned to represent the membership of x to $\tilde{A}$, and $u_{\tilde{A}}(x)$ is called the membership function of $\tilde{A}$. A fuzzy number $\tilde{A}$ is a normal and convex fuzzy subset of X. Here, the "Normality" implies that

$$\exists x \in \mathbb{R}, \quad \vee_x u_{\tilde{A}}(x) = 1$$

and "Convex" means that

$$\forall x_1 \in X, x_2 \in X, \quad \forall \alpha \in [0,1]$$

$$u_{\tilde{A}}(\alpha x_1 + (1-\alpha)x_2) \geq \min(u_{\tilde{A}}(x_1), u_{\tilde{A}}(x_2))$$

In reference [8], Chen represented a generalized trapezoidal fuzzy numbers $\tilde{A}$ as $\tilde{A} = (a,b,c,d;w)$, where $0 \leq w \leq 1$, and a,b,c and d are real numbers. The generalized trapezoidal fuzzy number $\tilde{A}$ is a fuzzy subset of the real line $\mathbb{R}$, whose membership function $u_{\tilde{A}}$ satisfies the following conditions:

(1) $u_{\tilde{A}}(x)$ is a continuous mapping from $\mathbb{R}$ to the closed interval $[0,w]$, $0 \leq w \leq 1$;

(2) $u_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a]$;

(3) $u_{\tilde{A}}(x)$ is strictly increasing on $[a,b]$;

(4) $u_{\tilde{A}}(x) = w$, for all $x \in [b,c]$, where w is a constant and $0 \leq w \leq 1$;

(5) $u_{\tilde{A}}(x)$ is strictly decreasing on $[c,d]$;

(6) $u_{\tilde{A}}(x) = 0$ for all $x \in [d, +\infty)$;

If $w = 1$, then the generalized fuzzy number $\tilde{A}$ is called a normal trapezoidal fuzzy number denoted $\tilde{A} = (a,b,c,d;w)$. If $a = b$ and $c = d$, then $\tilde{A}$ is called a crisp interval. If $b = c$, then $\tilde{A}$ is called a generalized triangular fuzzy number. If $a = b = c = d$, then $\tilde{A}$ is called a real number.

Fig. 1 shows two different generalized trapezoidal fuzzy numbers $\tilde{A} = (0.1, 0.2, 0.3, 0.4; 1.0)$, $\tilde{B} = (0.1, 0.2, 0.3, 0.4; 0.8)$. Compared with normal fuzzy numbers, the generalized

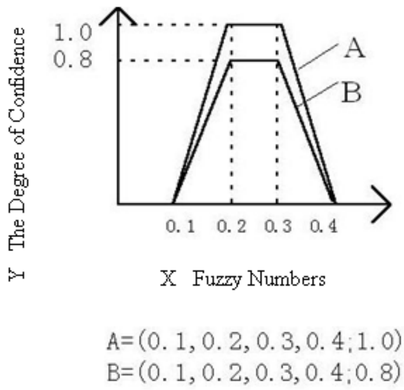

Figure 1. Note how the caption is centered in the column.

fuzzy numbers can deal with uncertain information in a more flexible manner. For example, in decision making situation, the values w_1 and w_2 represent the degree of confidence of the opinions of the decision-makers' $\tilde{A}$ and $\tilde{B}$, respectively, where $w_1 = 1.0$ and $w_2 = 0.8$.

B. The Existing Arithmetic Operations Between Generalized Fuzzy Numbers

Assume that there are two generalized trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)$ and $\tilde{B} = (b_1, b_2, b_3, b_4; w_B)$. The arithmetic operations between the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are as follows [7] [9] [10]:

Generalized fuzzy numbers addition $\oplus$:

$$\begin{aligned}\tilde{A} \oplus \tilde{B} &= (a_1, a_2, a_3, a_4; w_A) \oplus (b_1, b_2, b_3, b_4; w_B) \\ &= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; \min(w_A, w_B))\end{aligned} \quad (1)$$

Generalized fuzzy numbers subtraction $\odot$:

$$\begin{aligned}\tilde{A} \odot \tilde{B} &= (a_1, a_2, a_3, a_4; w_A) \odot (b_1, b_2, b_3, b_4; w_B) \\ &= (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1; \min(w_A, w_B))\end{aligned} \quad (2)$$

