

Fractional Modeling Method Research on Education Evaluation

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Abstract—Education assessment is one of important measures that will be to ensure and continuously improve the educational level and educational quality. A fractional order model method for education evaluation is proposed in this paper. Improved fractional Basset force model is referred to constitute the complex education evaluation model. The detailed descriptions are displayed through building block diagram. An algorithm for linear fractional order systems is described. The fractional evaluation model is composed of fractional order and common coefficient. Model parameters can be determined by a large number of actual data and mathematical statistics method. The proposed model was applied to actual course evaluation work of Capital Normal University. The practicability and effectiveness of the method have been validated.

Index Terms—fractional order, model, evaluation

I. INTRODUCTION

Post-Secondary Education Accreditation Council is set up in the United States in 1975. And this is the first institution of higher education assessment. In China educational assessment of colleges and universities began in the twentieth century, the eighties. Educational assessment is the efficient means that higher education institution realizes the higher education self-perfecting, self-regulation and self-improvement. Higher education assessment is one of important measures that will be to ensure and continuously improve the educational level and educational quality of China's institutions of higher

learning. It aims to improve the quality of instruction and promote teaching reform. There is more and more research on education evaluation recently. Reference [1] applies a growth modeling approach to the stability of teaching effectiveness. Reference [2] contributes to the conceptual and empirical distinction between appraisals of teaching behavior and self-reported competence acquirement within academic education evaluation. Reference [3] examined the effects of embedding special education instruction into pre-service general education assessment courses. Some methods had not considered the existence of objective weight, so that the result is too subjective. Because education evaluation is a complex nonlinear process that affected by many factors, traditional integer order calculus model is unable to accurately describe its action. Fractional order system is established on the idea of fractional order calculus and theory of fractional order differential equations, which is an extension to the conventional calculus problems.

It is well known that fractional order systems itself is an infinite dimensional filter due to the fractional order in the differentiator or integrator while the integer-order systems are with limited memory(finite dimensional). There has been a surge of interest in the possible engineering application of fractional order differentiation. Examples may be found in [4] and [5]. Some applications including automatic control are surveyed in [6]. The significance of fractional order theory is that it is a generalization of classical integral order theory, which could lead to more adequate modeling and more robust control performance. Fractional order systems could model various real materials more adequately than integer order ones and thus provide an excellent modeling tool in describing many actual dynamical processes[7]. Fractional model provides the scientific basis for prevention and treatment of satellite monitoring

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absorption rate [8]. The nematode movement can be simulated through fractional model [9]. It may be used for building “love” models using fractional-order system [10]. [11] modeled iron meteorites crystallization by fractional theory. And there are some people pay close attention to unemployment rates by means of fractional calculus [12].

Fractional model is a mathematical modeling approach based on fractional calculus, and it provides a powerful decision support and scientific basis for education evaluation. Fractional education evaluation model is proposed in this paper. Model parameters can be obtained by the corresponding actual data. It aims to reduce the complexity while improving the scientific validity of the assessment results based on the fractional model.

The remaining part of this paper is organized as follows. In Section II, mathematical foundation of fractional calculus is briefly introduced; in Section III, a fractional model method is presented for education evaluation; in Section IV, some practical examples are presented to verify the feasibility. Finally, conclusions are drawn in Section V.

II. BRIEF INTRODUCTION OF FRACTIONAL CALCULUS

Although the fractional order calculus is a 300-years old topic, the theory of fractional order derivative was developed mainly in the 19th century. References [13-18] provide a good source of references on fractional calculus.

Fractional calculus is a generalization of integration and differentiation to a fractional, or non-integer order fundamental operator ${}_a D_t^\alpha$, where a and t are the lower/upper bounds of integration and α the order of the operation.

$${}_a D_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & R(\alpha) > 0 \\ 1 & R(\alpha) = 0 \\ \int_a^t (d\tau)^{(-\alpha)} & R(\alpha) < 0 \end{cases} \quad (1)$$

which $R(\alpha)$ is the real part of α . Moreover, the fractional order can be a complex number as discussed in [19]. In this paper, we focus on the case where the fractional order is a real number.

