

Research on Parallel LU Decomposition Method and It's Application in Circle Transportation

Aimin YANG

College of Science, Hebei Polytechnic University, Tangshan 063009, China

Email : aimin_heut@163.com

Chunfeng LIU

College of Science, Hebei Polytechnic University, Tangshan 063009, China

Email : lcf403@163.com

Jincai CHANG

College of Science, Hebei Polytechnic University, Tangshan 063009, China

Email : jincai@heut.edu.cn

Xiaoqiang GUO

College of Science, Hebei Polytechnic University, Tangshan 063009, China

Email : gxqiang@heut.edu.cn

Abstract— This paper mainly designed a new method of using the Parallel LU Decomposition Algorithm for solving large-scale dense linear equations on the basis of the strategy of divide and rule, and analyzed the speedup and efficiency of the Parallel LU Decomposition Algorithm. In theory, it could improve the efficiency of problem-solving. In addition, the extension of Parallel LU Decomposition Algorithm, opened up a new idea of solving large-scale dense linear equations. This paper firstly introduced a solving method-LU decomposition algorithm of solving large-scale dense linear equations. And then described the related concepts and classification, the expression, the design and complexity metrics of the parallel algorithm etc. Put forwards the Parallel LU Decomposition Algorithm designed by the strategy of divide and rule. thus, conclusion that: In theory, it is not only more convenient and faster but reduces the computational complexity, which the parallel LU decomposition algorithm for solving large dense matrix. Finally, the parallel LU decomposition algorithm is used to solve a circling transportation problem.

Index Terms—LU Decomposition Algorithm, Parallel Algorithm, speedup and efficiency, inverse of matrix

I. INTRODUCTION

For a long time, in order to improve the speed of scientific computing, parallel computer develops rapidly. And make the large-scale scientific and engineering computing possible. Scientific computing is an important

field in the development of Artificial Intelligence and solving linear equations is the core of solving numerical calculation. Therefore, the speed optimization of solving large-scale linear equations has an extremely important significance to scientific computing.

Among modern scientific research, as their research field more and more widely, as well as the fast and accurate requirements of modern scientific research, so solving linear equations also need to improve efficiency. Solving linear equations question is the central issue in the field of matrix calculations. Among the many ways of solving linear equations, LU decomposition algorithm is a more common method. LU decomposition algorithm can be used to "high standard" analyze and can describe the elimination process of the Gauss, which increases the efficiency of solving linear equations. In order to further reduce the computing time of the LU decomposition algorithm, can use the parallel manner to parallel the matrix, and can analyze its speedup and efficiency theoretically while improving the algorithm to check whether this new algorithm can applied to practice for solving practical problems. Zhaoquan Cai, Wenhong Wei etc prospered a Parallel Algorithm based on a network of matrix multiplication, and analyze its speedup, efficiency, performance and scalability [1]; Zhaoquan Cai, Wenhong Wei, etc who proposed a network of architecture based on De Bruijn parallel matrix multiplication algorithm and analyze its speedup, efficiency, performance and scalability [1]; Long Tan, Jianzhong Li in Computer Science and Technology of college of Heilongjiang University, concrete realized the parallel computing of a matrix based on Mobile Agent parallel computing [2], Professor Yiming Chen of Yanshan University, Professor Chunfeng Liu, Ai-Min

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Corresponding author: Aimin Yang
Email: aimin_heut@163.com

Yang etc of Hebei Polytechnic University, and other studied in the improvement of basic numerical methods and parallelism, obtain a series of conclusions.

This paper firstly describes the basic content of LU decomposition algorithm and the parallel algorithm , by analyzing the inherent parallelism of LU decomposition algorithm, researched and designed a parallel LU decomposition algorithm on the basis of the strategy of divide and rule .and finally analyzed its speedup and efficiency theoretically . It provides a new method, thinking for the study of solving large-scale dense linear equations ,and is applied to solve circling transportation problems.

II. THE MEASURE STANDARD OF PARALLEL ALGORITHM

Speedup and efficiency are the two important indicators of assessing the pros and cons of a parallel algorithm, they are the most traditional evaluation criteria of a parallel algorithm ,reflected the advantages that obtained across using the parallel algorithms for solving practical problems on the parallel machine [7-10].

