

Image Denoising Algorithm Based on Dyadic Contourlet Transform

Hui Fan

Shandong Institute of Business and Technology, Yantai, China

Email: fanlinw@263.net

Yongliang Wang and Jinjiang Li

Shandong Normal University, Jinan, China

Email: hzliang99@126.com , lijinjiang@gmail.com

Abstract—This paper constructs a dyadic non-subsampled Contourlet transform for denoising on the image. The transformation has more directional subband, using the non-subsampled filter group for decomposing of direction, so it has the translation invariance, eliminated image distortion from Contourlet transform's lack of translation invariance. Non-subsampled filter reduces noise interference and data redundancy. Using the feature of NSCT translation invariance, multiresolution, multi-direction, and can according to the energy of NSCT in all directions and in all scale, adaptive denoising threshold. Experimental results show that compared to wavelet denoising and traditional Contourlet denoising, the method achieves a higher PSNR value, while preserving image edge details, can effectively reduce the Gibbs distortion, improve visual images.

Index Terms—component, Image denoising, Dyadic contourlet, Wavelet, Threshold value

I. INTRODUCTION

The main sources from which human obtain information the vision, so the image technology has been applied in various fields. As the rapid development of computer technology and network technology, the image processing technology has a great leap. Images will inevitably polluted by noise when it was collected and transported which seriously affected the recognition and process of image information.

According to the filtering method used, it can be divided into the linear filter and nonlinear filter; where according to the different signal domain, image denoising can be divided into spatial domain methods and transform domain methods. Space domain method directly processing pixel on the image plane; and transform domain methods first transformed by a mathematical transform, and then correct the factor in the transformation domain of the image, and then get the spatial domain image by the inverse transform.

Spatial filtering is one of the traditional image denoising method, can be divided into linear and nonlinear filtering forms, the difference is that the linear filter is a computing which meet the principle of superposition, and nonlinear filtering does not satisfy the principle of superposition. The most typical spatial linear

filtering using average filter, also known as linear smoothing filter, is an effective way to deal with Gaussian noise. In order to overcome average filter's shortcomings of average, simple and local, people proposed many filtering algorithm conserve details, for how to choose the neighborhood size, shape and direction, how to choose to number of points which participate in the average, the weighted coefficient of neighborhood points. Order statistics filter is the most common non-linear spatial filtering method, filtering output depends on the sort of neighboring pixels which surrounding the filter, sort the results, the statistical value decided which value instead of the center pixel value.

Frequency domain filtering is the other categories of traditional image enhancement methods, it is based on two-dimensional fast Fourier transform algorithm theory, and according to the features which general distribution of noise in the high frequency section, design filter system with the low-pass characteristics for image denoising. In the spatial linear filtering method discussed in the previous, the weighted action of a local window of pixels is actually equivalent to the convolution of image with a template; frequency domain filtering method is simple operation, intuitive features of physical significance. The process in frequency domain mostly be used in image de-noising nowadays, in which the Wavelet is most famous.

Generally speaking, the above discussion of the traditional denoising method unsatisfactory compromise in terms of noise reduction and preserving detail. Wavelet transform as a new time-frequency analysis method with the characteristics of multi-scale, multi-resolution analysis, offers a new and powerful tool for the signal processing. The rise of wavelet theory, is due to it has a good time-frequency localization properties and optimal approximation to the one-dimensional variation function class. The multi-resolution analysis has been widely used in digital signal processing and analysis, signal detection and noise suppression, and a variety of fast and efficient algorithms also contribute greatly to the application of wavelet analysis in the actual system. Wavelet transform can better express one-dimensional signal, and for two-dimensional signal, as two-dimensional wavelet [1] is

one-dimensional wavelet's tensor product, it has only very limited directional include horizontal, vertical and diagonal. Ordinary wavelet transform is usually not optimal in the high-dimensional case, so the other geometric analysis methods have been proposed, including Redgelet [2], Curvelet, Contourlet [3] and other methods. A way to deal with a features of a special type, but to other types the treatment effect is not satisfactory, for two-dimensional images, the two-dimensional wavelet is good at singularity and the spot, the Redgelet is good at line singularity, Contourlet is good at the description of the image's two-dimensional data, with characteristics of direction and anisotropic.