Generalized fuzzy numbers multiplication $\otimes$:

$$\begin{aligned}\tilde{A} \otimes \tilde{B} &= (a_1, a_2, a_3, a_4; w_A) \otimes (b_1, b_2, b_3, b_4; w_B) \\ &= (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4; \min(w_A, w_B))\end{aligned} \quad (3)$$

Generalized fuzzy numbers division $\varnothing$:

$$\tilde{A}\varnothing\tilde{B} = (a_1, a_2, a_3, a_4; w_A)\varnothing(b_1, b_2, b_3, b_4; w_B) \\ = (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1; \min(w_A, w_B)). \tag{4}$$

Liu [2] defined a new arithmetic operation called ratio (2009) in order to solve the problem that the generalized fuzzy numbers division by equation (4) is not always in [0, 1]. The generalized fuzzy numbers ratio is defined as follows:

$$\tilde{A}\varnothing\tilde{B} = (a_1, a_2, a_3, a_4; w_A)\varnothing(b_1, b_2, b_3, b_4; w_B) \\ = (a_1/b_1, a_2/b_2, a_3/b_3, a_4/b_4; \min(w_A, w_B)) \tag{5}$$

C. The Existing Similarity Measures Between Fuzzy Numbers

Assume that there are two trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$. Then, the degree of similarity $S(\tilde{A}, \tilde{B})$ between the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is calculated as follows [11]:

$$S(\tilde{A}, \tilde{B}) = \frac{\int_x (\min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\})dx}{\int_x (\max\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\})dx}. \tag{6}$$

The larger the value of $S(\tilde{A}, \tilde{B})$, the more the similarity measure between fuzzy numbers $\tilde{A}$ and $\tilde{B}$ .Because of the drawbacks of long computing time and the requirement that the fuzzy numbers should have a common intersection at some α -level cut, where $\alpha \in [0,1]$, this method is not widely used.

Chen (1998) [6] presented a similarity measure of two trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ as follows:

$$S(\tilde{A}, \tilde{B}) = 1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}. \tag{7}$$

Where $S(\tilde{A}, \tilde{B}) \in [0,1]$.

If $\tilde{A}$ and $\tilde{B}$ are two triangular fuzzy numbers, where $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$, then the degree of similarity $S(\tilde{A}, \tilde{B})$ between the triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be calculated as follows:

$$S(\tilde{A}, \tilde{B}) = 1 - \frac{\sum_{i=1}^{3} |a_i - b_i|}{3}. \tag{8}$$

The larger the value of $S(\tilde{A}, \tilde{B})$, the more the similarity measure between fuzzy numbers $\tilde{A}$ and $\tilde{B}$.

In [10], Hsieh and Chen proposed a similarity measure using the "graded mean integration representation distance", where the degree of similarity $S(\tilde{A}, \tilde{B})$ between fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be calculated as follows:

$$S(\tilde{A}, \tilde{B}) = \frac{1}{d(\tilde{A}, \tilde{B})}. \tag{9}$$

where $d(\tilde{A}, \tilde{B}) = \left|P(\tilde{A}) - P(\tilde{B})\right|$. $P(\tilde{A})$ and $P(\tilde{B})$ are the graded mean integration representations of $\tilde{A}$ and $\tilde{B}$. Further details of the graded mean integration representations are given in section III.

The larger the value of $S(\tilde{A}, \tilde{B})$, the more the similarity measure between fuzzy numbers $\tilde{A}$ and $\tilde{B}$.

In [12], Lee proposed a similarity measure between two trapezoidal fuzzy numbers, where the degree of similarity $S(\tilde{A}, \tilde{B})$ between fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be calculated as follows:

$$S(A, B) = 1 - \frac{\left\|\tilde{A} - \tilde{B}\right\|_{l_p}}{\|U\|} \times 4^{-1/p}. \tag{10}$$

where U is the universe of discourse,

$$\left\|\tilde{A} - \tilde{B}\right\|_{l_p} = \left(\sum_{i=1}^{4} \left(|a_i - b_i|\right)^p\right)^{1/p}. \tag{11}$$

and

$$\|U\| = \max(U) - \min(U). \tag{12}$$

The larger the value of $S(\tilde{A}, \tilde{B})$, the more the similarity measure between fuzzy numbers $\tilde{A}$ and $\tilde{B}$.

