

# Assigning Method for Decision Power Based on Linguistic 2-tuple Judgment Matrices

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**Abstract**—Decision power is very important in group decision making, which effects the final decision making result. When there exists uncertainty in group decision making, it is easy for an expert to express his/her preferences using fuzzy linguistic term such as ‘good’, ‘very good’. The linguistic 2-tuple representation model was selected to represent fuzzy linguistic term for its accuracy in representing fuzzy terms and less loss of information in processing. To obtain the objective decision power in group decision making where the experts express their opinions using linguistic 2-tuple judgment matrices, the paper put forward a new approach to assign decision power based on element consistency level. And the method for calculating the contribution degree of each expert was also provided. The proposed method thought of calculating the different weight to aggregate different element according to the element consistent level. An illustrated example was used to demonstrate the proposed method. And a GDSS based on the method was also developed to simplify the application in VB.NET. The GDSS was applied to building non-financial performance system for listed company in small and medium enterprise board in China, which showed its reasonability.

**Index Terms**—decision power, linguistic 2-tuple, group decision making, element consistency level

## I. INTRODUCTION

With the explosion of knowledge and information, the decision-making problem becomes more and more complicated, a decision-maker can not resolve it alone because of his/her limited experience and wisdom[1]. The decision-maker need consult with other decision-makers to seek more knowledge and information to deal with the complicated situations. Group decision-making basically solicits opinions from experts and combines these judgments into a coherent group decision[2]. In group decision making, an expert (is also called the decision maker) has different effect on the final decision making because of his/her capability and the information he or she can obtain [2,3].

Bidily put forward a method that experts were asked to evaluate other experts and got the weight of each expert.

Yang (2004) analyzed the decision power assigning method proposed by Bodily and thought it was difficult for the expert to assess each other, then he put forward a new method to designate the experts’ weight in which a stimulus was added to observe the members in group decision making and obtained the experts’ weight according to their responses [4]. Ye and Hong (2006) studied the method to classify the experts and assigning the weight to experts by building interval-valued attribute value and clustering them[5]. Yu and Fan (2006)proposed a new maximal tree clustering analysis method based on the traditional ideas of maximal tree clustering method and the dynamic semantic representation[6]. Fedrizzi (1992) developed a GDSS based on clustering to classify experts and gave the experts different weights according its clustering result[7]. Zhou and Wei (2006) judge the consistency level and consensus based on the distance of matrices given by experts and proposed a new method for deriving posterior weight based on reliability of expert’s fuzzy judgment matrix[8]. Chen and Fan(2005) made statistical analysis on fuzzy linguistic judgment matrix and concluded that the relative errors of elements in a consistency judgment matrix obeyed normal distribution with mean zero, then they sorted the judgment matrices given by experts and obtained the importance of each expert[9]. Liao, Li and Lei (2006) used linear planning method to solve assigning weight problem in incomplete multi-attribute group decision making[10]. Herrera-Viedma, Chiclana, Herrera and Alonso (2007) studied the method to designate experts’ weights based on additive consistency in incomplete group decision making environment[11].

The decision power assigning methods mentioned above were objective method, which obtained the experts’ weight from the information given by them. These methods gave each expert a fixed weigh to aggregate all elements in the judgment matrices into group decision making. There exists different element consistency level in the judgment matrix given by an expert, it will be more reasonable to calculating the different decision power for aggregating different

element. Thus, the paper proposed a new assigning method for decision power based on linguistic 2-tuple judgment matrix, which considered the element consistent level in a judgment matrix.

The structure of the paper was organized as follows: Part II introduced some preliminaries fuzzy group decision and linguistic 2-tuple representation model, part III described the method to calculate the element consistency level, part IV illustrated the method to aggregate the experts' opinions into group decision making based on element consistency level, part V put forward the method to compute the contribution degree of an expert, part VI was an example to demonstrate the proposed method, part VII was the GDSS developed to simplify the usage of it based on the proposed idea and the result of non-financial performance indicators for listed company in small and medium board using the GDSS, and part VIII gave the conclusion.

II. SOME PRELIMINARIES

Fuzziness exists because there is uncertainty in decision making condition. Here fuzzy group decision making means using fuzzy language terms to express experts' preferences in group decision making. Fuzzy language term can be represented by inter-valued fuzzy number, fuzzy triangle number, linguistic indices, linguistic 2-tuple representation model and other representation method [12,13]. The 2-tuple linguistic presentation model can avoid information loss in processing and computing linguistic information, and maintain accuracy and consistency of linguistic information [12]. Gong and Liu (2007) proposed fuzzy information fusion method based on linguistic 2-tuple representation model, which could transfer other fuzzy information expressed by fuzzy interval-value or fuzzy triangular number into linguistic 2-tuple representation model[13]. Through the transfer model, fuzzy information expressed by other representation model or linguistic 2-tuple representation model can be fused together. Therefore, study on decision power designating method based on linguistic 2-tuple representation model is practical and meaningful.