One important task of image denoising is to remove the noise as much as possible to retain the image edges and details [4]. In the denoising process, the rational selection of threshold is very important which directly affect the denoising. In 1992, Donoho and John stone proposed the method of wave threshold shrink and proved the optimal of Donoho threshold [5]. The wavelet threshold value is a nonlinear method, it is denoising by carries on processing the wavelet coefficient in the wavelet territory, its theory premise is thought that the image the wavelet coefficient is the obedience generalized Gaussian distribution, and big absolute amplitude value wavelet coefficient obtained mainly after signal translation, but small absolute amplitude value wavelet coefficient is mainly after the noise transformation obtains, so the small noise factor can be elimination through the hypothesis threshold value. However, the shrink threshold is the threshold limit, and is not the best shrink threshold, so that too many wavelet coefficients are set to 0, damage the image details. In 2002, Do.M.N proposed a multi-direction, multi-resolution image representation method [6-9], which is Contourlet transform theory. As a new multidimensional singularity analytical tool, Contourlet transform overcome the shortcomings of Wavelet transform's non-singularity optimal basis, is a new image denoising method.

Image denoising emergence along with the birth of image processing, is an ancient image processing issues, seeking an effective method for image denoising has been engaged in the work of the people; a wide variety of denoising methods has developed based on the actual characteristics of the image, the noise distribution of such statistics. With new theories and methods of mathematical are constantly emerging, provides these study new ideas and inject new vitality. Today, the image denoising theory and application of image processing is still a very active field of research.

This paper constructs a dyadic non-subsampled Contourlet transform for denoising on the image, the transformation has more directional subband, using the non-subsampled filter group for decomposing of direction, so has the translation invariance, eliminated image distortion from Contourlet transform's lack of translation invariance. Non-subsampled filter reduces noise interference and data redundancy. The experimental results show that the denoising algorithm can extract

useful information from input image effectively, get a clear denoised image.

II. DYADIC CONTOURLET TRANSFORM

A. Wavelet

Wavelet analysis is a time-frequency analysis, is developed from Fourier analysis, but it is better than Fourier analysis. Fourier analysis method as a classic had been widely used, but because it is a global transformation, reflects the time-domain signal contribution to the frequency, which means the Fourier transform integral kernel smoothing the mutant of signal components, can not determine the time location and intensity changes of signal transform, which can not express the nature of time-frequency localization, which is precisely the most fundamental and the most critical nature of non-stationary signal in the process of the practical application. Fourier analysis was carried out one of the reforms which put forward a series of non-stationary signal analysis and processing theory of new signal analysis, including the short time Fourier transform and wavelet transform. Although short time Fourier transform to some extent, overcome the defects which standard Fourier transform does not have the analytical capacity of local, but it also has its own shortcomings which can not overcome, that is, when the short window function determined, the shape of the window identified, so it is a single-resolution signal analysis. To change the resolution, you must re-select the window function.

The wavelet analysis method is ideological development of Fourier analysis, which is a window size fixed, but a time-frequency localization analysis which shape of the window, time window and frequency window can change: that in the low-frequency some have a high frequency resolution and low time resolution; in the high-frequency have a high time resolution and low frequency resolution, it is suitable for detection of transient anomal entrainment normal signal and show its composition, both in time and frequency domain has the capacity to show local signal characteristics, with a multi-resolution analysis characteristics which is known as the microscope of the signal.

Fourier transformation is a function which integral $f(t)(-\infty < t < +\infty)$ into another function $F(jw)$. Let the function $f(t)$ satisfied $\int_{-\infty}^{+\infty} |f(t)| dt < +\infty$, we define it's Fourier transformation as follows:

$$F(jw) = \int_{-\infty}^{+\infty} e^{-jw t} f(t) dt \quad (1)$$

When $f(t)$ satisfy the appropriate conditions, it has the inverse transformation:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(jw) e^{jw t} dw \quad (2)$$

So we can get the signal component of the total energy E and its relationship between the spectrum:

$$\begin{aligned}
 E &= \frac{1}{2} \int_{-\infty}^{\infty} |f(t)|^2 dt \\
 &= \int |F(jw)|^2 dw \\
 &= \int_0^{\infty} E(w) dw
 \end{aligned}
 \tag{3}$$

In which $E(w)$ is called Spectral density, shows the band signal components within the unit.

Wavelet transform inherited many strengths of Fourier transform, while overcome the weaknesses of Fourier transform lack of localized of in a certain extent.