The speedup of the Parallel systems is that for a given application, the speed of parallel algorithm or parallel program execution relative to the serial algorithm or the serial implementation of the procedures accelerated

multiples, defined as $S_p = \frac{T_1}{T_p}$,

The efficiency of parallel algorithms, is a concept of closely related to speed-up ratio, defined as $E_p = \frac{S_p}{P}$.

In type, T_1 expressed a running time that a sequential algorithm in a single processor, T_p expressed a running time that a parallel algorithm running on P units processor, P is the number of processor units.

Clearly, the speedup S marks the saving of running time in the parallel algorithm, and the efficiency E characterizes the overhead of parallel algorithm in calculating .It is noteworthy that the main affected factors of speedup and efficiency of the parallel algorithm, in addition to lack the sufficient parallelism and data communications of the algorithm itself, there is also access to conflict resolution and synchronization overhead, the latter two cases would give rise to the idle of some units processors .

III. THE PARALLEL LU DECOMPOSITION ALGORITHM OF SOLVING LINEAR EQUATIONS

Most of the scientific and engineering computing problems can be translated into the form of linear equations, so effectively solving the linear equations is very important to the scientific and engineering computing. All along, it is a basic algebra problem in the numerical calculation of solving matrix-related problems.

With the development of parallel computers, the improvement of problem solving speed and the expansion of the problem-solving scale, numerical calculation methods and numerical packages are changing, while the effective parallel solution to the corresponding linear equations cause a widespread concern. [11-14]

An algorithm has its inherently ordered, , must follow certain steps before they can finally get useful results. On the contrary, it will get the wrong results. Although the parallel algorithm allow certain operations can be implemented in order, but because the data results produce on the order has its own requirements, making parallel algorithm, in general, is still an orderly manner. This inner order determines the present parallel algorithms are generally derived from the serial algorithm, most of them obtained through the development of the serial algorithm parallelism, which is mainly methods used in this following paper. The following through analysis of the LU decomposition serial algorithm from the aspects of matrix multiplication to discuss the parallel computing based on fast matrix multiplication. [15-16]

3.1 The LU decomposition Seria algorithm

Assuming that there is linear equations $Ax = b$, methods of $A = LU$, according to decomposition algorithm [17],so

$$A = \begin{bmatrix} 1 & & & & & \\ l_{21}^{(1)} & 1 & & & & \\ l_{31}^{(1)} & l_{32}^{(2)} & \ddots & & & \\ \vdots & \vdots & & 1 & & \\ l_{n1}^{(1)} & l_{n2}^{(2)} & \dots & l_{n,n-1}^{(n-1)} & 1 & \end{bmatrix} \times \begin{bmatrix} u_{11}^{(1)} & u_{12}^{(1)} & \dots & u_{1,n-1}^{(1)} & u_{1n}^{(1)} \\ & u_{22}^{(2)} & \dots & u_{2,n-1}^{(2)} & u_{2n}^{(2)} \\ & & \ddots & \vdots & \vdots \\ & & & u_{n-1,n-1}^{(n-1)} & u_{n-1,n}^{(n-1)} \\ & & & & u_{nn}^{(n)} \end{bmatrix}$$

3.2 Solving the linear equations of decomposition algorithm.

Assuming that there is linear equations $Ax = b$, when matrix A is a large dense nonsingular matrix,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & \dots & a_{1,n-1} & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & \dots & a_{2,n-1} & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & \dots & a_{3,n-1} & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & & \vdots & \vdots \\ \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots \\ a_{n,n-1} & a_{n-1,2} & a_{n-1,3} & \dots & \dots & a_{n-1,n-1} & a_{n-1,n} \\ a_{n1} & a_{n2} & a_{n3} & \dots & \dots & a_{n,n-1} & a_{nn} \end{bmatrix}$$

resolve the matrix A by the methods of LU:

according to actual condition,decided to use single measures for 5-ton load truck.So total need $(55 + 30 + 80 + 35) \div 5 = 40$ trains.

If the availability of eight trucks, each vehicle need to run five round-trip to complete the transportation task.