Suppose there are n alternatives denoted as  $A=\{A_1, A_2, \dots, A_n\}$  in group decision making and m experts to make decision which is denoted as  $E=\{E_1, E_2, \dots, E_m\}$ . The experts use fuzzy linguistic terms to express their preferences on alternatives. And the fuzzy linguistic term set is composed of nine terms, which is denoted as  $S=\{s_0=\text{absolutely worse}, s_1=\text{extremely worse}, s_2=\text{much worse}, s_3=\text{worse}, s_4=\text{no difference}, s_5=\text{better}, s_6=\text{much better}, s_7=\text{extremely better}, s_8=\text{absolutely better}\}$ . And the linguistic terms were expressed by linguistic 2-tuple representation model.

A. Linguistic 2-tuple representation model and its operator

Suppose  $S=\{s_0, s_1, \dots, s_g\}$  be a set of labels assessed in a linguistic term set with odd elements, which has the following properties: ① ordered: when the index  $i \geq j$ ,

there must exist  $s_i \geq s_j$ ; ② a negation operator:  $Neg(s_i)=s_{g-i}$ ; ③ there exists a min and max operator:  $s_i \geq s_j$  means  $\max(s_i, s_j)=s_i$  and  $\min(s_i, s_j)=s_j$  [13].

Definition 1[14] Let  $\beta$  be the result of an aggregation of the indexes of a set  $S=\{s_0, s_1, \dots, s_g\}$ , for example, the result of a symbolic aggregation operation,  $\beta \in [0, g]$ .  $g+1$  is the cardinality of S. Let  $i = \text{round}(\beta)$  and  $\alpha = \beta - i$  be two values, such that,  $i \in [0, g]$  and  $\alpha \in [-0.5, 0.5]$  then  $\alpha$  is called a Symbolic Translation.

Definition 2[14] Let  $S=\{s_0, s_1, \dots, s_g\}$  be a linguistic term set and  $\beta \in [0, g]$  be a value representing the result of a symbolic aggregation operation, then the linguistic 2-tuple representation model that expresses the equivalent information to  $\beta$  can be obtained with the following function:

$$\nabla : [0, g] \rightarrow S \times [-0.5, 0.5]$$

$$\nabla(\beta) = (s_i, \alpha), \text{ with } \begin{cases} s_i, i = \text{round}(\beta) \\ \alpha = \beta - i, \alpha \in [-0.5, 0.5] \end{cases} \quad (1)$$

Where  $\text{round}(\cdot)$  is the usual round operation,  $s_i$  had the closest index label to  $\beta$ .

Proposition 1[14] Let  $S=\{s_0, s_1, \dots, s_g\}$  be a linguistic term set and  $(s_i, \alpha)$  be a linguistic 2-tuple representation model. There is always a  $\nabla^{-1}$  function, such that, from a 2-tuple it returns its equivalent numerical value  $\beta \in [0, g]$ , which is:

$$\begin{aligned} \nabla^{-1} : S \times [-0.5, 0.5] &\rightarrow [0, g] \\ \nabla^{-1}(s_i, \alpha) &= i + \alpha = \beta \end{aligned} \quad (2)$$

From definition 1, definition2 and proposition 1, we can conclude that the conversion of a fuzzy language term into a linguistic 2-tuple representation model consist of adding a value 0 as the symbolic translation, which is :

$$\theta(s_i) = (s_i, 0) \quad (3)$$

Operation model of linguistic 2-tuple representation model can be obtained according to the linguistic 2-tuple representation model.

(1) The negation operator [14]

$$Neg((s_i, \alpha)) = \nabla(g - (\nabla^{-1}(s_i, \alpha))) \quad (4)$$

(2) The aggregation operators

Definition 3[13] Let  $(s_1, \alpha_1), (s_1, \alpha_2), \dots, (s_n, \alpha_n)$  be a set with n linguistic 2-tuples, the average operator of linguistic 2-tuples  $\xi$  is:

$$\begin{aligned} \xi((s_1, \alpha_1), (s_2, \alpha_2), \dots, (s_n, \alpha_n)) &= (\bar{s}, \bar{\alpha}) \\ &= \nabla\left(\frac{1}{n} \sum_{i=1}^n \nabla^{-1}(s_i, \alpha_i)\right) \end{aligned} \quad (5)$$

Definition 4 [13] Let  $(s_1, \alpha_1), (s_1, \alpha_2), \dots, (s_n, \alpha_n)$  be a set with n linguistic 2-tuples and

$\omega = (\omega_1, \omega_2, \dots, \omega_n)$  be the related weighted vector with  $\sum_{i=1}^n \omega_i = 1$ , then the weighted average operator of linguistic 2-tuples  $\xi^\omega$  is

$$\begin{aligned} \xi^\omega((s_1, \alpha_1), (s_2, \alpha_2), \dots, (s_n, \alpha_n)) &= (\hat{s}, \hat{\alpha}) \\ &= \nabla(\sum_{i=1}^n \nabla^{-1}(s_i, \alpha_i) \omega_i) \end{aligned} \tag{6}$$