Caputo's fractional-order differentiation is defined by

$${}_a D_t^\alpha f(t) = \frac{1}{\Gamma(1-\gamma)} \int_a^t \frac{f^{(m+1)}(\tau)}{(t-\tau)^\gamma} d\tau \quad (2)$$

where $\alpha = m + \gamma$, m is an integer, and $0 < \gamma \leq 1$. Similarly, by Caputo's definition, the integral is described by

$${}_a D_t^\gamma f(t) = \frac{1}{\Gamma(-\gamma)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1+\gamma}} d\tau \quad (3)$$

As in the case for conventional calculus, fractional-order derivatives and integrals have the following qualities:

- (1) If $f(t)$ is an analytic function of the variable t , the derivative $D_t^\alpha f(t)$ is an analytic function of t and α .
- (2) The operation D_t^α and the usual derivative of order $n \in \mathbb{Z}^+$, $\alpha = n$ give the same result; The operation D_t^α and the usual n -fold integral with $n \in \mathbb{Z}^-$, $\alpha = n$ give the same result, and $D_t^0 f(t) = f(t)$.
- (3) The operator should be linear:

$$D_t^\alpha [af(t) + bg(t)] = aD_t^\alpha f(t) + bD_t^\alpha g(t) \quad (4)$$

- (4) For the fractional-order integrals of arbitrary order, it holds the additive law of exponents (semi-group property):

$$D_t^\alpha [D_t^\beta f(t)] = D_t^{\alpha+\beta} f(t) \quad (5)$$

Linear fractional-order differential equations are the fundamental governing equations. The linear fractional-order differential equation is defined as

$$a_n D_t^{\beta_n} y(t) + a_{n-1} D_t^{\beta_{n-1}} y(t) + \dots + a_1 D_t^{\beta_1} y(t) + a_0 D_t^{\beta_0} y(t) = b \quad (6)$$

Substituting fractional-order differentiation definition into the above equation, one may find that

$$\frac{\alpha_0}{h^{\beta_0}} \sum_{j=0}^{[(t-a)/h]} \omega_j^{(\beta_0)} y_{t-jh} + \dots + \frac{\alpha_m}{h^{\beta_m}} \sum_{j=0}^{[(t-a)/h]} \omega_j^{(\beta_m)} y_{t-jh} = b \quad (7)$$

where the binomial coefficients $\omega_j^{(\beta_i)}$ can still be evaluated recursively with

$$\omega_0^{(\beta_i)} = 1, \omega_j^{(\beta_i)} = \left(1 - \frac{\beta_i + 1}{j}\right) \omega_{j-1}^{(\beta_i)}, j = 1, 2, \dots \quad (8)$$

By slight rearrangement of the terms, the closed-form solution of the fractional-order differential equation can be obtained as

$$y_t = \frac{1}{\sum_{i=0}^n \frac{\alpha_i}{h^{\beta_i}}} \left[b - \sum_{i=0}^n \frac{\alpha_i}{h^{\beta_i}} \sum_{j=1}^{[(t-a)/h]} \omega_j^{(\beta_i)} y_{t-jh} \right] \quad (9)$$

Fractional evaluation model will be proposed based on the above theory in this paper. Model parameters can be obtained by the corresponding actual data. It aims to reduce the complexity while improving the scientific validity of the assessment results based on the fractional model.

III. FRACTIONAL MODEL

Here fractional model of education evaluation will be established based on the above fractional order systems. Educational evaluation is to assess the course system.

Firstly, each course is evaluated by the proposed method. Then course system evaluation result can be gotten depending on the course evaluation results. And then the curriculum and the construction could be analyzed. You can also compare the differences between courses.

Previous courses are assessed by the expert ratings. The human factors have an important effect, and the result is not so objective. In order to overcome the drawbacks, fractional modeling approach is adopted. Based on some obtained actual data, the effect of course evaluation can be taken through fractional model. It is the relative objective, realistic, and highly persuasive.