Definition 1. Let $f(t)$ is a square integrable function (denoted by $f(t) \in L^2(R)$), $\Psi(t)$ is called basic wavelet or mother wavelet function, then have the following equation:

$$\begin{aligned}
 WT_x(a,b) &= \frac{1}{\sqrt{a}} \int f(t) \Psi^* \left(\frac{t-b}{a} \right) dt \\
 &= \int f(t) \Psi^*_{a,b}(t) dt = \langle f(t), \Psi_{a,b}(t) \rangle
 \end{aligned}
 \tag{4}$$

In the above, the $WT_x(a,b)$ is known as the Wavelet Transform of $f(t)$, in which $a > 0$ is the scale factor, b is the displacement. Both a and b is continuous variable, so the equation is also called Continuous Wavelet Transform (CWT), b is the role of determining time position of $f(t)$, which is the time center. Scale factor a is the role of stretching the basic wavelet $\Psi(t)$. Figure 1 to Figure 3 shows the relationship between the wavelet function and factor a, b .

B. Dyadic wavelet

Discrete Dyadic wavelet is a special case of wavelet framework, in which wavelet base function has a characteristic of narrow band-pass filter, and also has a property of conservation of energy before and after the signal transform. Dyadic wavelet transform in time domain and space domain is continuous, it's only Dyadic discrete on the scales, and translational of time domain remains continuous change, thus has the same translation invariance with continuous wavelet transform, can be effective to carry out image noise detection, localization and classification.

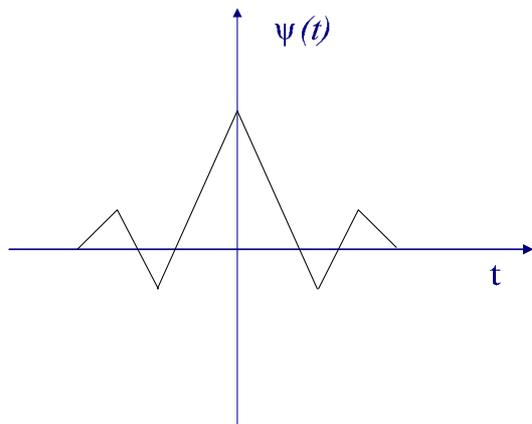


Figure 1. Basic Wavelet.

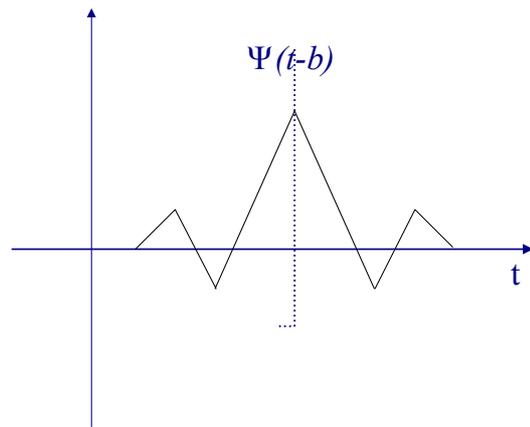


Figure 2. When $b > 0, a = 1$

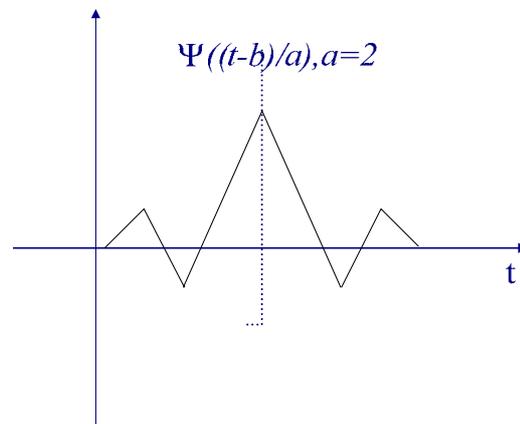


Figure 3. When b is constant, $a = 2$

The follow definitions are given to describe the Dyadic wavelet:

Definition 2. Function $\psi(t) \in L^2(R)$ is a one-dimensional Dyadic wavelet, if has constant $0 < A \leq B < \infty$, causes the equation below to be tenable:

$$A \leq \sum_{j \in \mathbb{Z}} |\widehat{\psi}(2^j \omega)|^2 \leq B
 \tag{5}$$

Definition 3. $\{\psi_1(x,y), \psi_2(x,y)\} \subset L^2(R^2)$ is an two-dimensional Dyadic wavelet, if has constant $0 < A \leq B < \infty$, causes the equation below to be tenable:

$$\begin{aligned}
 \forall \omega = (\omega_x, \omega_y) \in R^2 - \{(0,0)\} \\
 0 < |\widehat{\psi}_1(2^j \omega_x, 2^j \omega_y)|^2 + |\widehat{\psi}_2(2^j \omega_x, 2^j \omega_y)|^2 \leq B
 \end{aligned}
 \tag{6}$$