In [7], Chen and Chen proposed a similarity measure between two generalized fuzzy numbers using the centre-of-gravity (COG) points.

Assume that there are two generalized trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)$ and $\tilde{B} = (b_1, b_2, b_3, b_4; w_B)$. $0 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$, and $0 \le b_1 \le b_2 \le b_3 \le b_4 \le 1$. The COG points COG (A) and COG (B) of $\tilde{A}$ and $\tilde{B}$ can be denoted as (x_A^*, y_A^*) and (x_B^*, y_B^*), respectively. Then the degree of similarity $S(\tilde{A}, \tilde{B})$ between the generalized triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be calculated as follows [7]:

$$S(\tilde{A}, \tilde{B}) = \left[1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}\right] \times \left(1 - \left|x_{\tilde{A}}^* - x_{\tilde{B}}^*\right|\right)^{B(S_{\tilde{A}}, S_{\tilde{B}})} \times \frac{\min(y_{\tilde{A}}^*, y_{\tilde{B}}^*)}{\max(y_{\tilde{A}}^*, y_{\tilde{B}}^*)}. \tag{13}$$

where $B(S_{\tilde{A}}, S_{\tilde{B}})$ are defined as follows:

$$B(S_{\tilde{A}}, S_{\tilde{B}}) = \begin{cases} 1 & S_{\tilde{A}} + S_{\tilde{B}} > 0 \\ 0 & S_{\tilde{A}} + S_{\tilde{B}} = 0 \end{cases}. \tag{14}$$

where $S_{\tilde{A}}$ and $S_{\tilde{B}}$ are the lengths of the based of the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ respectively , defined as follows:

$$\begin{aligned} S_{\tilde{A}} &= a_4 - a_1 \\ S_{\tilde{B}} &= b_4 - b_1 \end{aligned} . \tag{15}$$

The similarity measure in [7] is proved to successfully determine the similarity measure of generalized fuzzy numbers in most situations. However, it cannot correctly deal with the situations when two different generalized fuzzy numbers have the same COG point. Thus, in [13], Deng proposed a similarity measure between two generalized trapezoidal fuzzy numbers based on radius-of-gyration ROG point to solve this problem:

Assume that there are two generalized trapezoidal fuzzy numbers $\tilde{A}=(a_1,a_2,a_3,a_4;w_A)$ and $\tilde{B}=(b_1,b_2,b_3,b_4;w_B)$. $0 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$, and $0 \le b_1 \le b_2 \le b_3 \le b_4 \le 1$. The ROG (A) and ROG (B) of $\tilde{A}$ and $\tilde{B}$ can be expressed as $(r_x^{\tilde{A}}, r_y^{\tilde{A}})$ and $(r_x^{\tilde{B}}, r_y^{\tilde{B}})$, respectively. Then the degree of similarity $S(\tilde{A},\tilde{B})$ between the generalized triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be calculated as follows [13]:

$$S(\tilde{A},\tilde{B}) = \left[1 - \frac{\sum_{i=1}^{4}|a_i - b_i|}{4}\right] \times \left(1 - \left|r_y^{\tilde{A}} - r_y^{\tilde{B}}\right|\right)^{B(S_{\tilde{A}},S_{\tilde{B}})} \times \frac{\min(r_x^{\tilde{A}}, r_x^{\tilde{B}})}{\max(r_x^{\tilde{A}}, r_x^{\tilde{B}})} . \tag{16}$$

where $B(S_{\tilde{A}}, S_{\tilde{B}})$ are defined as equation (14) and $S_{\tilde{A}}$ and $S_{\tilde{B}}$ are the lengths of the based of the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ respectively , defined as equation (15).

In [14], Shih-Hua Wei proposed a similarity measure between two generalized trapezoidal fuzzy numbers by combining the concepts of geometric distance, the perimeter and the height of generalized fuzzy numbers.

