Stochastic Programming Model Based on Compound Quantification and Its Application in Transportation Problems

Fachao Li, Li Wang
School of Economics and Management
Hebei University of Science and Technology, Shijiazhuang 050018, China
lifachao@tsinghua.org.cn, wangli198601@yahoo.com.cn

Abstract—As a kind of particular programming problem, transportation problem attracts much attention in many fields, such as energy development, materials management, etc. In this paper, after analyzing the essence of stochastic programming and the deficiencies of existing methods, propose a quasi-linear pattern based on expectation and variance for the satisfaction of the random constraints. Give a stochastic programming model (compound quantification model) with good operability, and establish its corresponding model in stochastic transportation problem. Its performance is discussed through an example. All these indicate the compound quantification model generalizes existing methods, and can solve the stochastic transportation problems with unknown distribution of the random variable under random environment. It is worthy to point out the solution reflects the consciousness of the decision maker, so it enriches methods of stochastic programming.

Index Terms—stochastic programming; expectation; stochastic transportation problem; compound quantification model; reliability coefficient

I. INTRODUCTION

Transportation problem is a kind of typical linear programming. Transporting of goods plays a key role in logistics, so it has practical significance to make transporting process reasonable in the overall planning and management of the whole logistic chain. Perfect methods, such as the tabular method etc., have been given to process numerical transportation problems, but for the non-numerical ones general method have not been found.

Actually, various uncertainties from objective environment (production, market and transportation conditions, etc.) and various uncertainties from subjective environment (judging of products, evaluating of social benefits, etc.) must be faced with for a transportation problem, so it belongs to complex system optimization. For this kind of optimization, different processing strategies for uncertain information will lead to different delivery schemes. At present, stochastic mathematics and fuzzy set can be considered as two relative good theories to process uncertain information. Stochastic methods are used to process the uncertainty caused by inadequate objective conditions, but fuzzy methods are used to process uncertainty that caused by the difference of subjective understanding. Thus, uncertainty transportation problems can be divided into two kinds: stochastic and fuzzy.


As the stochastic transportation problem is a kind of particular stochastic programming, its solution is closely related to that of stochastic programming problem. At present, there are three basic methods to solve the stochastic programming problem: 1) Expected value model. Its basic idea is that the random variable is centrally described by its expectation and then the stochastic programming could be converted into an ordinary one. 2) Chance-constrained programming (see Ref. [8]). Its basic idea is that stochastic constraints and objective functions are converted into ordinary ones through some reliability principles. 3) Dependent-chance programming (see Ref. [9]), its basic idea is related to maximizing the chance of stochastic event in uncertain environment and giving optimal decision. Though the three methods have attained good results in application (such as, Ref. [10] introduce the expected value models on Sugeno measure space, Ref. [11], used the chance-constrained model to the sizing of batteries for distributed power system, the bus dispatching model based on dependent-chance goal programming was established by Ref. [12], etc.), they could not effectively solve stochastic programming problems in complicated environment, the reasons are: 1) when the random fluctuation tend very strong, the expectation could not effectively represent and describe the random variable and the reliability of expected value model can’t be guaranteed; 2) when the randomness is too complex (that is to say, even the type of distribution of the random variable is difficult to obtain), the degree of complexity in computing will become too high to establish
feasible solution scheme for chance-constrained and dependent-chance programming.

Consider the above discussions, for stochastic programming problems, we analyze the essence of them and the shortages of existing methods, the main works are as follows: 1) propose a compound quantification strategy for chance constrains by analyzing the essence of constrains in chance-constrained model; 2) establish a compound quantification model for stochastic programming; 3) analyze the characteristics of the quantification model through an example, and the results indicate that our method can effectively applied to transportation problems.

In what follows, for convenience, let \( (\Omega, \mathcal{B}, P) \) be a probability space and \( \xi \) be a random variable on it, \( E(\xi) \) and \( D(\xi) \) denote the mathematical expectation and variation of \( \xi \), respectively.