In the above equation, $\widehat{\psi}$ is the Fourier transform of ψ . The Dyadic wavelet transform is defined as:

$$Wf(x,y) = \{W_{2^j}^1 f(x,y), W_{2^j}^2 f(x,y)\}, j \in \mathbb{Z}
 \tag{7}$$

In the above equation,
 $W_{2^j}^1 f(x, y) = f(x, y) * \bar{\psi}_{2^j}^{-1}(x, y)$,
 $W_{2^j}^2 f(x, y) = f(x, y) * \bar{\psi}_{2^j}^{-2}(x, y)$,
 and $\bar{\psi}_{2^j}^{-k}(x, y) = (-x, -y), k = 1, 2$.

The follow diagram is the decomposition of Dyadic wavelet:

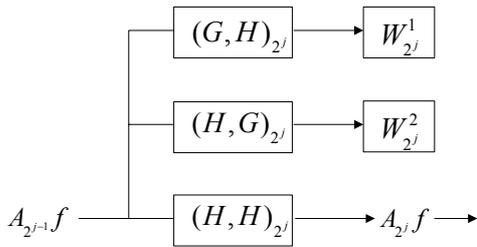


Figure 4. Dyadic wavelet decomposition

C. Construct dyadic Contourlet Transform

Contourlet inherited wavelet transform's multi-resolution feature of time-frequency analysis, and has a good characteristic of the direction dissimilarity, it can use less coefficients than the wavelet transform to represent a smooth curve. Contourlet transformation process use Laplace Pyramid(LP) to capture point discontinuities in the first, then use the direction filter to link the discontinuity points with the line structure, so get a sparse representation of the image, the overall similar to contour divided the image into the basic unit.

Laplacian pyramid filter (LP) is a multi-resolution analysis tools, whose intention is for compression encoding after sub-band decomposition of the image, has now become a very efficient signal processing and analysis tools. In the LP decomposition, first use (low pass) filter to low-pass filter the original signal, and then downsampling to obtain low-frequency signal that approximate the original signal components. Then sampled the low-frequency signal, and then use integrated filter high-pass filtering sampled signal, and difference high-pass filtered signal and the original signal, obtained high frequency band signal which after the LP decomposition.

Contourlet transform use double filter structure, first used Laplaeian Pyramid (LP) decomposition of the input signal to capture the singularity point, and then accord to the direction of information located the close singular points together into a profile section. Decomposition of each LP to generate a half resolution of the original signal low-frequency sub-band and a high-frequency sub-band whose signal the same with resolution, the high-frequency sub-band is the difference signal between the original signal and low-frequency subband's signal after sampled and filtered. Continue iterative decomposing low-frequency sub-band by LP transform, the original signal can be decomposed into a series of different scales low and high frequency sub-band. Subsequently, the high-frequency sub-band from decomposition of LP is

directional analysed by the Directional Fiter Banks(DFB).

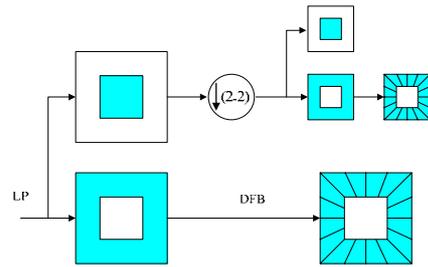


Figure 5. Contourlet transform structural

Article [10] use the non-subsampled Contourlet transform (NSCT) to resolve the Contourlet transform not meet the translation invariance, and disclosure and aliasing of frequency and other defects which caused by downsampling. Transform coefficients of each sub-band of NSCT the same as the original image size. NSCT is constituted by nonsubsampled pyramid (NSP) and nonsubsampled directional fiter banks (NSDFB), NSP is a two-channel non-sampling filter, NSP realize multi-scale features, NSDFB achieved multi-directional. NSP decomposed image at multi-scale, DFB then put bandpass signal obtained after decomposition decomposed into multiple directional subband, the number of directional subband can be any power of 2. This process can be repeated in iteration of low frequency sub-band.

The hierarchical structure of non-sampled pyramid is realized by multi-level iteration, and its complete reconstruction conditions are:

$$H_0(z)G_0(z) + H_1(z)G_1(z) = 1 \tag{8}$$

In which, $H_0(z)$ is the low-pass decomposition filter, $H_1(z)$ for the high-pass decomposition filter, $G_0(z)$ for the low-pass reconstruction filter, $G_1(z)$ for the high-pass reconstruction filter.