**B. Linguistic 2-tuple judgement matrix with additive consistency**

Definition 5 [15] Let  $P = (p_{ij}, \alpha_{ij})_{n \times n}$  be a linguistic 2-tuple comparison matrix and the element  $(p_{ij}, \alpha_{ij})$  represent the result of comparing two solutions. If the following propositions are right,

- (1)  $p_{ij} \in S; \alpha_{ij} \in [-0.5, 0.5]$
- (2)  $\nabla^{-1}(p_{ii}, \alpha_{ii}) = g / 2$  (7)
- (3)  $\nabla^{-1}(p_{ij}, \alpha_{ij}) + \nabla^{-1}(p_{ji}, \alpha_{ji}) = g$

$P$  is called a linguistic 2-tuple judgment matrix.

Definition 6 [15] Let  $P = (p_{ij}, \alpha_{ij})_{n \times n}$  be a linguistic 2-tuple judgment matrix, if  $\forall i, j, k \in I$ , elements in  $P$  has the properties of the formula (7), then  $P$  is called a linguistic 2-tuple judgment matrix with additive consistency.

$$\nabla^{-1}(p_{ij}, \alpha_{ij}) + \nabla^{-1}(p_{jk}, \alpha_{jk}) = \nabla^{-1}(p_{ik}, \alpha_{ik}) + g / 2, \forall i, j, k \in I \tag{8}$$

Theorem 1 [16] A linguistic 2-tuple judgment matrix with additive consistency  $P$  can be obtained from the following elements.

$$P_0 = ((p_{12}, \alpha_{12}), (p_{23}, \alpha_{23}), \dots, (p_{(n-1)n}, \alpha_{(n-1)n})) \tag{9}$$

**III. CALCULATE THE ELEMENT CONSISTENT LEVEL**

If the judgment matrix given by an expert is additive consistent, the formula(8) should be true, thus the element in it is identical with the indirect value based on additive consistency. In real decision making environment, however, it is difficult for an expert to give a judgment matrix that is completely additive consistent. One element in the given judgment matrix may have high similarity to its indirect value and another element may have low similarity to its indirect value because of the complexity and uncertainty of the problem and expert's limited knowledge. Therefore, it is unreasonable to give the expert a fixed weight when the judgment matrices are aggregated into group decision judgment matrix. It would be reasonable to differ the expert's weight according to the similarity between the element and its indirect value. To measure the similarity between an element and its indirect value, the concept of element consistent level was introduced.

If a linguistic 2-tuple representation judgment matrix  $P$  ( $P = [(p_{ij}, \alpha_{ij})]_{n \times n}$ ) is additive consistent, there exists

$$\nabla^{-1}(p_{ij}, \alpha_{ij}) = \nabla^{-1}(p_{kj}, \alpha_{kj}) - \nabla^{-1}(p_{ki}, \alpha_{ki}) + g / 2 .$$

The property can be used to compute the indirect value of an element in the judgment matrix.

Based on the two properties of an additive consistent linguistic 2-tuple judgment matrix  $\nabla^{-1}(p_{ij}, \alpha_{ij}) = \nabla^{-1}(p_{kj}, \alpha_{kj}) - \nabla^{-1}(p_{ki}, \alpha_{ki}) + g / 2$  and  $\nabla^{-1}(p_{ij}, \alpha_{ij}) + \nabla^{-1}(p_{ji}, \alpha_{ji}) = g$ , the following formulas can be reasoned out.

$$\begin{aligned} \nabla^{-1}(p_{ij}, \alpha_{ij}) &= \nabla^{-1}(p_{ik}, \alpha_{ik}) + \nabla^{-1}(p_{kj}, \alpha_{kj}) - g / 2 \\ \nabla^{-1}(p_{ij}, \alpha_{ij}) &= \nabla^{-1}(p_{kj}, \alpha_{kj}) - \nabla^{-1}(p_{ki}, \alpha_{ki}) + g / 2 \tag{10} \\ \nabla^{-1}(p_{ij}, \alpha_{ij}) &= \nabla^{-1}(p_{ik}, \alpha_{ik}) - \nabla^{-1}(p_{jk}, \alpha_{jk}) + g / 2 \end{aligned}$$

Therefore, the indirect valued of an element in a linguistic 2-tuple judgment with additive consistency can be calculated through the neighbor elements. There are different elements in the judgment matrix can be used to compute an element's indirect value, thus, to assessment the indirect values comprehensively, the RMM(Row Mean Method) is used to calculate the indirect value of an element. The following formulas are induced from the formula(10).