II. COMPOUND QUANTIFICATION MODEL FOR STOCHASTIC PROGRAMMING

A. Overview

Stochastic programming is the key point in many actual problems such as production planning and resources distributing, its general form is as follows:

\[
\begin{align*}
\max f(x, \xi), \\
\text{s.t. } g_j(x, \xi) &\leq 0, j = 1, 2, \cdots, m. 
\end{align*}
\]

(1)

Here, \( x = (x_1, x_2, \cdots, x_n) \) is the decision vector, \( \xi = (\xi_1, \xi_2, \cdots, \xi_n) \) is a random vector on space \( (\Omega, \mathcal{B}, P) \), \( f(x, \xi) \) and \( g_j(x, \xi) \) are all random variable functions.

Expected value model: Mathematical expectation, is a common way to centrally describe the possible values of a random variable. If we use the expected value to replace it approximately, then (1) can be converted into the following:

\[
\begin{align*}
\max E(f(x, \xi)), \\
\text{s.t. } E(g_j(x, \xi)) &\leq 0, j = 1, 2, \cdots, m. 
\end{align*}
\]

(2)

Generally, we call (2) the expected model. When the fluctuation degree of the random variable tends higher, the expected value could not describe the variable effectively, so the optimal solution obtained using model (2) can’t often satisfy the requirement of decision.

Chance-constrained model: For a stochastic programming, the constraints can not be often satisfied absolutely, so if some satisfaction degree (say reliability) is used to deal with the constraints and objective functions, then model (1) can be converted into the following one:

\[
\begin{align*}
\max \bar{f}(x), \\
\text{s.t. } P(f(x, \xi) \geq \bar{f}(x)) \geq \alpha, \\
P(g_j(x, \xi) \leq 0) \geq \beta_j, j = 1, 2, \cdots, m.
\end{align*}
\]

(3)

Generally, we call (3) the chance-constrained model. Here, \( \beta, \alpha \in [0, 1] \) represent the reliability for the constraints and objective functions. \( P(A) \) represents the probability of event \( A \). Compared with (2), model (3) reflects the function controlling the decision quality beforehand, but there exist the following two shortages: ① complexity degree in computing is high; ② the values of \( \beta_j \) such that model (3) has solution can not be known beforehand.

Dependent-chance programming: For the allocation of resources in random environment, the following stochastic programming model (4) can be established through maximizing the probability of dependent stochastic event:

\[
\begin{align*}
\max P(h_k(x, \xi) \leq 0, k = 1, 2, \cdots, q), \\
\text{s.t. } g_j(x, \xi) &\leq 0, j = 1, 2, \cdots, p.
\end{align*}
\]

(4)

Generally, we call (4) the dependent-chance programming. Here, \( \xi \) is a given random environment, \( g_j(x, \xi) \leq 0, j = 1, 2, \cdots, p \) are constrains related to \( \xi \), \( h_k(x, \xi) \leq 0, k = 1, 2, \cdots, q \) are related events to \( \xi \). This model has good interpretability, but it is necessary to know the distribution of \( \xi \), and it is difficult to achieve the computation of \( P(h_k(x, \xi) \leq 0, k = 1, 2, \cdots, q) \).

The above discussion shows that existing stochastic programming models have their own limitations that are difficult to overcome, in which establishing an operable method for processing objectives and constraints is the core to solve stochastic programming problem. For the evaluation of objective function value in the decision process, we consider not only the size of objective function value, but also the uncertainty. Therefore, we can evaluate the objective function value by expected value according to model (2). When processing constraint, we should simplify judgment method of the constraints as possible. In the following, we will give the processing method of constraint.