Figure 6 is the non-subsampled sampling Pyramid filter group third-level decomposition structure schematic. Through this group of filter, the image is divided into many each one two-dimensional low frequency subband and many two-dimensional high frequency subband.

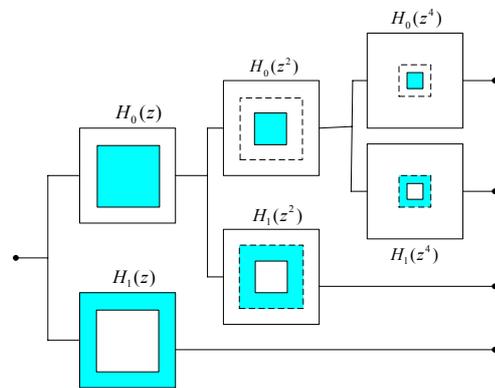


Figure 6. Three level NSP decomposition diagram

NSDFB adopted a set of two-channel nonsubsampled filter banks, decomposed in the direction of each layer, and then use all the filters of quincunx matrix $Q = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ top-sampled directional filter banks, as the directional filter of next layer's directional decomposition. NSDFB decomposition schematic shown as Figure 7, $U(z)$ is filter.

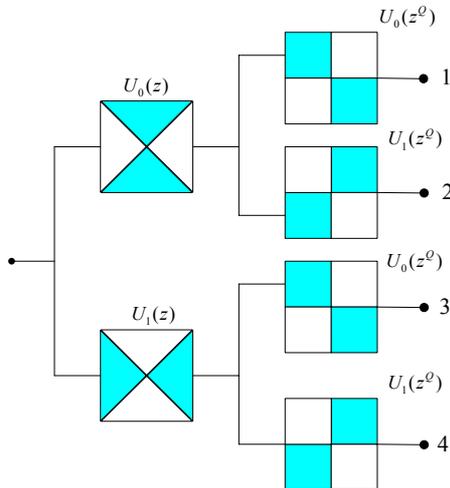


Figure 7. NSDFB decomposition

Traditional Contourlet transform use the Laplace transform, transform by cascading direction filter in the basis of Laplace transform, so obtained the frequency domain decomposition in different directions, then has the frequency domain multi-directional. However, the frequency domain information of traditional Contourlet transform is not fully utilized, while for solve the problem of frequency domain translation invariance, using non-subsampled directional filter, there is a large redundancy. This paper structure a new Dyadic Contourlet Transform(DCT) based on traditional Contourlet transform construction, the main structural idea is: first, transform image into the dyadic wavelet, then put the transformed high frequency sub-band (two) for NSDFB decomposition, in which the low-frequency sub-band to conduct the dyadic wavelet transform again, continue this iterative process. The idea shown in Figure 8.

Compared with the traditional Contourlet transform, dyadic Contourlet have more directional subband transform, while decomposition of the direction using NSDFB, so Contourlet transform has shift invariance which can be widely used in image denoising and image fusion. Figure 9 - Figure 10 is the decomposition of dyadic Contourlet transform, the parameter is set to: nlevels = [1, 3]; pfilter = 'maxflat'; dfilter = 'dmaxflat7'. Decomposition as shown below, where is the sample images in Figure 9, Figure 10 is the first layer of dyadic Contourlet transform decomposition.

