

A New Model for the Structure of Leaves

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Abstract— This paper proposes a new model for the shapes and details of simple and compound leaves in plants. The layout of leaf is modeled via branching structure generated using B-Splines. The proposed model controls the relationships that govern the shape through the marginal and venation of the considered simple leaf or leaflet. The developed algorithm starts by taking four points that represent the out boundary of a simple leaf or a leaflet of compound one. Then, the algorithm optimizes the value of these parameters in order to get the best representation of the considered leaf/leaflet. The proposed model is tested with acceptable results for various shapes of leaf and compound leaves.

Index Terms—shape modeling, leaves structure, and B-Spline.

I. INTRODUCTION

One of the challenges in computer graphics is to create the geometry of objects in an intuitive and direct way, while allowing for interactive manipulation of the resulting shapes. The synthetic modeling for natural shapes is a challenging problem in computer graphics and related fields. While the modeling of plant structures is well-researched, the modeling of individual plant organs has relatively explored by Xfrog, a commercially available plant modeling system by greenworks [1].

This paper present an interactive method for modeling simple and compound leaves that captured by a branching structure generated using a B-spline curve of degree four [2]. The modeling process begins with number of control points of a leaf, which can be defined interactively using a curve editor. The generated curve is then defined interactively or computed automatically using a B-spline curve that preserves topological relations between leaflets during subsequent manipulation. The leaf surface is constructed by a leaf skeleton consisting of two boundary curves and a cross-section line between the first and the last control points. It can be bent and complemented with a relief, if these features are found in the real leaf. This can be done by creating sub-curve segments joining between a sequence of points on the cross-section line and the corresponding points on the curve that generated for the boundary of the leaf.

In Section 2, we describe the structure of a leaf and review previous work on the modeling of leaves. We then describe our proposed model in Section 3. Some

experimental results with discussions are given in Section 4. We conclude the paper and further research in Section 5.

II. BACKGROUND AND RELATED WORK

A. Leaf Shape Structure

A useful summary of the terminology for describing leaf shape is given by Judd *et al.* [3]. A typical simple leaf consists of a *leaf blade*, *margin*, *vein*, *base*, and *apex*. The *blade* is the entire leaf unit. Sometimes this is made up of several smaller leaflets. The *margin* is the term used to describe the edge of the leaf. Leaf venation is the pattern of *veins* in the blade of a leaf. The *Base* is the name given to the part of the blade that is closest to the stem. The *Apex* is the tip of the leaf. Fig. 1 illustrates these parts that consist of the leaf model. There are more terms to describe other parts of a leaf, but these five are enough to start with plant identification.

In contrast to the simple leaves, *compound* leaves have

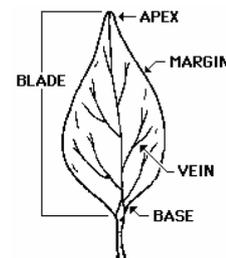


Figure 1. Leaf structure.

blades partitioned into separate subunits called *leaflets*. In this paper we consider compound leaves, assuming that their venation can be modeled at the level of individual leaflets.

B. Previous work

Most researchers try to generate curled shapes of plant leaves. Prusinkiewicz *et al.* [4] provided a detailed representation of combining interaction and parameterized algorithms for realistic plant modeling and scene creation involving plants, including curled leaves. Mundermann *et al.* [5] proposed a method for modeling lobed leaves; curled surface of leaves could be generated by using free deformation in their framework. Hong *et al.* [6] proposed an interactive method for modeling curled leaf surface. But it could involve excessive manual interactions for generating a desired curled shape of leaf

by using their method. Studies on curvature of plant leaves from biophysical perspective have raised the question of what role, if any, genes play in controlling the curvature of leaves [7]. Yet there are some researchers who study waved or wrinkled pattern in leaves with physical analysis [8].