B. Compound Quantification Model

Quasi-linearization of chance-constrains: The key point of the chance-constrained model is that the random constraints are converted into ordinary ones through a given reliability. Processing uncertain constraints become possible theoretically, but the high complexity degree in computing make it is difficult to realize the solution. In this section, we mainly discuss the simplification of chance-constrains. By

\[
P(g(x, \xi) \leq 0) \geq \beta \Rightarrow \frac{g(x, \xi) - E(g(x, \xi))}{\sqrt{D(g(x, \xi))}} \leq \frac{-E(g(x, \xi))}{\sqrt{D(g(x, \xi))}},
\]

if we set \( \Psi = [g(x, \xi) - E(g(x, \xi))]/\sqrt{D(g(x, \xi))} \), and let \( \Psi_\beta \) denote the \( \beta \)-quantile of \( \Psi \) (that is, \( P(\Psi \leq \Psi_\beta) \geq \beta \)), then

\[
P(g(x, \xi) \leq 0) \geq \beta \Leftrightarrow \Psi(\Psi_\beta) \leq 1 - \beta.
\]

(5)

Noticing that \( E(\Psi) = 0, D(\Psi) = 1 \), we know that \( \Psi_\beta \) is the standard \( \beta \)-quantile of \( \Psi \). Due to the standard quantile of usual distributions have certain regulation (see Table I), thus, if we regard \( \Psi_\beta \) as a constant \( C \), then chance-constrain \( P(g(x, \xi) \leq 0) \geq \beta \) can be simplified as...
Obviously, the complexity degree in computing tends to be lower, therefore if we use (6) instead of the chance-constrain $P(g(x, \xi) \leq 0) \geq \beta$, then the operability of the stochastic programming will be greatly enhanced.

### TABLE I. STANDARD QUANTILES OF SEVERAL DISTRIBUTIONS

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Expectation</th>
<th>Variance</th>
<th>Parameter values</th>
<th>$\beta$</th>
<th>$\Psi_{\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(\mu, \sigma^2)$</td>
<td>$\mu$</td>
<td>$\sigma^2$</td>
<td>$\mu, \sigma^2$</td>
<td>0.75</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9</td>
<td>1.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.95</td>
<td>1.65</td>
</tr>
<tr>
<td>$U(a, b)$</td>
<td>$\frac{a+b}{2}$</td>
<td>$\frac{(a+b)^2}{12}$</td>
<td>$a, b$</td>
<td>0.75</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.95</td>
<td>1.56</td>
</tr>
<tr>
<td>$Exp(\lambda)$</td>
<td>$\lambda^2$</td>
<td>$\lambda^{-2}$</td>
<td>$\lambda$</td>
<td>0.75</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.95</td>
<td>1.99</td>
</tr>
<tr>
<td>$t(n)$</td>
<td>0</td>
<td>$\frac{a}{n-2}$</td>
<td>$(n &gt; 2)$</td>
<td>$n=5$</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.95</td>
<td>1.56</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.75</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.95</td>
<td>1.61</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.75</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.95</td>
<td>1.62</td>
</tr>
<tr>
<td>$\chi^2(n)$</td>
<td>$n$</td>
<td>$2n$</td>
<td></td>
<td>$n=5$</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9</td>
<td>1.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.95</td>
<td>1.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.75</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9</td>
<td>1.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.95</td>
<td>1.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.75</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9</td>
<td>1.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.95</td>
<td>1.88</td>
</tr>
<tr>
<td>$F(m, n)$</td>
<td>$\frac{n}{n-2}$</td>
<td>$\frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$</td>
<td>$m, n$</td>
<td>$m=5$</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.95</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.95</td>
<td>1.56</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.95</td>
<td>1.75</td>
</tr>
</tbody>
</table>

Figure 1. Probability density curves of several common distributions
In Table I, \( N(\mu, \sigma^2) \) denotes the normal distribution with parameters \( \mu \) and \( \sigma^2 \), \( U(a, b) \) denotes the uniform distribution with parameters \( a \) and \( b \), \( \text{Exp}(\lambda) \) denotes the exponential distribution with parameter \( \lambda \), \( t(n) \) denotes the \( t \)-distribution with freedom degree \( n \), and \( \chi^2(n) \) denotes the \( \chi^2 \)-distribution with freedom degree \( n \), and \( F(m, n) \) denotes the \( F \)-distribution with freedom degree \( m \) and \( n \).