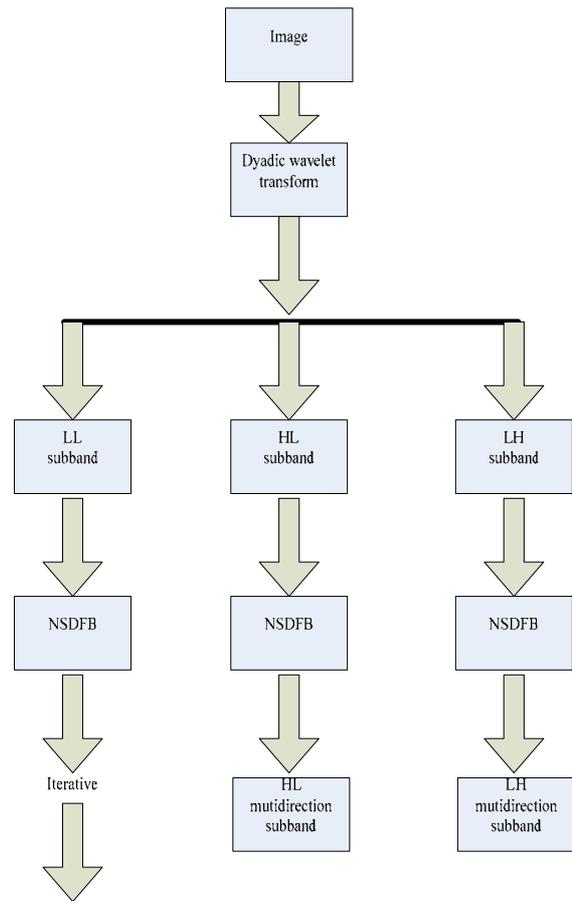


Figure 8. Decomposition of DCT

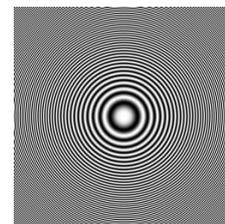


Figure 9. Example image

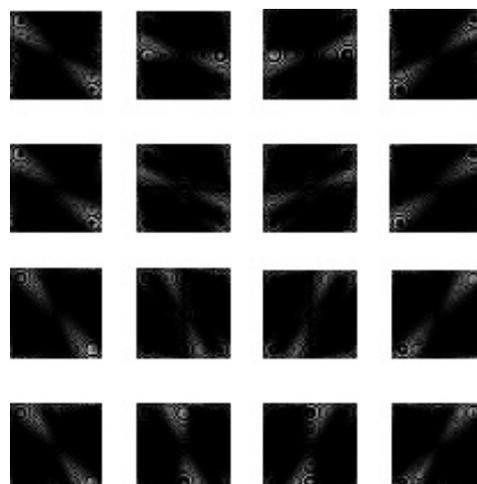


Figure 10. Decomposition of the first layer

III. IMAGE DENOISING BASED ON DCT

The image degradation model in the spatial can be expressed as:

$$g(x, y) = h(x, y) * f(x, y) + n(x, y) \tag{9}$$

where, $f(x, y)$ is the true picture without interference, $h(x, y)$ is the point spread function, $n(x, y)$ is the image noise, $g(x, y)$ is the degraded image we ultimate observed, $*$ is convolution.

Compared with the wavelet transform, DCT choose the direction of more flexible, and transform domain size of the scale is the same original images. In the the conditions of the same scale and direction, DCT coefficients is less than wavelet coefficients [11]. This paper based on the superior performance of DCT, combined with multi-scale threshold theory,proposed a new image denoising method.

The noisy image in the DCT domain can be express as:

$$d_k^j = c_k^j + n_k^j \tag{10}$$

where d_k^j , c_k^j and n_k^j represent coefficient(which scale is k and in the jth sub-band) of the noise image,original image and noise after DCT.

The denoising process is to restore the primitive image's DCT coefficient from the image contain noise whose DCT coefficient is d_k^j , and maintains the characteristic of primitive image DCT coefficient c_k^j , optimized the standard deviation. The threshold denoising is a misalignment denoising method which easy to realize and has a good effect, its merit is the noise may be suppressed perfectly, and the peak that reflected the primitive character is well retained.

To the wavelet threshold denoising algorithm, the threshold value is important. If the threshold is too small, then the image after denoising residue too more noise; if the threshold is too large, the important image features will be filtered out, causing deviation. Intuitively, for a given wavelet coefficient, the greater the noise, the threshold will be. To this end, the researchers presented many excellent threshold selection methods according to characteristics of the wavelet coefficients. The earliest threshold denoising method is the VisuShrink method proposed by Donoho, is also called the general threshold value denoising law. The threshold current can be divided into global adaptive threshold and local adaptive threshold. Among them, the global threshold for all layers of the wavelet coefficients or wavelet coefficients within the same layer are unified; and local adaptive threshold is based on factors surrounding the current local situation to determine the threshold.

Different to the global threshold values, local adaptive threshold mainly through test at some point or a local feature, according to the principle of flexibility to determine the coefficient is the noise or signal, in order to achieve a balance between denoising and retention signals, Sometimes these principle is not only necessarily consideration from a coefficient's absolute value, but also

from other aspects. Today, the threshold selection method of study is still in progress, there are still new threshold formulas have been raising, but usually the threshold is based on the needs of practical application, by determining the appropriate criteria, and through possible thresholds optimization to choose.

In the process of threshold value denoising, the threshold function manifests the different processing strategy as well as the different estimate method to DCT coefficient mold which surpass or lower than the threshold value. The commonly used threshold function is the hard threshold and the soft threshold [12].