A. Each Course Evaluation

Fractional model of education evaluation can be modeled by improved Basset force and fractional model in this paper. Basset force mainly describes the process that the ball moves in a straight line. And when the ball sinks into the viscous fluid, two-phase flow in the actual ball movement is not linear motion. The force is impacted by other particles movement. The force should be connected with particle size, particle and fluid density ratio and fluid pulsation frequency so on. The process is multi-factor process. Refer to the model of fractional Basset force, based on the character of education evaluation and a lot of relevant data, fractional education evaluation model for one course can be modeled.

$$\left(\frac{9}{1+2\cdot\lambda}\right)^{\alpha} D^{\alpha}x(t) + bD^{\beta}x(t) + aDx(t) + x(t) = 1 \quad (10)$$

where $x(t)$ expresses the final evaluation result of the course. Fractional coefficient α shows characteristics of the teachers. Whether teachers have the experience, passion, quality and evaluation of previous classes so on? And it also includes the teachers teaching methods and means. Fractional factor β shows the overall characteristics of the students. It includes the students understanding of the course, interest and the past accumulation of knowledge so on. Other coefficients are determined by teachers and school students. They express that students are main body and teacher is the organizer. And it fully reflects that the masters of classroom are students. The primary role of teachers is to guide and answer questions in the classroom. And it should not be spoon-fed the traditional speaking. Students' active learning should be mainly part. Coefficient λ shows the evaluation of students for the course. It is mainly confirmed by recognition degree and benefit from the course of students. And it fully reflects student initiative. The coefficient a expresses the evaluation of course teachers. It is mainly determined based on the teaching content, teaching hours and teaching arrangement aspect. Coefficient b expresses students and teachers for the objective evaluation of teaching and learning environment. It is decided by the credits set, the college supporting degree for the program and class conditions so on.

The fractional simulation block diagram is used for the proposed fractional model[20]. Through building

Simulink model, the numerical solution of fractional order nonlinear calculus equation can be obtained directly. A fractional calculus module has been mainly adopted. In fractional calculus model, a modified approximation method is introduced[21]. Based on series expansion and recurrence, the continuous rational transfer function is

$$G(s) = (\omega_b \omega_h)^{\alpha} \left(\frac{ds^2 + b\omega_h s}{d(1-\alpha)s^2 + b\omega_h s + d\alpha} \right) \prod_{k=-N}^N \frac{1+s/\omega_k}{1+s/\omega_k'} \quad (11)$$

where,

$$\omega_k' = \left(\frac{b}{d}\right)^{\frac{2k-\alpha}{2N+1}} \omega_h^{\frac{N+k+\frac{1}{2}(1-\alpha)}{2N+1}} \omega_b^{\frac{N-k+\frac{1}{2}(1+\alpha)}{2N+1}} \quad (12)$$

$$\omega_k = \left(\frac{b}{d}\right)^{\frac{2k+\alpha}{2N+1}} \omega_h^{\frac{N+k+\frac{1}{2}(1+\alpha)}{2N+1}} \omega_b^{\frac{N-k+\frac{1}{2}(1-\alpha)}{2N+1}} \quad (13)$$

where $2N+1$ is the order of approximation, and b, d are improvement factor. Here $b=10$, $d=9$, $N=3$, and the pre-specified frequency range is $\omega_b=0.001$, $\omega_h=1000$.

B. Course System Evaluation

Linear fractional order system is adopted for each course assessment result. Output is the final result of the evaluation system that we expect. The fractional order is important. The different course evaluation results are introduced as orders of fractional order systems. Other parameters are determined by the characteristics of curriculum.

$$a_1 D^{\alpha_1} y(t) + a_2 D^{\alpha_2} y(t) + \dots + a_n D^{\alpha_n} y(t) + a_0 y(t) = 1 \quad (14)$$

One coefficient a_i shows the characteristic parameters of the course. And α_i is the above evaluation result. Through a comprehensive study for college courses, we can select

$$a_0 = 1, a_i = 10 \cdot \alpha_i, i = 1, 2, \dots, n \quad (15)$$

Then the above fractional model can be expressed as:

$$10\alpha_1 D^{\alpha_1} y(t) + 10\alpha_2 D^{\alpha_2} y(t) + \dots + 10\alpha_n D^{\alpha_n} y(t) + y(t) = 1 \quad (16)$$

Based on the above algorithm, the numerical solutions to the linear fractional-order differential equation can be obtained with a MATLAB function. Results of curve can be achieved by the established fractional model.