$$cp_{ij}^{p1} = \frac{\sum_{\substack{k=1 \\ k \neq i, j}}^n (\nabla^{-1}(p_{ik}^p, \alpha_{ik}^p) + \nabla^{-1}(p_{kj}^p, \alpha_{kj}^p) - g / 2)}{n - 2} \tag{11}$$

$$cp_{ij}^{p2} = \frac{\sum_{\substack{k=1 \\ k \neq i, j}}^n (\nabla^{-1}(p_{kj}^p, \alpha_{kj}^p) - \nabla^{-1}(p_{ki}^p, \alpha_{ki}^p) + g / 2)}{n - 2} \tag{12}$$

$$cp_{ij}^{p3} = \frac{\sum_{\substack{k=1 \\ k \neq i, j}}^n (\nabla^{-1}(p_{ik}^p, \alpha_{ik}^p) - \nabla^{-1}(p_{jk}^p, \alpha_{jk}^p) + g / 2)}{n - 2} \tag{13}$$

The preceding three formulas can be used to calculate the indirect value of an element in a linguistic 2-tuple judgment matrix, the indirect valued can be expressed as follows.

$$cp_{ij}^p = \frac{cp_{ij}^{p1} + cp_{ij}^{p2} + cp_{ij}^{p3}}{3} \tag{14}$$

If the given judgment matrix given by an expert was not additive consistent, the indirect value of an element calculated by formula may not be in the scope of  $[0, g]$ . To deal with such situation, the formula(14) was revised as the following formula.

$$cp_{ij} = \max(\min(\frac{cp_{ij}^{p1} + cp_{ij}^{p2} + cp_{ij}^{p3}}{3}, g), 0) \tag{15}$$

The indirect value of an element in a linguistic 2-tuple judgment matrix is calculated based on its additive consistency. The element similarity between the element and its indirect value can be obtained through the distance between the element and its indirect value.

Definition 7 Let  $P = (p_{ij}, \alpha_{ij})_{n \times n}$  be a linguistic 2-tuple judgment matrix, if  $\forall i, j, k \in I$ . The element similarity between the element  $p_{ij}$  and its indirect value  $cp_{ij}$  is called the element consistent level, which can be denoted as  $cl_{ij}$ .

$$cl_{ij} = 1 - \frac{|cp_{ij} - \nabla^{-1}(p_{ij})|}{g} \quad (16)$$

$CL = (cl_{ij})_{n \times n}$  is the element consistent level matrix of a linguistic 2-tuple judgment matrix, which can be used to designate the weight in aggregating the experts' options into group decision.

IV. AGGREGATE THE EXPERTS' OPINIONS BASED ON ELEMENT CONSISTENCY LEVEL

There are many methods to aggregate the experts' opinions into group decision. The following three approaches are considering linguistic 2-tuple representation model and the element consistent level.

A. Aggregating method based on 2-tuple IOWA

Definition 8 Suppose  $f : R^n \rightarrow R$ , if

$$f(a_1, a_2, \dots, a_n) = \sum_{i=1}^n \omega_i b_i \quad (17)$$

where  $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$  is a vector related to f,

$\sum_{i=1}^n \omega_i = 1, \omega_i \in [0,1]$ , and  $b_i$  is the  $i$ th largest element in  $a_1, a_2, \dots, a_n$ , the function f is called n dimension OWA(Ordered Weighted Average) operator[18].

The different assignment of w represents the expert's character in decision making. Yager defined a concept called orness to gauge the optimistic degree of the expert[19,20]. The value of it is between 0 and 1, the larger the orness, the more optimistic the expert.

$$orness(\omega) = \frac{1}{n-1} \sum_{i=1}^n (n-i)\omega_i \quad (18)$$

In view of min operator, max operator and average operator of OWA, their orness values are  $orness([0,0, \dots, 1]) = 0$ ,  $orness([1,0, \dots, 0]) = 1$ ,

$$orness([\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}]) = 0.5, \text{ respectively.}$$

Suppose  $(p_1, \alpha_1), (p_2, \alpha_2), \dots, (p_n, \alpha_n)$  are linguistic 2-tuple representation model and are the inverse operators of them, then  $F(P, \beta, \omega)$  is called 2-TUPLE IOWA operator.

$$F(P, \beta, \omega) = \sum_{i=1}^n \nabla^{-1}(p_i^*, \alpha_i^*) \otimes \omega_i \quad (19)$$

Sequence  $\beta_1^*, \beta_2^*, \dots, \beta_n^*$  is the ordered sequence of  $\beta_1, \beta_2, \dots, \beta_n$  and  $\nabla^{-1}(p_i^*, \alpha_i^*)$  is the inverse operator of  $(p_i^*, \alpha_i^*)$ , which is the related linguistic 2-tuple representation model of  $i$ th element in  $\beta_1^*, \beta_2^*, \dots, \beta_n^*$ .