Assume that there are two generalized trapezoidal fuzzy numbers $\tilde{A}=(a_1,a_2,a_3,a_4;w_A)$ and $\tilde{B}=(b_1,b_2,b_3,b_4;w_B)$. $0 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$, and $0 \le b_1 \le b_2 \le b_3 \le b_4 \le 1$. Then the degree of similarity $S(\tilde{A},\tilde{B})$ between the generalized triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be calculated as follows [14]:

$$S(\tilde{A},\tilde{B}) = \left[1 - \frac{\sum_{i=1}^{4}|a_i - b_i|}{4}\right] \times \frac{\min(q(\tilde{A}), q(\tilde{B})) + \min(w_{\tilde{A}}, w_{\tilde{B}})}{\max(q(\tilde{A}), q(\tilde{B})) + \max(w_{\tilde{A}}, w_{\tilde{B}})} . \tag{17}$$

where $q(\tilde{A})$ and $q(\tilde{B})$ are defined as follows:

$$\begin{aligned} q(\tilde{A}) &= \sqrt{(a_1-a_2)^2 + w_{\tilde{A}}^2} + \sqrt{(a_3-a_4)^2 + w_{\tilde{A}}^2} \\ &\quad +(a_3-a_2)+(a_4-a_1) , \\ q(\tilde{B}) &= \sqrt{(b_1-b_2)^2 + w_{\tilde{B}}^2} + \sqrt{(b_3-b_4)^2 + w_{\tilde{B}}^2} \\ &\quad +(b_3-b_2)+(b_4-b_1) , \end{aligned} . \tag{18}$$

$q(\tilde{A})$ and $q(\tilde{B})$ denote the perimeters of the generalized trapezoidal fuzzy numbers.

III. A New Fuzzy Risk Analysis Method

A. New Arithmetic Operations Between Generalized Fuzzy Numbers

In this section, we briefly review basic concepts of arithmetic operations between generalized fuzzy numbers presented by CH.-CH. CHOU [15]. According to [15], a generalized fuzzy number $\tilde{A} = (a,b,c,d;w)$ is generally a trapezoidal fuzzy number. If $b = c$, then $\tilde{A}$ becomes a triangular fuzzy number. The graded mean integration is a effective representation of fuzzy numbers. In [15], CH.-CH. CHOU give the meaning of the graded mean integration which given by Chen and Hsieh [16], and propose a new arithmetical principle and a new arithmetical method for the arithmetical operations on fuzzy numbers.

Assume that there are two generalized trapezoidal fuzzy numbers $\tilde{A}=(a_1,a_2,a_3,a_4;w_A)$ and $\tilde{B}=(b_1,b_2,b_3,b_4;w_B)$. According to [16], the graded mean integration of $\tilde{A}$ and $\tilde{B}$ is as follows:

$$P(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6} . \tag{19}$$

$$P(\tilde{B}) = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6} . \tag{20}$$

If $\tilde{A}$ and $\tilde{B}$ are triangular fuzzy numbers, where $\tilde{A} = (a_1, a_2, a_3; w_A)$ and $\tilde{B} = (b_1, b_2, b_3; w_B)$, then the graded mean integration representations $P(\tilde{A})$ and $P(\tilde{B})$ of $\tilde{A}$ and $\tilde{B}$, respectively, are defined as follows:

$$P(\tilde{A}) = \frac{a_1 + 4a_2 + a_3}{6} . \tag{21}$$

$$P(\tilde{B}) = \frac{b_1 + 4b_2 + b_3}{6} . \tag{22}$$

The arithmetic operations between the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ based on the graded mean integration are reviewed from CH.-CH. CHOU [15] as follows:

$$P(\tilde{A} \oplus \tilde{B}) = P(\tilde{A}) + P(\tilde{B}) . \tag{23}$$

$$P(\tilde{A} \otimes \tilde{B}) = P(\tilde{A}) \cdot P(\tilde{B}) . \tag{24}$$

From equation (19)-(24), we can see that w is not taken into consideration. Thus, we present new arithmetic operations between generalized fuzzy numbers as follows:

Assume that there are two generalized trapezoidal fuzzy numbers $\tilde{A}=(a_1,a_2,a_3,a_4;w_A)$ and $\tilde{B}=(b_1,b_2,b_3,b_4;w_B)$. Representing $(a_1,a_2,a_3,a_4;w_A)$ with $(P(\tilde{A});w_A)$, we can rewrite $\tilde{A}$ as

$$\tilde{A}=(P(\tilde{A});w_A). \quad (25)$$

Similarly,

$$\tilde{B}=(P(\tilde{B});w_B). \quad (26)$$

1) Generalized fuzzy numbers addition $\oplus$ *:*

$$\begin{aligned}\tilde{A}\oplus\tilde{B}&=(P(\tilde{A});w_A)\oplus(P(\tilde{B});w_B)\\&=(P(\tilde{A})+P(\tilde{B});\frac{w_A+w_B}{2})\end{aligned}. \quad (27)$$

2) Generalized fuzzy numbers subtraction $\odot$ *:*

$$\begin{aligned}\tilde{A}\odot\tilde{B}&=(P(\tilde{A});w_A)\odot(P(\tilde{B});w_B)\\&=(P(\tilde{A})-P(\tilde{B});\frac{w_A+w_B}{2})\end{aligned}. \quad (28)$$

3) Generalized fuzzy numbers multiplication $\otimes$ *:*

$$\begin{aligned}\tilde{A}\otimes\tilde{B}&=(P(\tilde{A});w_A)\otimes(P(\tilde{B});w_B)\\&=(P(\tilde{A})\cdot P(\tilde{B});\sqrt{w_A\cdot w_B})\end{aligned}. \quad (29)$$

4) Generalized fuzzy numbers division $\varnothing$ *:*

$$\begin{aligned}\tilde{A}\varnothing\tilde{B}&=(P(\tilde{A});w_A)\varnothing(P(\tilde{B});w_B)\\&=(\frac{P(\tilde{A})}{P(\tilde{B})};\frac{\min(w_A,w_B)}{\max(w_A,w_B)})\end{aligned}. \quad (30)$$

B. A New Similarity Measure Between Fuzzy Numbers

Assume that there are two generalized trapezoidal fuzzy numbers $\tilde{A}=(a_1,a_2,a_3,a_4;w_A)$ and $\tilde{B}=(b_1,b_2,b_3,b_4;w_B)$. The graded mean integration of $\tilde{A}$ is $P(\tilde{A})$ and the graded mean integration of $\tilde{B}$ is $P(\tilde{B})$. Then, the degree of similarity $S(\tilde{A},\tilde{B})$ between the generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is calculated as follows:

$$S(\tilde{A},\tilde{B})=\frac{1}{1+\left|P(\tilde{A})\cdot w_A-P(\tilde{B})\cdot w_B\right|}. \quad (31)$$

The proposed similarity measure has the following properties:

(1) $S(\tilde{A},\tilde{B})=1$ if two generalized trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are identical.

(2) $S(\tilde{A},\tilde{B})=S(\tilde{B},\tilde{A})$;

IV. Fuzzy Risk Analysis Based On the Proposed Similarity Measure

In this section, we use the proposed similarity measure of generalized fuzzy numbers to deal with the fuzzy risk analysis problems. Let us consider the structure of risk analysis shown in Fig.2. According to [7, 17], each subcomponent A_i is evaluated by two evaluating items, i.e. $\tilde{R}_i$ and $\tilde{W}_i$, where $\tilde{R}_i$ denotes the probability of failure of the subcomponent A_i , $\tilde{W}_i$ denotes the severity of loss of the subcomponent A_i , and $1\le i\le 3$. According to[17], all of the evaluating items are linguistic terms (i.e. "high", "medium", "low"…,etc.) to represent the values