IV. COMPARATIVE SIMULATIONS

Taken the recent course evaluation as an example, there are three courses in software engineering system.

They are Software Testing Technology, Software Testing Practice and Software Engineering Practice respectively. Firstly each course is analyzed.

A. Software Testing Technology Evaluation

Conventional course evaluation score of Software Testing Technology is 78 out of a hundred. And then the course is evaluated by the proposed fractional model in this paper. There are two fractional coefficients in the above fractional model, which ranges between 0 and 1.

The coefficient α shows characteristics of the teachers. The larger the value of fractional coefficients shows that teachers have the higher overall quality of, the better characteristics and ways. And it is converse if the value is smaller. The fractional coefficient can be determined grounded on the above factors, expert ratings and data analysis so on. And there is $\alpha = 0.8$. The fractional order coefficient β is decided due to the past content knowledge and overall level of students to understand. The coefficient is relatively large if the student has accumulated a strong knowledge of related courses with active learning. It is converse if the value is smaller. According to the previous data analysis of the students selected the course, sorting and statistics, mathematical statistics and other conventional methods, the fractional coefficient can be gotten $\beta = 0.6$.

The rest factors should be confirmed by teaching teachers and school students all together. The scope of these three factors is 1 to 10. Coefficient λ is evaluated from the perspective of the students for course. It is mainly reflected on students' satisfaction for the curriculum and interest for course content. The value is large shows that students are satisfied with the curriculum and interested in course content. And they believe that teachers have very good teaching ways and means. It is converse if the value is smaller. The value is 6 through investigating the overall students in the classroom. The coefficients a is mainly determined by teachers and curriculum. The larger coefficient indicates that teachers think that the course is enough for students and the content is appropriate for students, whereas the smaller the value. The coefficient value is 9 in this course. Coefficient b is confirmed from the perspective of students and teachers for teaching objective configuration. In other words, it reflects that support degree of external factors for the course. And it includes the institutions support and the school classroom environment. The larger coefficient indicates that the course has been attached importance to and various external factors are favorable for the course. While it is converse if the value is smaller. The coefficient is taken as 8 under the joint participation of all students and Teaching Committee.

Based on the above data and related methods, the fractional program evaluation model can be obtained:

$$\left(\frac{9}{1+2 \times 6}\right)^{0.8} D^{0.8} x(t) + 8 D^{0.6} x(t) + 9 D x(t) + x(t) = 1 \quad (17)$$

Simulation model can be built by using the above algorithm, as shown in Fig. 1.

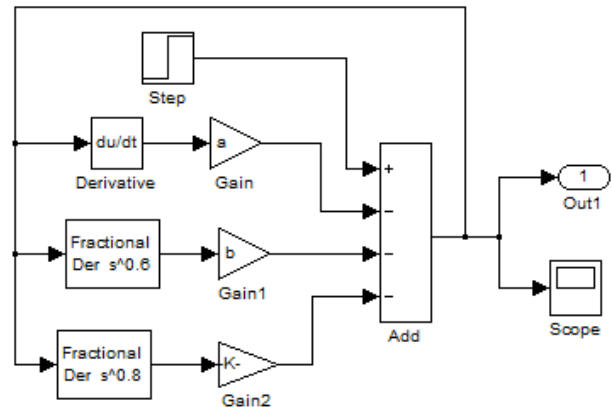


Figure 1. Simulink model

According to the algorithm proposed in this paper, the corresponding course curve can be obtained with MATLAB, as shown in Fig. 2.