**Remark 1** The above discussion tells us that constant \( C \) in (6) is a parameter reflecting the satisfaction degree of constraint. The larger/smaller the value of \( C \) is, the higher/lower the attention degree paid to the constraint will be.

**Remark 2** From Table I we can find that: 1) the standard \( \beta \)-quantile of normal distribution and that of \( t \)-distribution are almost about the same; 2) except for \( F \)-distribution, the standard 0.9-quantiles of other distributions are almost about the same; 3) when \( \beta \) takes values from 0.5 to 0.75, the standard \( \beta \)-quantile \( \Psi_{\beta} \) has significant differences for different distributions; 4) the standard 0.95-quantile for all those distributions does not exceed 2; 5) the standard 0.75-quantiles for normal distribution, \( t \)-distribution and \( \chi^2 \)-distribution are all between 0.5 and 0.7, and those for \( F \)-distribution with \( m < n \) and exponential distribution between 0.35 and 0.45, while that for uniform distribution is significantly larger than that for others.

The above analysis indicates that distribution feature of \( g(x, \xi) \) should be considered in the selecting of \( C \) in (6), for details in Part C. In order to emphasize the role of \( C \), we say \( C \) quasi-linearization reliability coefficient (reliability coefficient in short) of constraint \( g(x, \xi) \leq 0 \).

**Compound quantification model:** Using the discussion in above, if both the objective function and the constraints contain random variables, then the objective function can be described using expected value as in (2), and the the degree of satisfaction for constraints be described as in (6). Thus model (1) can be converted into the following one:

\[
\begin{align*}
\text{max} & \quad E(f(x, \xi)) \\
\text{s.t.} & \quad E(g_j(x, \xi)) + C_j \sqrt{D(g_j(x, \xi))} \leq 0, \quad j = 1, 2, \ldots, m.
\end{align*}
\]

**Remark 3** ① The above analysis indicates that model (7) is the extension of both ordinary numerical programming model and the expected value model (2), and with better structural characteristics and strong interpretability. Therefore model (7) provides a theoretical platform for solving stochastic programming problems. For different problems, we can select different reliability coefficients \( C \) to embody and describe different decision consciousness; ② Expectation and variance separately centrally describes the size and uncertainty of stochastic variable from different ways, that is, \( (E(\xi), D(\xi)) \) is a kind of compound quantification value of \( \xi \), \( E(\xi) \) is a main index describing the size of the value of \( \xi \) from the whole, \( D(\xi) \) is a secondary index that reflects that whether \( E(\xi) \) can approximately represents \( \xi \), in order to distinguish from other methods, we call (7) compound quantification model for stochastic programming (1).

**C. The determine strategy of reliable coefficient**

Combining with the above discussion, we can determine the reliability coefficient \( C \) in the random constraints \( g(x, \xi) \leq 0 \) by the following steps:

**Step 1** Determine the distribution characteristics of \( g(x, \xi) \) through certain ways: theoretical analysis or statistical analysis.

**Step 2** Combine the distribution characteristics of \( g(x, \xi) \) (several probability density curves are given in Fig. 1), Fig. 1 and the risk demand for decision-making process to determine the reliability coefficient \( C \).