The hard threshold is defined as follow:

$$\hat{d} = \begin{cases} d_i & |d_i| \geq T \\ 0 & else \end{cases} \quad i = 1, 2, \dots, j \tag{11}$$

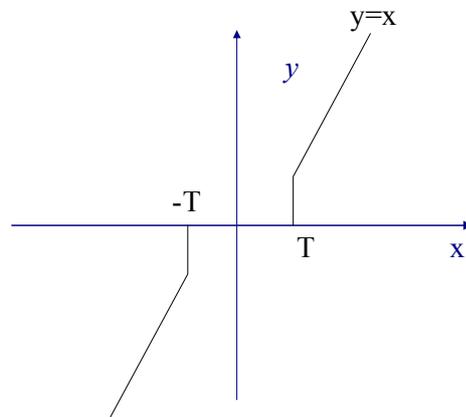


Figure 11. The curve of hrad thresholding function

The soft threshold is defined as follow:

$$\hat{d} = \begin{cases} \text{sgn}(d_i)(|d_i| - T) & |d_i| \geq T \\ 0 & else \end{cases} \quad i = 1, 2, \dots, j \tag{12}$$

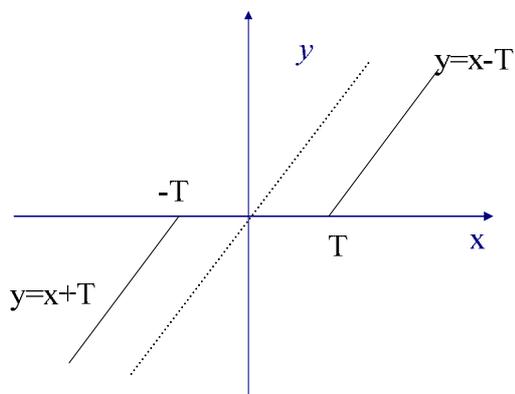


Figure 12. The curve of soft thresholding function

In which d_i is the high frequency coefficient of the original DCT and \hat{d}_i is DCT high frequency coefficient after the thresholding. T is the threshold value. The hard threshold value may retain the picture edge well, but after

restructuring, will present the image distortion, the soft threshold value method denoising effect relatively smoother, but will usually lead to the edge detail fuzziness. This paper will adopt the hard threshold value denoising plan which is easy and feasible.

Regarding the threshold value denoising method, how the correct selection of threshold value will affect the image denoising effect directly. According to the DCT's characteristic, this paper adopts the following local adaptive threshold value selection method: due to the energy distribution is different of the different scale and different subband after multistage DCT of noise image, the coefficient is mainly the pictorial information in the low scale, the proportion of the noise occupies is small, so when denoising the threshold value should reduce suitably, however, in the high scale, the proportion which the noise occupies is big, the pictorial information reduces, therefore when denoising the threshold value suitable carries on the enlargement.

The implementation steps of this algorithm are as follow:

(1) Put the noise image into DCT transform, obtained the transform coefficients of different directions and different scales;

(2) Stratified hard threshold process to the transform coefficients;

(3) Inverse transforms of the processed coefficients and then get the denoised image.

IV. RESULTS

To verify the effect of this method for denoising, select 2 standard image which size is 512x512 pixels as the test image, and compare denoising results of the proposed method with the traditional Bayes denoising method in a variety of strength under the Gaussian white noise. In the experiment, the parameter is set to: nlevels = [1, 3]; pfilter = 'maxflat'; dfilter = 'dmaxflat7'. This paper adopts the following multi-scale threshold [13] determination method:

$$T_k = \sigma \sqrt{2 \ln(N)} \times 2^{(k-K)/2} (k = 0 \dots K-1) \tag{13}$$

Where N is the number of image pixels, K is the number of the DCT total scale, σ is the size of the noise, k is the scale level. σ obtained by median estimate, $\sigma = \text{Median}(|d_1|) / 0.6745$, in which d_1 is the the high-frequency coefficients of the first layer of noisy images obtained by wavelet decomposition. Statistical results show as Table 1.