Generalized cylinders [9] were originally applied to tree branch modeling by Bloomenthal [10], and have often been used to model plant organs since then. A generalized cylinder is formed by sweeping a generating curve, which determines the organ's cross-section, along a trajectory that defines the organ's axis [11]. The generating curve may be closed, as is typically the case for stems, or open, as for thin leaves. Furthermore, it can be scaled according to a profile curve and may change shape while being swept. Several researchers have used generalized cylinders to model leaves supported by a single axis [12], [13]. More complex leaves have been modeled by constructing a leaf surface using spline patches [10], [14], or by calculating an implicit contour around a branching structure [15].

Y. Rodkaew et al. [16] introduce Particle Transportation System. It is extended to model marble patterned color using color diffusion process.

Recently, Adam Runions et al. [17] proposed an algorithm for synthesizing leaf venation patterns. The algorithm is based on the biologically plausible hypothesis that venation results from an interplay between leaf growth, placement of auxin sources, and the development of veins. Shenglian Lu et al. [18] present a venation skeleton-driven method for modeling and animating plant leaf wilting.

The first computational model of venation patterns was a four-substance reaction-diffusion model proposed by Meinhardt [19]. A reaction-diffusion model leads to interesting patterns that emerge from a medium that is initially homogeneous and continuous. Two or more morphogens diffuse throughout the medium and interact with one another, resulting in the formation of patterns. Partial differential equations are used to mathematically model a diffusion-reaction system. Meinhardt [20] proposed a model to construct a network structure in a homogeneous environment. Another model of branching involves Gottlieb's geometrical model [21] in which the space expands uniformly. A grid system is used to recursively construct branches based on distance. The grid is initially 2x2 cells, and new branches result from connecting grid cell centers to the structure, provided the distance between the cell center and the structure is sufficiently large [22]. The space is then scaled and each cell is divided into a smaller 2x2 grid, and the process repeats until the desired level of detail is obtained.

III. LEAF SHAPE MODELING

Fig. 2 gives an overview how our proposed method works. Our approach to leaf shape generation is divided into two steps. The first step is for the leaflet distribution and the second is for their venations. The leaf is constructed by symmetrically a generating curve between

the starting and the ending points of the leaf. In the simplest case, the generating curve is a straight line segment. At the end of the modeling process, the surface can be extended to 3D by deforming (turning, bending, and twisting) the curve.

The object's venation is a branching structure. To represent it, we use the principle of the skeleton and use the axis as a *primary vein*. All veins that branching from the primary vein are called *secondary veins*. For our needs, we develop an interface for generating interactively venation skeleton from a leaf skeleton. As Fig. 2 shows the leaf skeleton can be obtained by using curve generator; a venation skeleton is then generated from the leaf skeleton, and each vein is segmented by a curve. The bending of this curve depends on the value and the sign of the parameter η . The result demonstrates a generated venation skeleton consisting of one main vein and number of attached secondary veins.

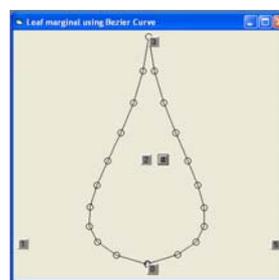


Figure 2. Leaf marginal generated by B-Spline curve editor

The leaf models discussed so far can be generated in two steps. The first step generates their marginal and the second generates their venation. A predefined starting leaf structure is two points. The line connected between these two points divides the leaf into two symmetric parts right and left sub-leaf. This line, called *axis*, became a central notion of leaf structure. One of these parts is created by connecting the two points by generated curve. The other part can generate by the same way using symmetric points with respect to the axis. The layout of this curve is captured by using a B-spline curve of degree four. The first and the last one identify the starting and ending of the curve and the two middle point's acts as a control points which can be defined interactively the curve and hence the leaf margin. Fig. 2 illustrates this process and Section 3.1 shows how to generate this curve as the leaf margin.