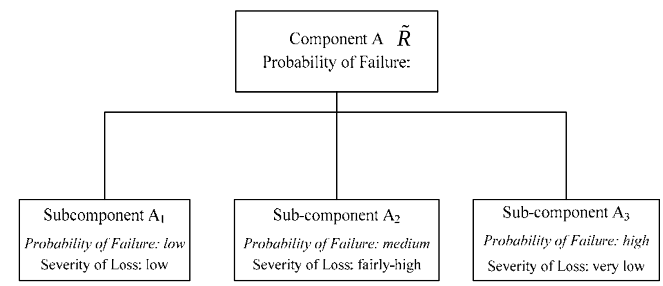

Figure 2. Structure of fuzzy risk analysis[17]

of $\tilde{R}_i$ and $\tilde{W}_i$. Table 1 illustrates the linguistic terms and their corresponding generalized trapezoidal fuzzy numbers. Assume that there is a component A consisting of n subcomponents $A_1, A_2,\cdots A_n$, and assume that each subcomponent is evaluated by two evaluating items "probability of failure" $\tilde{R}_i$ and "severity of loss" $\tilde{W}_i$. The fuzzy risk analysis algorithm is presented as follows [7].

Step1: Use the fuzzy weighted mean method and the generalized fuzzy number arithmetic operations to integrate the evaluating items $\tilde{R}_i$ and $\tilde{W}_i$ of each subcomponent A_i , where $1\le i\le n$, to get the total risk $\tilde{R}$ of the component A shown as follows:

TABLE I.
A NINE-MEMBER LINGUISTIC TERM SET

Linguistic Terms	**Generalized Fuzzy Numbers**
absolutely low	(0.0,0.0,0.0,0.0;1.0)
very low	(0.0,0.0,0.02,0.07;1.0)
low	(0.04,0.1,0.18,0.23;1.0)
fairly low	(0.17,022,0.36,0.42;1.0)
medium	(0.32,0.41,0.58,0.65;1.0)
fairly high	(0.58,0.63,0.80,0.86;1.0)
high	(0.72,0.78,0.92,0.97;1.0)
very high	(0.93,0.98,1.0,1.0;1.0)
absolutely high	(1.0,1.0,1.0,1.0;1.0)

$$\widetilde{R} = \left(\sum_{i=1}^{n} \widetilde{W_i} \otimes \widetilde{R_i} \right) \varnothing \left(\sum_{i=1}^{n} \widetilde{W_i} \right). \quad (32)$$

Step 2: Use the proposed similarity measure in section III to evaluate the degree of similarity between the fuzzy number $\tilde{R}$ and each linguistic term shown in Table I. Translate the fuzzy number $\tilde{R}$ into a linguistic term, which has the largest degree of similarity.

According to equation (19), (20), (25), (26), the generalized fuzzy numbers can also be expressed as is shown in Table II.

TABLE II.
A NINE-MEMBER LINGUISTIC TERM SET EXPRESSED BY THE PROPOSED METHOD

Linguistic Terms	Generalized Fuzzy Numbers
absolutely low	(0;1.0)
very low	(0.0183;1.0)
low	(0.138;1.0)
fairly low	(0.2917;1.0)
medium	(0.4917;1.0)
fairly high	(0.7166;1.0)
high	(0.8480;1.0)
very high	(0.9816;1.0)
absolutely high	(1.0;1.0)

In the following, we use two examples [17] to show how to deal with the fuzzy risk analysis problem by the proposed fuzzy risk analysis method.

A. Example 1

Consider the structure of risk analysis shown in Fig.2, where the component A consists of three subcomponents A_1, A_2 and A_3 , and we want to evaluate the probability of failure $\tilde{R}$ of the component A .Table III shows the linguistic values of the two evaluating items $\tilde{R}_i$ and $\tilde{W}_i$ of the subcomponents A_1, A_2 and A_3 , respectively, where the linguistic values are represented by generalized trapezoidal fuzzy numbers as shown in Table II.

Based on (19), (20), (27), (29), (30), (32), Table I, Table II, the probability of failure of the component can be evaluated as follows:

$$\begin{aligned}
\widetilde{R} &= \left(\sum_{i=1}^{n} \widetilde{W_i} \otimes \widetilde{R_i} \right) \varnothing \left(\sum_{i=1}^{n} \widetilde{W_i} \right) \\
&= [(0.138;1.0) \otimes (0.138;1.0) \\
&\quad \oplus (0.492;1.0) \otimes (0.717;1.0) \\
&\quad \oplus (0.848;1.0) \otimes (0.0183;1.0)] \\
&\quad \varnothing [(0.138;1.0) \oplus (0.717;1.0) \\
&\quad \oplus (0.183;1.0)] \\
&= (0.4456;1.0)
\end{aligned}$$