After the element consistency level is calculated, the comparison on a pair of alternative can be aggregated based on each expert's consistency level. The consistency level of experts  $cp_{ij}^1, cp_{ij}^2, \dots, cp_{ij}^m$  is sorted into the descend ordered sequence  $cp_{ij}^{1*}, cp_{ij}^{2*}, \dots, cp_{ij}^{m*}$ . The judgment matrix represented the group's preference on the alternatives is  $GP = (gp_{ij})_{n \times n}$ , which is obtained based on 2-tuple IOWA operator.

$$\begin{aligned} gp_{ij} &= F(((p_{ij}^1, \alpha_{ij}^1), (p_{ij}^2, \alpha_{ij}^2), \dots, (p_{ij}^m, \alpha_{ij}^m)), \\ & (cp_{ij}^1, cp_{ij}^2, \dots, cp_{ij}^m), (\omega_1, \omega_2, \dots, \omega_m)) \quad (20) \\ &= \sum_{k=1}^m \nabla^{-1}(p_{ij}^k, \alpha_{ij}^k) \otimes \omega_k \end{aligned}$$

where  $cp_{ij}^1, cp_{ij}^2, \dots, cp_{ij}^m$  indicates the consistency level of the experts' preference on alternative pair  $(X_i, X_j)$ , the higher element consistency level is, the higher weigh the expert gets. And the orness(w) is greater than 0.5.

B. Aggregating method based on element consistent level

The aggregating method based on 2-tuple IOWA give the expert higher weight if he/she gets high element consistency level. It does not think of the distance of the element consistency level between the experts, while the element consistency level of an expert may be almost same with the another expert's element consistency level, while the weights designate to them are different. To resolve such situation, an aggregating method based on element consistency level was put forward.

The expert's weight to aggregate his/her options on the pair of alternatives can be obtained based on element consistency level.

$$w_{ij}^k = \frac{cp_{ij}^k}{\sum_{k=1}^m cp_{ij}^k}, i \neq j \quad (21)$$

As the weight is assigned based on element consistency level, the experts' preferences on alternatives can be aggregated into group preference.

$$GP = (gp_{ij})_{n \times n}, \quad P_{ij} = \nabla(\sum_{k=1}^m p_{ij}^k \cdot w_{ij}^k) \quad (22)$$

The method shows the idea that the expert gets higher weight to aggregate his/her preference over the pair of alternatives if the related element gets higher consistency level. It also thinks of the difference between the element consistency level of experts. If the element consistent

level of two experts are similar, the weight assigned to them are also similar.

C. Aggregating method based on fuzzy quantifier and 2-tuple IOWA

There are many quantifier words in human language such as all, some, about 5, almost, at least half and so on. In traditional Logic there are universal quantifier and existential quantifier to express all and some, respectively, which is not enough to represent other fuzzy human language terms. To bridge the formula system and human language, Zadeh (1983) proposed the concept of fuzzy quantifier [21]. He used fuzzy set to describe fuzzy quantifier and classified the fuzzy quantifier as absolute fuzzy quantifier and relative fuzzy quantifier. Absolute fuzzy quantifier are quantifiers related to numbers, such as about 5, is greater than 5. And relative fuzzy quantifier can be represented by fuzzy set, such as almost, at least half. Yager defined formula for all, is great than  $\alpha$  and

$$\text{about } \alpha, \text{ which is } Q\left(\frac{i}{n}\right) = \begin{cases} 0, \frac{i}{n} < \alpha \\ 1, \frac{i}{n} = 1 \end{cases},$$

$$Q(r) = \begin{cases} 0, r < \alpha \\ \frac{r - \alpha}{n - \alpha}, r \geq \alpha \end{cases} \text{ and } Q(r) = \exp\left(-\left(\frac{r - \alpha}{\delta}\right)^2\right)$$

respectively.

Relative fuzzy quantifier has the property:  $Q(0) = 0, \exists r \in [0,1](Q(r) = 1)$ . The most common membership function for Regular Increasing Monotone(RIM) Quantifier is as follows[22].

$$Q(r) = \begin{cases} 0, r < a \\ \frac{r - a}{b - a}, a \leq r < b \\ 1, r \geq b \end{cases} \quad (23)$$

The weight in 2-Tuple OWA operator can be constructed by fuzzy quantifier operator. Suppose Q be the RIM fuzzy quantifier operator, the weight assigned to the expert  $E_i$  is  $w_k$ .

$$w_k = Q\left(\frac{\sum_{k=1}^i \mu_{\sigma(k)}}{T}\right) - Q\left(\frac{\sum_{k=1}^{i-1} \mu_{\sigma(k)}}{T}\right), T = \sum_{k=1}^s \mu_{\sigma(k)} \quad (24)$$

where Q is the Rim fuzzy quantifier defined by Yager  $Q(x) = x^\alpha, \alpha \geq 0$ , and  $\alpha = 1/2$  can be used as the operator for fuzzy quantifier ‘almost’ (Herrera studied the fuzzy quantifier ‘almost’ defined by Yager and found that  $\alpha = 2$  lead to inconsistency while  $\alpha = 1/2$  can not only keep the fuzzy quantifier regular increasing monotone but also keep the higher consistency. [22]).