Leaf modeling using structure-based approach is very difficult. The leaf venation structure is extremely complex. In this paper, the model exhibits a hierarchical organization of the veins. This can be done on two stages. The first stage is for selecting some points on the leaf margin that was generated from the first step. The second stage is for identifying some points on the axis by dividing it into equidistant subintervals or selects the first point as a center for all veins. The first level of leaf venation can be obtained by joining every point on the leaf axis with the same index of corresponding point on the leaf margin.

Various venation structures can be found in plant architecture. It is believed that venation patterns correlate closely with the taxonomic groups of plants and the shapes of leaves. Hickey [23] has given a classification for the leaf venation patterns, in which pinnate venation and actinodromous venation are two commonly found categories.

A. Leaf Margin

Let $S = \{Q_1, Q_2, \dots, Q_n\}$ be the sequence of n points (n even and $n \geq 2$) that construct leaf skeleton. Every two consecutive points of S define the half boundary of leaf or leaflet. The connection of Q_1 and Q_n define the primary vein of the leaf. In the simplest case ($n=2$), the generating model is a simple leaf. Fig. 3 shows the skeleton of compound leaf.

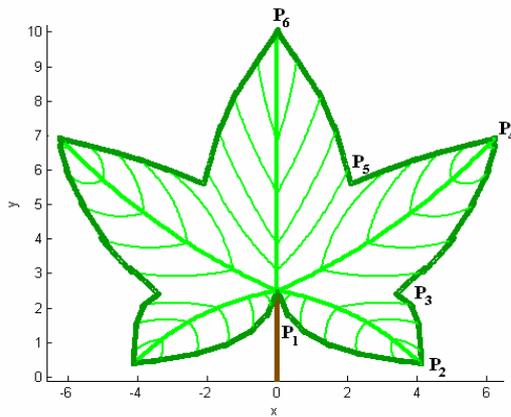


Figure 3. The skeleton of compound leaf ($n=6$)

The input of the leaf (or leaflet) is two points $Q_i = P_0$ and $Q_{i+1} = P_3$ which refer to the starting and the ending of the leaf model. It is obvious that the curvature and the blinding of the curve that passes from P_0 to P_3 depend on the position of the two middle points P_1 and P_2 (Fig. 2). A method to generate the position of these two points $P_i, i = 1, 2$ is described in the following formula:

$$P_i(X_i, Y_i, Z_i) = (X_j - D \frac{d_j^2}{m+1}, Y_j - D \frac{md_j^2}{m+1}, Z_j + \eta)$$

where m is the slope of the line P_0P_3 , D indicates the direction of the curvature if is positive (or negative) then the curvature on the left side (or right side) of the line P_0P_3 , and d_j represents the perpendicular distance between P_j and P_i in the plane $P_0, P_j, \text{ and } P_3$. The parameter η is the perpendicular distance in z -axis direction if is positive (or negative) then the direction above (or below) of the plane $P_0, P_j, \text{ and } P_3$. The position of a new point $P_j = (X_j, Y_j, Z_j)$ located on the line P_0P_3 , determined by the weight w_{j1} and w_{j2} of the two points P_0 and P_3 respectively, would then be:

$$P_j = w_{j1}P_0 + w_{j2}P_3; \quad j = 1, 2$$

such that $w_{j1}, w_{j2} \in \mathfrak{R}, \quad w_{j1} + w_{j2} = 1$

Fig. 4 (a) and (b) illustrates this process to generate the leaf margin that generated by using B-Spline curve of order four.