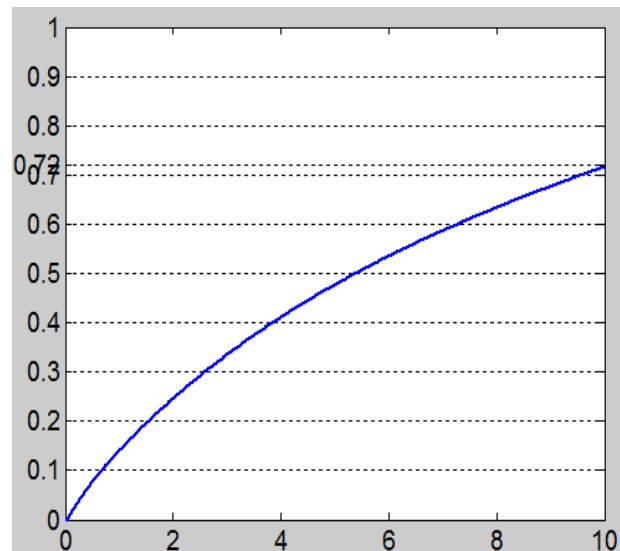


Figure 2. Output curve of Software Testing Technology

The result obtained by the proposed method is close to 0.72 in this figure. It is equivalent to 72 percentile points. And it is slightly inferior to the results of traditional assessment methods. Through a lot of surveys and interviews, it can be found that this method is more objective for course evaluation and avoids a lot of human factors.

B. Software Testing Technology Evaluation

In Software Testing Technology, the coefficient α shows characteristics of the teachers. And it can be determined grounded on the above factors, expert ratings and data analysis so on. And there is also $\alpha = 0.8$. The fractional order coefficient β is decided due to the past content knowledge and overall level of students to understand. According to the previous data analysis of the students selected the course, sorting and statistics, mathematical statistics and other conventional methods,

the fractional coefficient can be gotten $\beta = 0.7$. Coefficient λ is mainly reflected on students' satisfaction for the curriculum and interest for course content. Here is $\lambda = 7$.

The coefficients a is 8 in this course. Coefficient b reflects that support degree of external factors for the course. And it includes the institutions support and the school classroom environment. The coefficient is taken as 8 under the joint participation of all students and Teaching Committee.

Then fractional model is as follows:

$$\left(\frac{9}{1+2 \times 7}\right)^{0.8} D^{0.8} x(t) + 8 D^{0.7} x(t) + 8 D x(t) + x(t) = 1 \quad (18)$$

Simulation model can be built by using the above algorithm, as shown in Fig. 3.

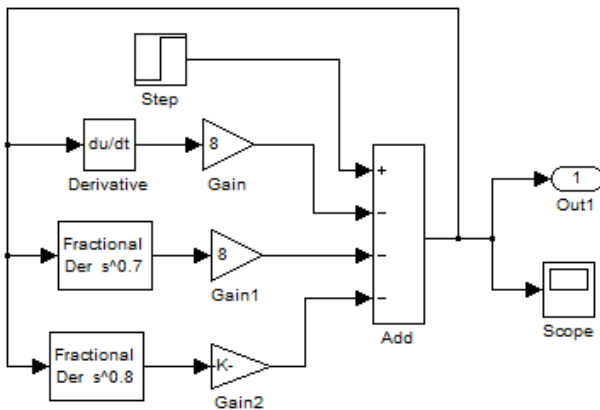


Figure 3. Simulink model

The final result curve of the evaluation can be gotten by building Simulation model block, as shown in Fig. 4. The result is 0.825.