Thus, the aggregating method based on fuzzy quantifier and 2-tuple IOWA can integrate experts’ opinions into group decision using induced ordered weight average operator, which sort the element

consistency level and use fuzzy quantifier to obtain the weight.

$$gp_{ij} = F(((p_{ij}^1, \alpha_{ij}^1), (p_{ij}^2, \alpha_{ij}^2), \dots, (p_{ij}^m, \alpha_{ij}^m)), (cp_{ij}^1, cp_{ij}^2, \dots, cp_{ij}^m), (\omega_1, \omega_2, \dots, \omega_m)) \\ = \sum_{k=1}^m \nabla^{-1}(p_{ij}^k, \alpha_{ij}^k) \otimes \omega_k \quad (25)$$

$$w_k = Q\left(\frac{\sum_{k=1}^i \mu_{\sigma(k)}}{T}\right) - Q\left(\frac{\sum_{k=1}^{i-1} \mu_{\sigma(k)}}{T}\right), T = \sum_{k=1}^s \mu_{\sigma(k)}$$

$$Q(r) = x^{\frac{1}{2}}$$

V. THE CONTRIBUTION DEGREE OF AN EXPERT

When an expert’s opinion is aggregated into group decision, the weight of each element in the judgment matrix may not be identical because of the different consistency of elements in the judgment matrix. Thus, the expert does not obtain a fixed weight. To measure the contribution degree of an expert in the group decision process, the paper used the following mechanism: aggregate the other experts’ option into the group decision  $GP^{-k}$  without expert  $E_k$  and compare it with the group decision GP made by all experts, the distance between GP and  $GP^{-k}$  can show the contribution degree of the expert  $E_k$ . The detailed process to calculate the contribution degree of experts can demonstrated as follows:

Step 1 calculate the group decision  $GP^{-k}$  without the expert  $E_k$ . The formula can be expressed as :

$$GP^{-k} = (gp_{ij}^{-k})_{n \times n} \\ gp_{ij}^{-k} = \nabla\left(\sum_{s=1, s \neq k}^m p_{ij}^s \cdot w_{ij}^s\right) \quad (26)$$

Step 2 compute the distance between  $GP^{-k}$  and GP, and it can be denoted as  $d(GP^{-k}, GP)$ .

$$d(GP^{-k}, GP) = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n d_{ij}, d_{ij} \\ = |\nabla^{-1}(gp_{ij}^{-k}) - \nabla^{-1}(gp_{ij})| \quad (27)$$

Step3 define the contribution degree according to  $d(GP^{-k}, GP)$ .

$$C^k = \frac{d(GP^{-k}, GP)}{\sum_{k=1}^m d(GP^{-k}, GP)} \quad (28)$$

The preceding steps were used to calculate the contribution degree of each expert, which should reflect the importance and the contribution of an expert in the group decision making.

VI. A DEMONSTRATED EXAMPLE

A project to build the non-financial performance of listed companies in SME board was carried last year. The

indicators such as the organization capability, customer relation management, quality of employee, the sustainability were used to assess the non-financial performance of the listed companies and denoted them as  $X=\{X1, X2, X3, X4\}$ . And a dean of Loan Department in China Bank, a CFO in a SME company, a CFO in a Security Agency, and a senior professor whose research interest focused on enterprise performance were invited to give their preference on the above indicators. The experts were asked to express their opinions in fuzzy language term. And the mentioned method was used to compute the weight of the indicators. The calculation process can be illustrated as follows:

(1) The preference on the non-financial performance indicators were expressed by using fuzzy terms, the fuzzy terms set  $S=\{s_0=\text{absolutely worse}, s_1=\text{extremely worse}, s_2=\text{much worse}, s_3=\text{worse}, s_4=\text{no difference}, s_5=\text{better}, s_6=\text{much better}, s_7=\text{extremely better}, s_8=\text{absolutely better}\}$ . And got the following judgment matrices.

$$P^1 = \begin{bmatrix} - & (s_3,0) & (s_7,0) & (s_8,0) \\ (s_5,0) & - & (s_6,0) & (s_7,0) \\ (s_1,0) & (s_2,0) & - & (s_5,0.1) \\ (s_0,0) & (s_1,0) & (s_3,-0.1) & - \end{bmatrix},$$

$$P^2 = \begin{bmatrix} - & (s_5,0) & (s_7,0) & (s_2,0) \\ (s_3,0) & - & (s_5,0) & (s_3,0) \\ (s_1,0) & (s_3,0) & - & (s_2,0) \\ (s_6,0) & (s_5,0) & (s_6,0) & - \end{bmatrix},$$

$$P^3 = \begin{bmatrix} - & (s_5,0) & (s_7,0) & (s_5,0) \\ (s_3,0) & - & (s_5,0) & (s_3,0) \\ (s_1,0) & (s_3,0) & - & (s_2,0) \\ (s_3,0) & (s_5,0) & (s_6,0) & - \end{bmatrix},$$

$$P^4 = \begin{bmatrix} - & (s_2,0) & (s_5,0) & (s_3,0) \\ (s_6,0) & - & (s_2,0) & (s_3,0) \\ (s_3,0) & (s_6,0) & - & (s_2,0) \\ (s_5,0) & (s_5,0) & (s_6,0) & - \end{bmatrix}$$