### III. COMPOUND QUANTIFICATION MODEL FOR STOCHASTIC TRANSPORTATION PROBLEM

**A. Formalized description for stochastic transportation problems**

The transportation problem is a kind of typical programming problem, which can be expressed in the general ways: For a given goods, there are \( m \) sources \( A_1, A_2, \ldots, A_m \) with outputs \( a_1, a_2, \ldots, a_m \), respectively, and \( n \) destinations \( B_1, B_2, \ldots, B_n \) with sales \( b_1, b_2, \ldots, b_n \), respectively, \( c_{ij} \) is the freight of unit goods from source to destination, determine the delivery scheme of the goods so that the total freight is the smallest.

Let \( x_{ij} \) \((i=1, 2, \ldots, m, j=1, 2, \ldots, n) \) be the number of goods delivered from source \( A_i \) to destination \( B_j \), then the mathematical model of transportation problem can be expressed as:

\[
\begin{align*}
\text{min} & \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \\
\text{s.t.} & \quad \sum_{j=1}^{n} x_{ij} \leq a_i, \quad i = 1, 2, \ldots, m, \\
& \quad \sum_{i=1}^{m} x_{ij} \geq b_j, \quad j = 1, 2, \ldots, n, \\
& \quad x_{ij} \geq 0, \quad i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.
\end{align*}
\]

(8)

For model (8), if \( a_i, b_j \) and \( c_{ij} \) are all real variables, we regard it as numerical transportation problem, and if they are variables with uncertainty, we regard it as uncertain transportation problem. In particular, if \( a_i, b_j \) and \( c_{ij} \) are all random variables/fuzzy numbers, we call the model (8) stochastic/fuzzy transportation problem. For numerical transportation problem, the optimal delivery scheme could be obtained by the tabular method. For the uncertain one, there is no perfect solution method because the order of uncertain information can not be determined. In what follows we only focus on the stochastic transportation problem.

**B. Compound quantification model for stochastic transportation problem**

Stochastic transportation problem is a kind of particular stochastic programming problem, so we can convert stochastic transportation problem (8) into the following compound quantification model (9) using model (7).
That is,
\[
\begin{align*}
\min & \sum_{i,j} E(c_{ij}) x_{ij} \\
\text{s.t.} & \sum_{j} x_{ij} = a_i, C_i \end{align*}
\] (9)
\[
\begin{align*}
\sum_{i} x_{ij} = b_j, D_j \end{align*}
\]
\[
\begin{align*}
E(\sum_{i,j} x_{ij} + b_j + C_i) \leq 0, j = 1, 2, \ldots, n,
\end{align*}
\]
\[
\begin{align*}
x_{ij} \geq 0, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.
\end{align*}
\]
Here, \(C_i, C_j\) for every destination, \(i = 1, 2, \ldots, m\), \(j = 1, 2, \ldots, n\) are reliability coefficients of supply constrains and demand constrains, respectively. Obviously, (9) is a linear programming problem of ordinary numerical type, we can use conventional methods to solve it.

IV. CASE ANALYSIS

In this section, we combined a transportation problem with random demand and supply, we consider how to establish a compound quantification model.

Example 1 Given goods with three sources \(A_1, A_2, A_3\) and four destinations \(B_1, B_2, B_3, B_4\). The output, sales and freight are shown in Table II. Here, \(\xi_1, \xi_2, \eta_1, \eta_2\) represents the demands for four destinations, \(\eta_1, \eta_2, \eta_3\) are supplies for three sources, in which the demands and supplies in the recent 20 weeks are shown in Table III. Try to determine the optimal delivery scheme for this transportation problem.

If we regard demands \(\xi_1, \xi_2, \eta_1, \eta_2, \eta_3\), as random variables, then the above transportation problem is a stochastic transportation problem. However, the exact distribution of \(\xi_1, \xi_2, \eta_1, \eta_2, \eta_3\) is unknown, thus, this problem can not be solved by model (2), (3) and (4). In the following, we consider the transportation problem through model (9).