According to the proposed similarity measure in equation (31), we can calculate degree of similarity between the fuzzy number $\tilde{R} = (0.4456;1.0)$ which represent the probability of failure and the linguistic item "low" which is $(0.138;1.0)$ according to Table II:

$$\begin{aligned}
S(\widetilde{R}, low) &= \frac{1}{1 + \left| P(R) \cdot w_R - P(low) \cdot w_{low} \right|} \\
&= \frac{1}{1 + \left| 0.4456 \cdot 1 - 0.138 \cdot 1 \right|} \\
&= 0.7648
\end{aligned}$$

In this way, the degrees of similarity between the generalized trapezoidal fuzzy number $\tilde{R}$ and each linguistic term shown in Table II can be evaluated as follows: $S(\widetilde{R}, absolutely\ low) = 0.6981$, $S(\widetilde{R}, very\ low) = 0.7006$, $S(\widetilde{R}, low) = 0.7648$, $S(\widetilde{R}, fairly\ low) = 0.8666$, $S(\widetilde{R}, medium) = 0.9559$, $S(\widetilde{R}, fairly\ high) = 0.7867$, $S(\widetilde{R}, high) = 0.7131$, $S(\widetilde{R}, very\ high) = 0.6510$, $S(\widetilde{R}, absolutely\ high) = 0.6433$. It is obvious that the fuzzy number $\tilde{R} = (0.4456;1.0)$ is much close to the linguistic term "medium". In another word, the probability of failure of the component A is medium. This result coincides with the one presented in [17].

To show the efficiency of our method, a comparision with Chen and Chen's COG method(g), Deng's ROG method(p) and Wei's method(1) is made. The results are illustrated in Table III.

As can be seen from Table III.The result coincides with the ones presented in Chen and Chen's COG method(g), Deng's ROG method(p) and Wei's method(1).

TABLE III.
THE RESULT OF THE PROPOSED METHOD COMPARED WITH CHEN AND CHEN'S METHOD

Linguistic terms	Chen and Chen's COG method	Deng's ROG method	Wei's method	The proposed method
absolutely low	0.1565	0.2821	0.3235	0.6981
very low	0.1962	0.3127	0.3494	0.7006
low	0.3226	0.4704	0.4571	0.7648
fairly low	0.5092	0.7157	0.5921	0.8666
medium	0.7056	0.9072	0.6538	0.9559
fairly high	0.5828	0.5160	0.5960	0.7867
high	0.4545	0.3525	0.5267	0.7131
very high	0.2937	0.2038	0.3977	0.6510
absolutely high	0.2391	0.1818	0.3703	0.6433

B. Example 2

This example takes the degree of confidence of the decision-maker's opinions into consideration (example

5.2 in Chen and Chen 2003[7]). Consider the structure of risk analysis shown in Fig.2, where the component A consists of three subcomponents A_1, A_2 and A_3, and we want to evaluate the probability of failure $\tilde{R}$ of the component A. Assume that there are three decision-makers $E1, E2, E3$, and to evaluate the probability of failure of the three subcomponents A_1, A_2 and A_3 as shown in Table IV, where the value w_{ij} denotes the degree of confidence that decision-maker evaluates the probability of failure $\tilde{R}_{ij}$ of subcomponent A_i, where

TABLE IV.
LINGUISTIC VALUES OF THE PROBABILITY OF FAILURE AND THE DEGREE OF CONFIDENCE OF THE THREE SUB-COMPONENTS EVALUATED BY THREE DECISION-MAKERS

Sub-component	**Decision-makers**		
	E_1	E_2	E_3
A_1	$\tilde{R}_{11} = low$ ($w_{11} = 1.0$)	$\tilde{R}_{12} = medium$ ($w_{12} = 0.8$)	$\tilde{R}_{13} = fairly-low$ ($w_{13} = 0.9$)
A_2	$\tilde{R}_{21} = fairly-high$ ($w_{21} = 0.6$)	$\tilde{R}_{22} = fairly-low$ ($w_{22} = 0.7$)	$\tilde{R}_{23} = medium$ ($w_{23} = 0.8$)
A_3	$\tilde{R}_{31} = medium$ ($w_{31} = 0.7$)	$\tilde{R}_{32} = high$ ($w_{32} = 0.9$)	$\tilde{R}_{33} = high$ ($w_{33} = 0.8$)

$1 \le i \le 3$ and $1 \le j \le 3$.