(2) Calculate the element consistency level in a judgment matrix based on additive consistency. The four judgment matrices obtained its related element consistent level matrices.

$$CP^1 = \begin{bmatrix} - & 0.75 & 0.88 & 0.88 \\ 0.75 & - & 0.88 & 0.88 \\ 0.88 & 0.88 & - & 1 \\ 0.88 & 0.88 & 1 & - \end{bmatrix},$$

$$CP^2 = \begin{bmatrix} - & 0.94 & 0.75 & 0.69 \\ 0.94 & - & 0.94 & 0.88 \\ 0.75 & 0.94 & - & 0.81 \\ 0.69 & 0.88 & 0.81 & - \end{bmatrix}$$

$$CP^3 = \begin{bmatrix} - & 0.88 & 0.94 & 0.94 \\ 0.88 & - & 0.94 & 0.94 \\ 0.94 & 0.94 & - & 1 \\ 0.94 & 0.94 & 1 & - \end{bmatrix},$$

$$CP^4 = \begin{bmatrix} - & 0.56 & 0.69 & 0.88 \\ 0.56 & - & 0.50 & 0.94 \\ 0.69 & 0.50 & - & 0.81 \\ 0.88 & 0.94 & 0.81 & - \end{bmatrix}$$

(3) Using aggregating method based on element consistent level to aggregate the experts' opinion into group decision. And we obtained the group decision following matrix.

$$GP = \begin{bmatrix} - & (s_4,-0.02) & (s_7,-0.42) & (s_5,-0.35) \\ (s_4,0.02) & - & (s_5,-0.19) & (s_4,-0.03) \\ (s_1,0.42) & (s_3,0.19) & - & (s_3,-0.17) \\ (s_3,0.35) & (s_4,0.03) & (s_5,0.17) & - \end{bmatrix}$$

Using RMM(Row Mean Method), the integrated assessment using linguistic 2-tuple representation model on each indicator was calculated and the comprehensive value of the organization capability, customer relation management, quality of employee, the sustainability was  $(s_5,0.07)$ ,  $(s_4,0.25)$ ,  $(s_2,0.48)$  and  $(s_4,0.18)$  respectively.

(4) Suppose the expert's ( $E_k$ ) opinion is not adopted in the group decision process, the other experts' opinions were aggregate into group decision  $GP_{-k}$ . The distance between the group decision  $GP_{-k}$  without the expert  $E_k$  and the group decision  $GP$  shows the expert's effect on the decision.

$$GP^1 = \begin{bmatrix} - & (s_4,0.29) & (s_6,0.42) & (s_4,-0.12) \\ (s_4,-0.29) & - & (s_3,0.34) & (s_3,0) \\ (s_2,-0.42) & (s_4,-0.37) & - & (s_2,0) \\ (s_5,-0.47) & (s_5,0) & (s_6,0) & - \end{bmatrix}$$

$$GP^2 = \begin{bmatrix} - & (s_4,-0.46) & (s_6,0.45) & (s_5,0.33) \\ (s_4,0.46) & - & (s_5,-0.27) & (s_4,0.27) \\ (s_2,-0.45) & (s_3,0.27) & - & (s_3,0.07) \\ (s_3,-0.33) & (s_4,-0.23) & (s_5,-0.07) & - \end{bmatrix}$$

$$GP^3 = \begin{bmatrix} - & (s_4,-0.42) & (s_6,0.41) & (s_5,-0.49) \\ (s_4,0.42) & - & (s_5,-0.27) & (s_4,0.3) \\ (s_2,-0.41) & (s_3,0.27) & - & (s_3,0.14) \\ (s_3,0.49) & (s_4,-0.3) & (s_5,0.14) & - \end{bmatrix}$$

$$GP^4 = \begin{bmatrix} - & (s_4,0.41) & (s_7,0) & (s_5,0.23) \\ (s_4,-0.41) & - & (s_5,0.32) & (s_4,0.3) \\ (s_1,0) & (s_3,-0.32) & - & (s_3,0.07) \\ (s_3,-0.22) & (s_4,-0.3) & (s_5,-0.07) & - \end{bmatrix}$$

The experts' contribution degree were obtained according to the distance between the exclude  $E_k$  group decision  $GP_{-k}$  and the complete group decision  $GP$ , Its detailed values were demonstrated in Table1.