If we regard the values in Table III as a group of sample values of \(\xi_1, \xi_2, \xi_3, \xi_4, \eta_1, \eta_2, \eta_3\), their approximations of mathematical expectation and variance are: \(E(\xi_1) = 4, D(\xi_1) = 2; E(\xi_2) = 6; D(\xi_2) = 4.6; E(\xi_3) = 1.65; E(\eta_1) = 8.25, D(\eta_1) = 4.19, E(\eta_2) = 10, D(\eta_2) = 2, E(\eta_3) = 11, D(\eta_3) = 1.6, E(\eta_4) = 13, D(\eta_4) = 1.6\). If we denote \(x_{ij}\) as the deliveries from \(A_i\) to \(B_j\), then we can establish the mathematical model of this transportation problem by the model (10):

\[
\begin{align*}
\min & \sum_{i,j} E(c_{ij}) x_{ij} \\
\text{s.t.} & \sum_{j} x_{ij} = a_i, C_i \end{align*}
\] (10)
\[
\begin{align*}
\sum_{i} x_{ij} = b_j, D_j \end{align*}
\]
\[
\begin{align*}
E(\sum_{i,j} x_{ij} + b_j + C_i) \leq 0, j = 1, 2, \ldots, n,
\end{align*}
\]
\[
\begin{align*}
x_{ij} \geq 0, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.
\end{align*}
\]

In the following, we determine the reliability coefficients \(C_i, i = 1, 2, 3, 4, 5, 6, 7\).

Step 1 Get the probability density histograms according to the demands from \(B_1, B_2, B_3, B_4\) and the outputs from \(A_1, A_2, A_3\), using Matlab7.0 (see Fig. 2).

Step 2 Determine \(C_i\) according to decision consciousness, \(i = 1, 2, 3, 4, 5, 6, 7\).

From Fig. 1 and Table I, we can see the distributions of \(\xi_1, \xi_2, \eta_1, \eta_2\) should be Uniform, Normal, Normal, \(\chi^2\)-distribution, respectively, and \(\eta_1, \eta_3, \eta_4\) should be Uniform, Normal, Normal, respectively. Here, both demands and supplies are random variables, so they need processing by (7). The bigger reliability of demands, the more sales, the bigger reliability of supplies, the less outputs.

Table II. Output, Sales and Freight in Each Source and Destination

<table>
<thead>
<tr>
<th>Sales</th>
<th>(B_1)</th>
<th>(B_2)</th>
<th>(B_3)</th>
<th>(B_4)</th>
<th>(\eta_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\xi_1)</td>
<td>2</td>
<td>9</td>
<td>10</td>
<td>7</td>
<td>(\eta_1)</td>
</tr>
<tr>
<td>(\xi_2)</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>(\eta_2)</td>
</tr>
<tr>
<td>(\xi_3)</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>(\eta_3)</td>
</tr>
<tr>
<td>(\xi_4)</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>(\eta_4)</td>
</tr>
</tbody>
</table>

Table III. Demand for Every Destination in Recent 20 Weeks

<table>
<thead>
<tr>
<th>(B_1)</th>
<th>(B_2)</th>
<th>(B_3)</th>
<th>(B_4)</th>
<th>(\eta_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\xi_1)</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>(\xi_2)</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>(\xi_3)</td>
<td>5</td>
<td>3</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>(\xi_4)</td>
<td>8</td>
<td>6</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>(\eta_1)</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>(\eta_2)</td>
<td>11</td>
<td>13</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>(\eta_3)</td>
<td>12</td>
<td>14</td>
<td>13</td>
<td>11</td>
</tr>
</tbody>
</table>
Figure 2. Probability density histograms about demands of each destination and supplies of each source

Then the values of $C_i$ could be determined combining Fig. 1 and risk demand. For example, if the decision-maker demands that the satisfaction for $A_3$ should be not less than 0.95, and the reliability of other destinations and sources should be not less than 0.75, then the values of $C_i$ can be determined as follows: $C_1 = 0.87$, $C_2 = 0.87$, $C_3 = 1.65$, $C_4 = C_5 = C_6 = C_7 = 1.0$, (see Case 6, in Table IV); if the decision-maker demands that the satisfaction for $B_1$ and $B_2$ should be not less than 0.95, and the reliability of other destinations be not less than 0.75, then the values of $C_i$ will be $C_1 = 0.87$, $C_2 = 0.68$, $C_3 = 0.68$, $C_4 = 1.56$, $C_5 = 0.8$, $C_6 = 1.65$, $C_7 = 0.9$ (see Case 20, in Table IV).