Given four points $P_0, P_1, P_2,$ and P_3 in 3D space the cubic B-Spline curve [2] is defined as

$$P(t) = at^3 + bt^2 + ct + P_0$$

where t ranges from 0 (the start of the curve, P_0) to 1 (the end of the curve, P_3). The vectors a, b, c are given as follows:

$$c = 3(P_1 - P_0)$$

$$b = 3(P_2 - P_1) - c$$

$$a = P_3 - P_0 - c - b$$

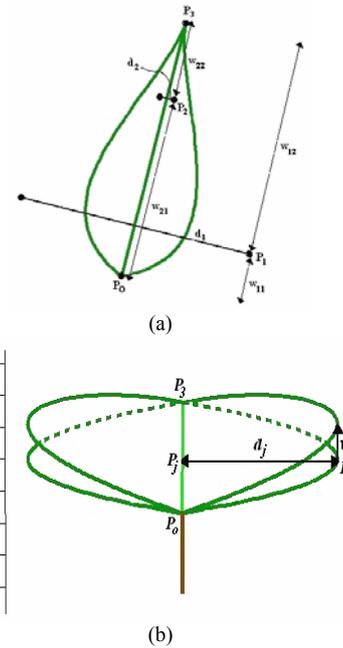


Figure 4. (a) Leaf boundary generated using P_1 and P_2 as the control points according to the curvature value $w_{j1}, w_{j2} \in \mathfrak{R}$, and (b) the blinding parameters η

During the generation of the points of leaf margin, the algorithm will select sequence of points every δ subinterval. It is possible to determine the starting index parameter σ to identify the starting index of points that generated on the leaf margin. These points are called *marginal-control points*, will be the terminal touch of the secondary veins that will generated from the primary vein as will describe in the next section. In addition, it is possible to join every two consecutive points of these points as a new B-Spline curve. This is a situation in

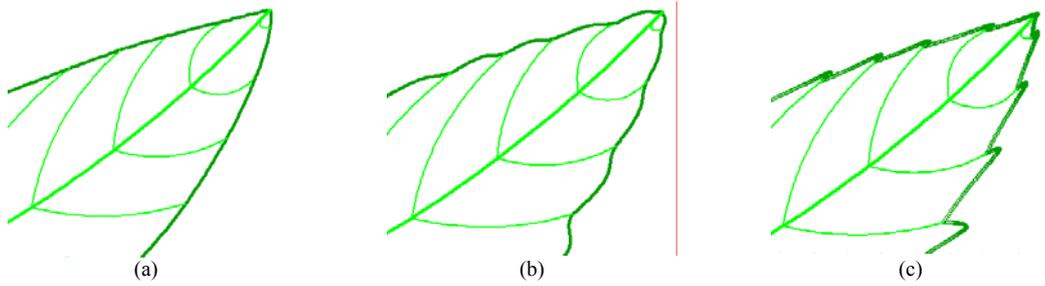


Figure 5. Different types of the leaf margin according the value of the parameter (a) $\mu = 0$ (b) $\mu = 1$, (c) $\mu = 2$.

which the margin leaf may have an arbitrary boundary attribute that augmented by a serrated edge capability. Fig. 5 illustrates different types of the leaf margin according to the value of the parameter μ .

If N is the number of points that generated from P_0 to P_3 on the leaf margin then the following algorithm can generate the marginal-control points:

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Function GetControlPoints(  $P_0, P_1, P_2, P_3, \delta, \sigma, N$  )
    return Control_Points
    TA  $\leftarrow$  0   TC  $\leftarrow$  1   Inc = (TC - TA)/N   i, j  $\leftarrow$  1
    for T = TA  $\rightarrow$  TC step Inc
        P(x, y, z)(i)  $\leftarrow$  a point on the curve ( $P_0, P_1, P_2, P_3, T$ )
        if (mod(i,  $\delta$ ) = 0 and (i >  $\sigma$ )) || (i =  $\sigma$ )
            Control_Points(j)  $\leftarrow$  P(x, y, z)(i)
            j  $\leftarrow$  j + 1
        i  $\leftarrow$  i + 1
    plot3(P(x, y, z))
    