Table 1 The contribution degree of each expert

	Expert $E_1$	Expert $E_2$	Expert $E_3$	Expert $E_4$
Contribution degree	0.40	0.207	0.168	0.225

Therefore, the expert  $E_1$  was most important decision maker in the group decision making, his contribution degree was 0.4. The expert  $E_3$  was the least important decision maker, his contribution degree of expert  $E_3$  was only 0.168.

VII. GDSS BASED ON LINGUISITC 2-TUPLE REPRESENTATION MODEL

The designating method for decision power designating based on linguistic 2-tuple representation model is calculated according to element consistent level. The calculation of the method is relatively complicated. To simplify the application of it, a GDSS (group decision support system ) is necessary, which can make the coordinator's job in group decision easier and enhance the application of it in practical group decision making.

The GDSS includes the following function models: initialization of parameters, the process of group decision making, query of decision data. The parameters initialization model initializes information such as experts, alternatives or indices, fuzzy linguistic terms set, data storage area. Group decision making process model deals with input data of expert's preference, calculation of decision making data. Query model provides

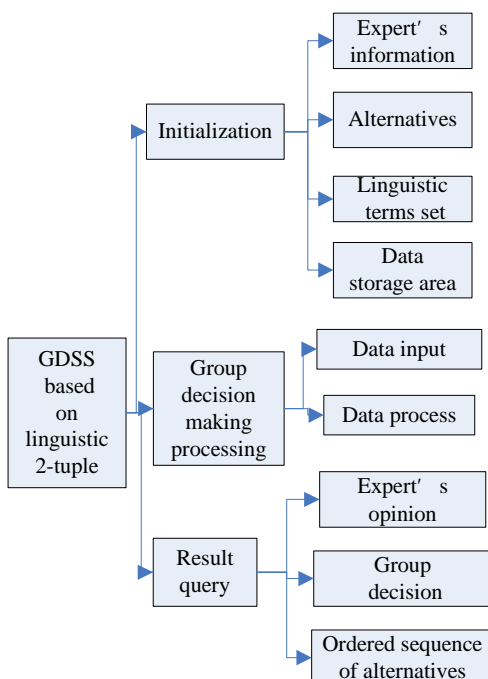


Figure 1 The modules of the GSDD

information querying about expert's preference, the element consistency level, group decision making result, the expert's contribution degree and the superior ordered sequence. The functional model of the GDSS can be demonstrated as Fig1.

Through interview and questionnaire investigation, the non-financial indicators for assessing performance of listed company in small and medium enterprise board in China were obtained. And the GDSS was used to deal with the related calculations and compute the weight of the indicators of the non-financial performance system for listed company in small and medium enterprise (SME) board. The detailed indicators and its relative weigh is demonstrated in Table 2.

Used the evaluation system proposed, 77 listed companies in SME board in Yangzi river Delta economic zone in eastern China were assessed. The assessment using this system was compared with other evaluation system and result testified the effectiveness of the evaluation system for listed company in SME board, which proved soundness of the proposed method

VIII. CONCLUSION

A new method was provided to designate the decision power of each expert under group decision-making based on linguistic 2-tuple representation. The basic idea for designating the decision power is that the consistency of each element is not same, thus the weight to aggregate expert's alternative comparison for each decision-maker should not be given the same priority. Based on the element consistent level, the responding individual aggregating method, the measurement for contribution degree of the decision-makers were proposed. And a demonstrated example was used to illustrate the idea.

Table 2 the non-financial performance system for listed company in SME board

Organization (0.2795)	Setting of department	0.077
	Modernization of management	0.086
	Quality of manager	0.067
	Frequency of accident	0.040
Customer management (0.2241)	Market ratio	0.053
	Satisfaction degree of customer	0.049
	Retain ratio of customer	0.052
	Efficiency of delivery	0.028
Employee's capability (0.1786)	Quality or product	0.042
	Education of employee	0.036
	Satisfaction degree of employee	0.028
	Team spirit	0.033
Sustainability (0.3214)	Ratio of employee's advice	0.018
	Loyalty of employee	0.032
	Training of employee	0.031
	Reputation	0.099
	creativity	0.091
	Fixed cost	0.054
	Energy saved and Pollution reduction	0.077

To make the calculation easier, the group decision-making support system under 2-tuple linguistic representation model was developed based on the researched theory. In acquiring the performance framework for small and medium entrepreneur board listed companies, the linguistic 2-tuple matrices which were given by experts from different companies. And the decision power of each expert on an element was calculated according to the additive consistency of the element. The final weight of each non-financial performance indicator was calculated according to the proposed method. The performance evaluation framework was build for small and medium entrepreneur board listed company and the weight of each indicator was obtained used the developed GDSS. Assessment for 77 listed companies in SME board in Yantz River economic zone was done and result testified the soundness of the method.

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