According to [7], the average probability of failure $\tilde{R}_i = (a_i, b_i, c_i, d_i; w_i)$ of sub-component A_i is calculated as follows:

$$\tilde{R}_i = \frac{\sum_{j=1}^{3} \tilde{R}_{ij}}{3} = \left(\frac{\sum_{j=1}^{3} a_{ij}}{3}, \frac{\sum_{j=1}^{3} b_{ij}}{3}, \frac{\sum_{j=1}^{3} c_{ij}}{3}, \frac{\sum_{j=1}^{3} d_{ij}}{3}; \frac{\sum_{j=1}^{3} w_{ij}}{3}\right) \quad (33)$$

Based on Table IV and (19), (25), (33), we can get

$\tilde{R}_1 = (0.3072; 0.9)$;

$\tilde{R}_2 = (0.5; 0.7)$;

$\tilde{R}_3 = (0.7295; 0.8)$

The probability of failure $\tilde{R}$ of component A can be calculated based on (19), (20), (27), (29), (30), (32) and Table II:

$$\begin{aligned}\tilde{R} &= \left(\sum_{i=1}^{3} \tilde{W}_i \otimes \tilde{R}_i\right) \varnothing \left(\sum_{i=1}^{3} \tilde{W}_i\right) \\ &= [(0.3072;0.9) \otimes (0.138;1.0) \\ &\quad \oplus (0.5;0.7) \otimes (0.7166;1.0) \\ &\quad \oplus (0.7295;0.8) \otimes (0.0183;1.0)] \\ &\quad \varnothing [(0.138;1.0) \oplus (0.7166;1.0) \\ &\quad \oplus (0.0183;1.0)] \\ &= (0.4743;0.8933)\end{aligned}$$

Similar to the above example, we can calculate the degree of similarity between the fuzzy number $\tilde{R} = (0.4743;0,8933)$ and the linguistic items in Table II. The results are illustrated in Table V. In table V, a comparison with Chen and Chen's COG method(g), Deng's ROG method(p) and Wei's method(1) is also presented.

TABLE V.
THE RESULT OF THE PROPOSED METHOD COMPARED WITH CHEN AND CHEN'S METHOD

Linguistic terms	**Chen and Chen's COG method**	**Deng's ROG method**	**Wei's method**	**The proposed method**
absolutely low	0.0927	0.1947	0.3218	0.7024
very low	0.1366	0.2514	0.3495	0.7115
low	0.2070	0.3437	0.4640	0.7778
fairly low	0.3245	0.4961	0.6113	0.8834
medium	0.4683	0.6200	0.6933	0.9363
fairly high	0.4318	0.3653	0.6499	0.7735
high	0.3395	0.2501	0.5753	0.7021
very high	0.2591	0.1689	0.4330	0.6419
absolutely high	0.1816	0.1296	0.4026	0.6344

As can be seen from Table V, the risk $\tilde{R} = (0.4743;0,8933)$ of component A can be evaluated as linguistic term "Medium", which coincides with the ones presented in Chen and Chen's COG method, Deng's ROG method and Wei's method.

V. CONCLUSION

In this paper, we present a new fuzzy risk analysis method based on generalized fuzzy numbers. Firstly, we define new arithmetic operations between generalized fuzzy numbers. Then, we propose a new method to measure the degree of similarity between generalized fuzzy numbers. We apply the new arithmetic operations and proposed similarity measure to deal with fuzzy risk analysis problems. The results coincide when comparing with Chen and Chen's method [7] in Example 1 and Example 2. Since the representation of a fuzzy numbers is simpler in equation (25), (26), the proposed fuzzy risk analysis method is easier and more efficient when dealing with the risk analysis problems.

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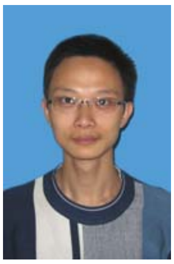

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