Obviously, Model (9) is a linear programming problem, so we can solve it by LINGO. To analyze the performance of Model (10), the corresponding delivery schemes for different $i$, $i = 1, 2, 3, 4, 5, 6, 7$ are shown as Table IV.

The above analysis and results indicate that: ① Model (9) has good explanatory and low computational complexity; ② When the reliability coefficient changes, the optimal solution and objective function value will change correspondingly, even have significant difference, which means that model (9) can reflect effectively decision consciousness (i.e., strategies to deal with uncertainty) in decision making; If the reliability of demand and supply are both big, then it will not have optimal solution (Case 8), and if both are small, then it causes wasting resources; ④ If the reliability of demands is smaller, then that of supplies can be bigger correspondingly, such as Case 1-5 and Case 9-10, the reliability of demands is smaller, and reliability of supplies is not less than 0.95, instead, the
the essence of stochastic programming and the deficiencies of transportation problems. Moreover, we establish a compound quantification model for stochastic transportation problems and we discuss its performance from different angles.

V. CONCLUSIONS

For stochastic transportation problems, by analyzing the essence of stochastic programming and the deficiencies of existing methods, we propose a quasi-linear pattern based on expectation and variance. Then we give an operable stochastic programming pattern—compound quantification model, which can be used to solve stochastic transportation problems. Moreover, we establish a compound quantification model for stochastic transportation problems and we discuss its performance from different aspects through an example. The results indicate that the compound quantification model can solve effectively stochastic transportation problems under random environment, with the distributions of random variables are unknown. This method not only covers existing results, but also has simpler operability, better explanatory, and lower computational complexity, etc. And the consciousness facing with uncertainty can also be incorporated in the decision making, so the discussion in this paper lays the foundation for giving programming methods under complex environment.

ACKNOWLEDGMENTS

This research is supported by the National Natural Science Foundation of China (70671034, 70871036) and the Natural Science Foundation of Hebei Province (F2006000346).

REFERENCES


Fa-Chao Li was born in Shijiazhuang, China, in 1962, and received the B.S. degree and the M.S. degree at the Department of Mathematics from Hebei University, Baoding, P.R.China, in 1983 and in 1994, respectively. He received the Ph.D. degree in 2000 from the Department of Mathematics at Harbin University of Industry and Technology, Haerbin, P.R.China. And from 2001 to 2003, he did postdoctoral research work in the Department of Control Science and Engineering, Tsinghua University, Beijing, P.R.China. His research interests include fuzzy information metric, structural characteristic of fuzzy number, uncertain information processing, evolutionary computation, data mining, machine learning and optimization scheduling.

From 1983 to 1987, he taught in Hebei Architectural and Civil Engineering Institute, and since 1987, he has been with Hebei University of Science and Technology, where he is currently a professor and the Vice President of School of Economy and Management. His previous publication is “Oprations Research”, science press, 2006. His current interests is uncertain programming methods and theories based on effect.

Prof. Li is the chairman of IEEE in Shijiazhuang, the director in oprations research of China and vice director in machine learning academy of Hebei Province.

Li Wang was born in Shijiazhuang, China, in 1986, and received the B.S. degree in 2008, and continues pursing the M.S. degree at the Department of Economics and Management from Hebei University of Science and Technology, Shijiazhuang, P.R.China. Her research interests include stochastic and fuzzy information processing, intelligence optimization and optimal decision.