TABLE I. PSNR OF DIFFERENT ALGORITHMS

Denosing method	PSNR of pic 1	PSNR of pic 2
Original image	21.18	22.03
Wavelet denosing	27.23	28.14
Tradition contourlet denosing	28.31	29.16
Dyadic contourlet denosing	29.79	30.46

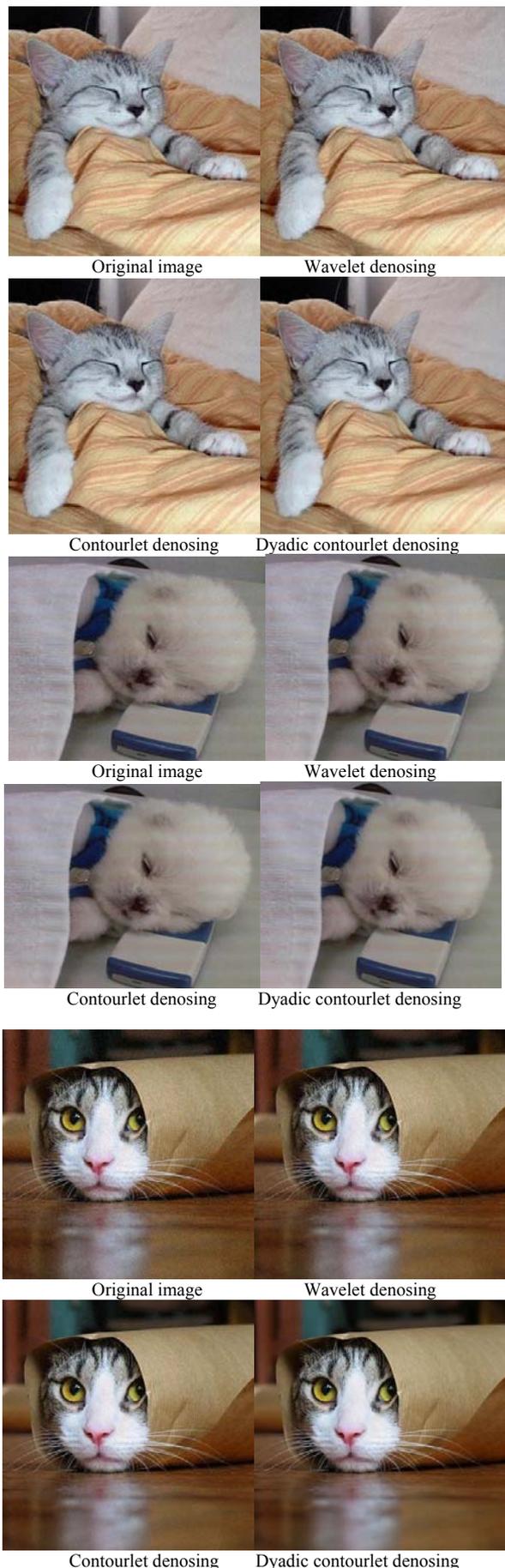


Figure 13. The experimental results of different algorithm

From the experimental results can be seen, for different test images and different intensity of noise, the method presented in this paper in terms of PSNR or visual effects are to some extent better than traditional wavelet denoising and traditional contourlet denoising, and the performance of the proposed method is relatively stable. Can be seen from the graph, while preserving image detail, this method effectively removes Gibbs distortion which like hair filamentous.

V. CONCLUSION

As the dyadic wavelet and non-sampling Contourlet have translation invariance, so you can avoid the image distortion generated by lack of translation invariance, and Contourlet also can capture image information in the multi-scale and multi-directional. This paper presents a image denoising algorithm based on dyadic contourlet transform. Using the feature of NSCT translation invariance, multiresolution, multi-direction, and can according to the energy of NSCT in all directions and in all scale, adaptive denoising threshold. Experimental results show that compared to wavelet denoising and traditional Contourlet denoising, the method achieves a higher PSNR value, while preserving image edge details, can effectively reduce the Gibbs distortion, improve visual images. How to further enhance the PSNR value and improve the image effect is the next step of research in strong noise environment.

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Hui Fan born in 1962, and he is a professor of school of computer science & technology, Shandong Institute of Business and Technology. He is a senior member of China Computer Federation and IEEE. He awarded a Ph.D degree in computer science at TaiYuan University of technology in 2007. His research interests include virtual reality, information visualization, digital image processing.

Yongliang Wang born in 1987. He received the bachelor's degree in computer science & technology from Shandong Normal University. Now he is studied for his M.S. degree in computer science at Shandong Normal University. He is a CCF student member. His research interests is image processing.

Jinjiang Li born in 1978, associate professor and Ph.D. He received the master's degree in applied computer from TaiYuan University of technology in 2004. From 2004 to 2006, he was an assistant engineer at institute of computer science & technology of Peking University. He awarded a Ph.D. degree in computer from Shandong University in 2010, and now work at Shandong Institute of Economic & Technology. His research interests include computer graphics, computer-aided geometry design and digital image processing.