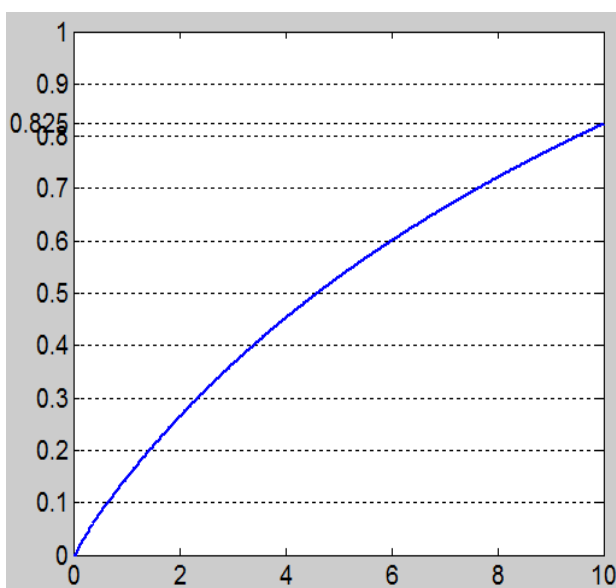


Figure 4. Output curve of Software Testing Practice

C. Software Engineering Practice Evaluation

Course parameters of Software Engineering Practice can also be obtained. The coefficient α can be determined grounded on the above factors, expert ratings and data analysis so on. And there is $\alpha = 0.9$. The fractional order coefficient β is decided due to the past content knowledge and overall level of students to understand. According to the previous data analysis of the students selected the course, sorting and statistics, mathematical statistics and other conventional methods, the fractional coefficient can be gotten $\beta = 0.8$, and it is high. Coefficient λ is evaluated from the perspective of the students for course. It is mainly reflected on students' satisfaction for the curriculum and interest for course content. The value is 8 through investigating the overall students in the classroom. The coefficients a is mainly determined by teachers and curriculum. And it is 7 in this course. Coefficient b is confirmed from the perspective of students and teachers for teaching objective configuration. And it includes the institutions support and the school classroom environment. The larger coefficient indicates that the course has been attached importance to and various external factors are favorable for the course. It is 9 in this course.

So the fractional model is:

$$\left(\frac{9}{1+2 \times 8}\right)^{0.9} D^{0.9} x(t) + 9 D^{0.8} x(t) + 7 D x(t) + x(t) = 1 \quad (19)$$

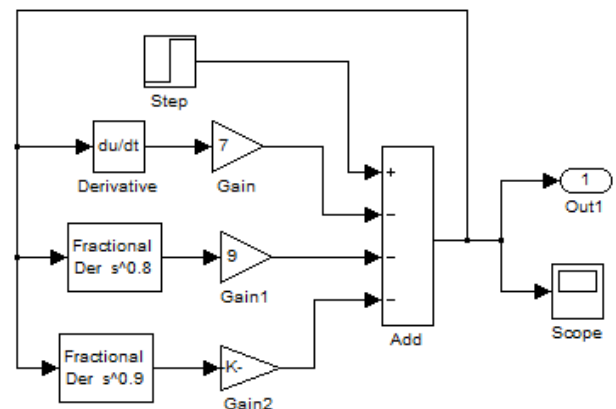


Figure 5. Simulink model

Simulation model can be built as shown in Fig. 5. Through building Simulation block, the final result curve is as shown in Fig. 6. The result is 0.905.