```

B. Leaf Venation

For our needs, we develop an interface for generating interactively venation skeleton from a leaf skeleton. The skeleton can be constructed interactively. However, a user must make sure that the skeleton meets a set of somewhat non-intuitive criteria to prevent the density and self-intersections in the generated branching. For this reason, the user may prefer to have the skeleton generated automatically.

We implemented a skeleton generator based on the primary vein. The same technique that used in the margin leaf can be used to generate the primary vein. During the generation of the points of primary vein, the algorithm will select sequence of points every δ subinterval. These points are called *vein-control points*. The secondary veins are created by joining every vein-control points with the corresponding index of the marginal-control points on the two sides of the leaf margin. It is possible to determine

the starting index parameter σ to identify the starting index of points that generated on the primary vein. In some cases, there is no need to the first m points on the vein and will be neglected to join the marginal-control points. Fig. 6 illustrates this case with the first leaflet.

In addition, the distribution points along the primary vein can construct according to the value of β such that $P_3 \leftarrow \frac{(\beta - 1)}{\beta} P_0 + \frac{1}{\beta} P_3$. If $\beta = 1$, the vein-control points will distribute along all points of the primary vein. If $\beta = 2$, the vein-control points will distribute along the half of the primary vein and so on. When $\beta \rightarrow \infty$ ($P_3 \leftarrow P_0$), all generated points on the primary vein will be on the first point $P_0(X_0, Y_0)$. The connection of the secondary veins depends on the blinding parameter η . Fig. 7 illustrates the leaf model with different values of the parameter of distribution points β .

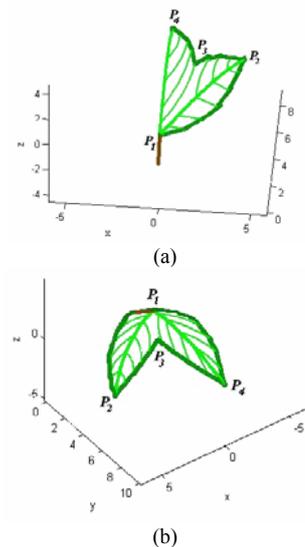


Figure 6. Some secondary veins will be neglected to join the marginal-control points according to the parameter σ .

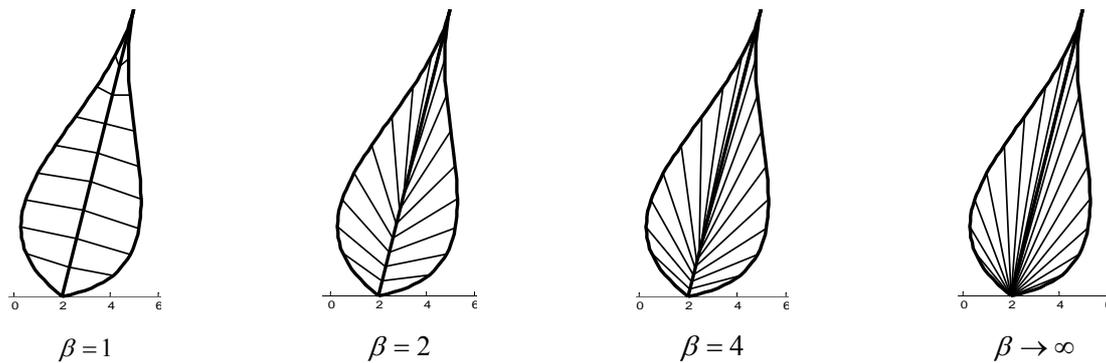


Figure 7. Leaf models with different values of β , the parameter of points distribution along the primary vein.

IV. EXPERIMENTS AND COMPARISONS

The process of generating leaf margin and leaf venation skeleton involve several manual interactions including defining the start point and end point for the leaf (simple leaf) or for each leaflet (compound leaf), and specifying parameters for the blinding and the curvature of leaf margin. Note that the venation skeleton can be created according to actual needs. Users can decide how many secondary veins are attached to the primary vein, and how these veins cross the primary vein.

There are a few parameters governing the generation of leaf model. They are divided into two groups: leaflet distribution and their venations. The distribution pattern determines the initial positions of each leaflet or vein length (P_0 and P_3) and their marginal including the curvature ($w_{j_1}, w_{j_2} \in \mathfrak{R}, w_{j_1} + w_{j_2} = 1, j = 1, 2$), the blinding (η), the width of leaf blade ($d_j, j = 1, 2$) of each leaflet and the property of the leaf margin if its margin (edge) forms a smooth arc ($\mu = 0$), a toothed arc ($\mu > 0$) if the margin has small protrusions. The parameters determining the venation have the same pervious parameters with different values and in addition how the secondary veins are attached to the main vein ($\beta > 0$). Table I shows the necessary values of the parameters for each leaflet according to its order in the half leaf in count clockwise direction. These values that the user may prefer to have the skeleton generated automatically. Fig. 8

shows the development of leaf structure using these parameter values with different number of leaflets. The user will enter the number of points of the leaf, marginal arc type and secondary veins distribution as displayed in Table II. Fig. 9 illustrates the leaves constructed in Fig. 8 have textures similar to the real leaves.

V. CONCLUSIONS AND FUTURE WORK

We have presented a model for modeling leaf surface. This model creates a leaf surface by driving the leaf margin and a venation skeleton. The venation skeleton can be created from primary vein and the secondary veins. The proposed method to generate venation skeleton makes it easy to create highly realistic leaf appearance models. Currently generating the venation skeleton is manual and interactive in our framework, and with a parametric method.

The leaf parameter variation model presented in this paper is an example of a model that provides intuitive control for simulating of some shapes of plant leaves. The orientation of the generating curve is adjusted to properly capture the shape of the leaf blade near the extremities and branching points of the skeleton. An exciting area for future work is the development of nonsymmetrical leaf in 3D using functions that control turning, bending, and twisting of contents of the leaf.

TABLE I.
THE PARAMETER VALUES OF SIMPLE LEAF (N=2) AND COMPOUND LEAF ($n \geq 4$)

Leaf Type	#Leaflet	Leaflet Index	Sub leaflet Index	Margin leaflet					Venation leaflet				
				Curvature		Blinding	starting index	subinterval	Curvature		Blinding	starting index	subinterval
				d_1	d_2	η	σ	δ	d_2	d_2	η	σ	δ
Compound $k=4, 6, \dots, n-2$	1	$P_1P_2P_3$	P_1P_2	1.2	1.1	-0.3	20	20	0.3	0.3	0.5	5	10
			P_3P_2	0.8	0.7	-0.3	20	20	0.3	0.3	0.5	20	10
	$i=k/2$	$P_{k-1}P_kP_{k+1}$	$P_{k-1}P_k$	0.5	0.4	-0.3	$\frac{\sigma_{i-1}}{2}$	$10i$	0.2	0.2	0.5	10	10
			$P_{k+1}P_k$	0.3	0.5	-0.3	$\frac{\sigma_{i-1}}{2}$	$10i$	0.2	0.2	0.5	10	10
$n/2$	$P_{n-1}P_n$	$P_{n-1}P_n$	0.8	0.7	-0.3	5	20	0	0	0.5	5	10	
Simple	1	P_1P_2	P_1P_2	2	0.5	-0.3	40	20	0	0	0.5	7	7

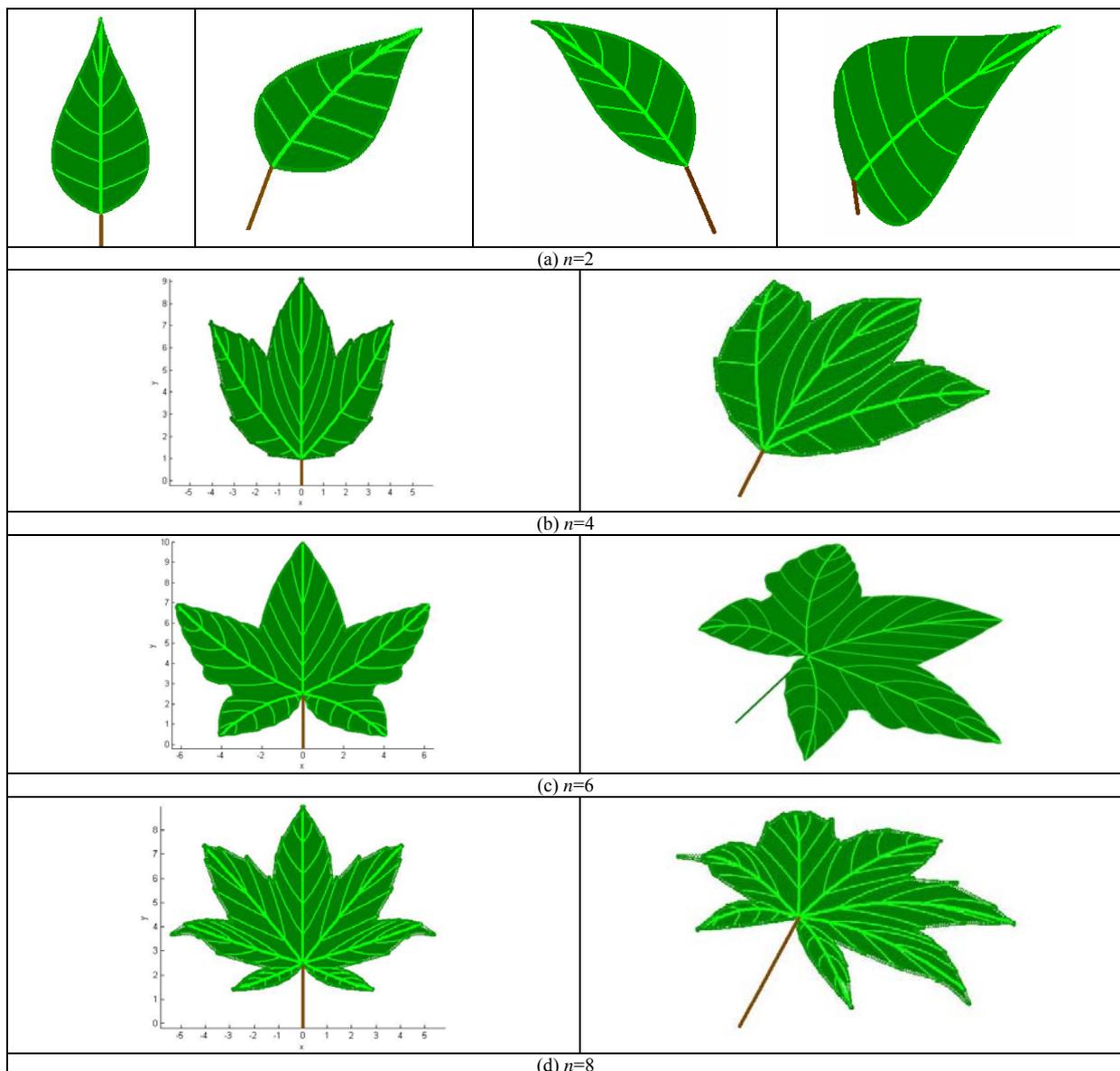


Figure 8. Leaves model with different shape according to the parameters values (Table II).

Table II.
THE PARAMETERS VALUES OF FIGURE 9(A)-(D)

Figure 9	Number of points	Marginal Arc type	Secondary Veins Distribution
	n	μ	β
(a)	2	0	1
(b)	4	2	1
(c)	6	1	1
(d)	8	2	1

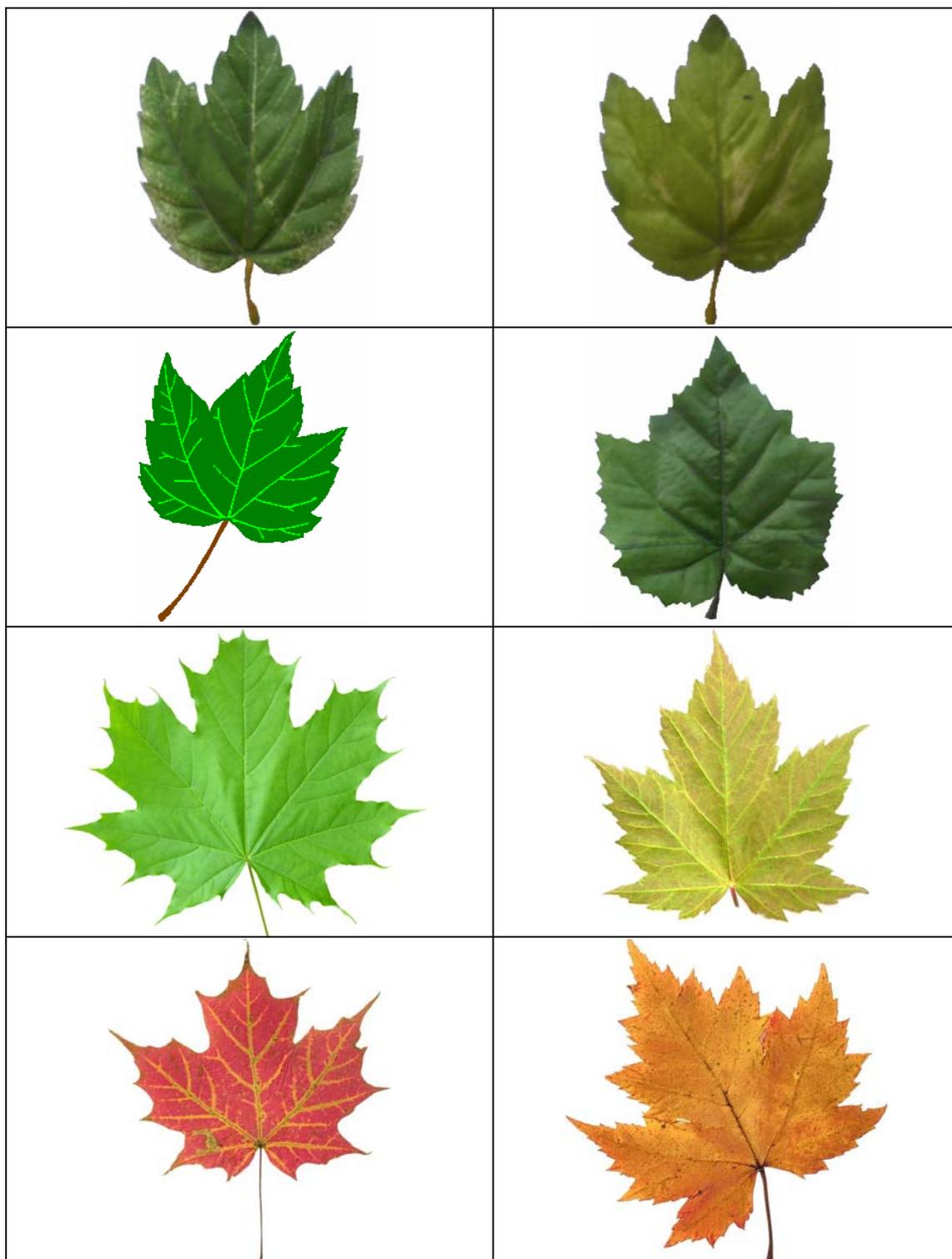


Figure 9. The leaves have textures similar to those in Figure 8.

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