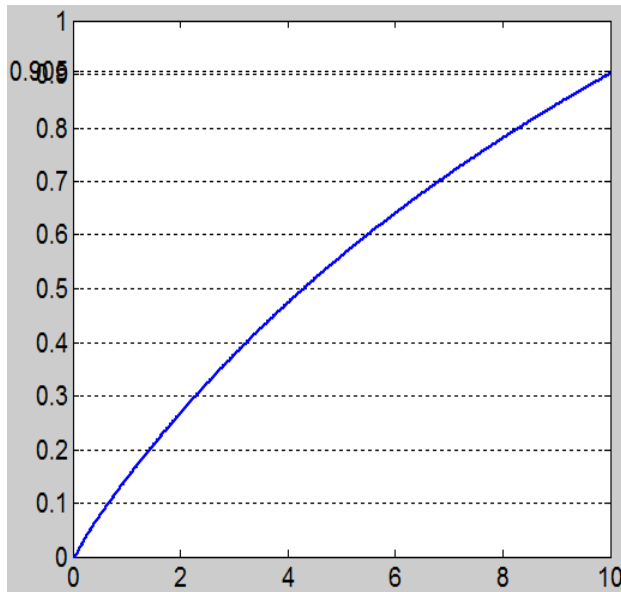


Figure 6. Output curve of Software Engineering Practice

Analysis: Teachers are not the most important for curriculum and student evaluation in education evaluation. In contrast the overall quality characteristic of the students is the most important for the course evaluation. From the view of above three courses, the course evaluation result is high when the teacher did not think highly of their own but the overall quality of students is high and student evaluation is good, such as Software Engineering Practice. While the final result of the evaluation is not good when the teacher evaluation is high whereas the evaluation of the students is low. It can be shown in this evaluation model system that external factors and students play an important role and the proportion of teachers is not large. The actual data and results could be more convincing. It is fully consistent with the actual teaching activities. And it is shown that the fractional order model is valid.

D. Course System Evaluation

The fractional order linear model is taken based on each course evaluation result. Output is the evaluation of the course system.

$$10\alpha_1 D^{\alpha_1} y(t) + 10\alpha_2 D^{\alpha_2} y(t) + 10\alpha_3 D^{\alpha_3} y(t) + y(t) = 1 \quad (20)$$

There is a fractional order system founded on the above three course evaluation results

$$9.1D^{0.91} y(t) + 8.3D^{0.83} y(t) + 7.2D^{0.72} y(t) + y(t) = 1 \quad (21)$$

The result is the output $y(t) = 0.78$, as shown in Fig. 7.

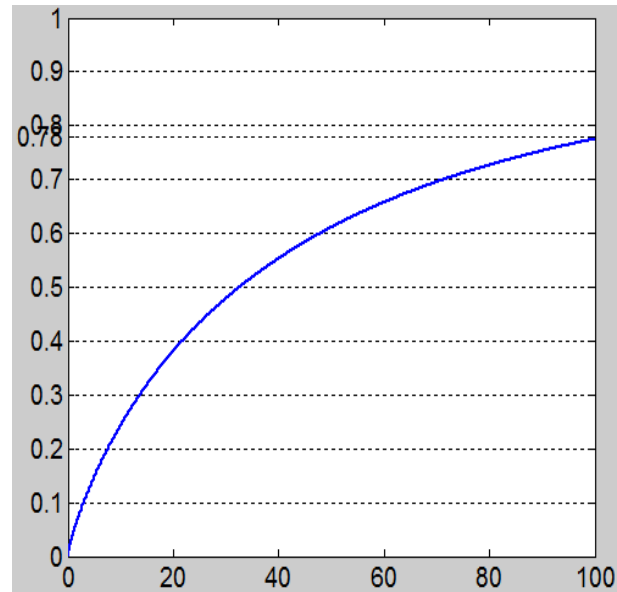


Figure 7. Output curve of course system evaluation

Analysis: It is obviously that the poor evaluation result has a great influence on the overall evaluation program in the course evaluation system. And it is one of the characteristics of the proposed model. It should be taken into account the effect of each course not the average. It aimed at improving the overall quality and not just pursuing the optimal assessment of individual course. And it will improve the overall standard of teaching rather than creating one or two quality courses. So students will get the most benefit.

The results are consistent with the actual situation, and there is more obvious difference in evaluation results. It is clear that the proposed method is practical and effective.

V. CONCLUSION

Education evaluation is a complex multi-factorial process, and it is not modeled by integer order model accurately. While fractional order system can model the complex process. And the corresponding curves can be shown in the model block. A fractional modeling method of course evaluation is proposed in this paper. The fractional model is based on the linear fractional order systems. The coefficients of the model are ascertained by a large number of related data. Results of the assessment can be obtained through MATLAB program. At last, the method was applied to actual course assessment instance of Capital Normal University Information Engineering College. And result indicates that this method is highly efficient for solving real-world problems.

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