In transportation planning, under the premise of truck unloaded the return of goods, can consider to use local stability distribution, the dispatcher to avoid running according to the distribution of the transportation departments to provide a list, can be related to the following transport routes.

Dispatching officers need consider is in each truck unloaded again after the goods arrive which place,can return toloading,the overall transport benefit obtianed. According to the method, below return vehicle scheduling schemes.

Table 2 Transport to take charge sheet

| Delivery /Receiving Point | A ₁ | A ₂ | A ₃ | A ₄ | The number of start |
|---------------------------|----------------|----------------|----------------|----------------|---------------------|
| B ₁ | 3 | 8 | 9 | 6 | 11 |
| B ₂ | 10 | 7 | 12 | 4 | 6 |
| B ₃ | 5 | 3 | 6 | 8 | 16 |
| B ₄ | 7 | 9 | 4 | 1 | 7 |
| The number of receive | 11 | 6 | 16 | 7 | |

Table 3 Backhaul traffic flow program

| | A ₁ | A ₂ | A ₃ | A ₄ | |
|----------------|----------------|----------------|----------------|----------------|----|
| B ₁ | | | 11 | | 11 |
| B ₂ | 1 | | 5 | | 6 |
| B ₃ | 9 | | | 7 | 16 |
| B ₄ | 1 | 6 | | | 7 |
| | 11 | 6 | 16 | 7 | |

Upon examination,MaxS=33100 (\$100) is the optimal scheduling bus, such, can make the return vehicle scheduling general maximal profit[28].

According to the Freight traffic flow plan and return programs, organizations loop transport, can get four cycles cyclic route, namely:

1. A₁ ⇒ B₁ ⇒ A₃ ⇒ B₃ ⇒ A₁ (A₁ front line has already appeared,this circuit has been closed , Under similar)
2. A₁ ⇒ B₁ ⇒ A₃ ⇒ B₃ ⇒ A₄ ⇒ B₄ ⇒ A₁
3. A₁ ⇒ B₁ ⇒ A₃ ⇒ B₃ ⇒ A₄ ⇒ B₄ ⇒ A₂ ⇒ B₂ ⇒ A₁
- 4., A₂ ⇒ B₂ ⇒ A₃ ⇒ B₃ ⇒ A₄ ⇒ B₄ ⇒ A₁

Then, each route cycle times ,can satisfy the need transportation?

Suppose four lines' cycles were x₁, x₂, x₃, x₄, Is to be determined according to cycle transport program

Transportation scheme ,It has linear equations :

$$\begin{cases} x_1 + x_2 + x_3 = 11 \\ x_3 + x_4 = 6 \\ x_1 + x_2 + x_3 + x_4 = 16 \\ x_2 + x_3 + x_4 = 7 \end{cases}$$

By the parallel LU decomposition algorithm can get

$$(x_1 \ x_2 \ x_3 \ x_4)^T = (9 \ 1 \ 1 \ 5)$$

Namely, four line cycles followed by nine times, one times, one times, five times. It can satisfy the transport (11,6,16,7) requirements. Cycle routes and frequency, concrete implementation, consider the circular route starting and ending points and the interface between the two circular routes, it must be the garage to transport the location to determine the distance.

This chapter presents the transportation in linear programming, the transportation problem of circular solving linear equations using parallel LU decomposition algorithm for solving the solution, improve the speed of linear equations. To improve the speed of transport circulation problems to solve, and improve work efficiency.

V CONCLUSIONS AND FUTURE WORKS

In this thesis ,the first introduced to solve the linear equations LU decomposition algorithm and parallel algorithms of knowledge, then put forward a new algorithm - Parallel LU decomposition algorithm .with serial LU decomposition algorithm which embraced parallel and divide and rule, solving linear equations are dense an improved algorithm .with the method of solving linear equations of large populous conducted a detailed description.

In addition, based on parallel LU decomposition algorithm is efficiency of the accelerated and efficiency analysis. theoretically explain parallel LU decomposition algorithm and the serial LU decomposition algorithm, the high speed, solving problem solving efficiency is improved.

Finally, this paper expounds $n \times n$ order dense matrix inverse matrix of the process,and use the parallel LU decomposition algorithm for solving the transportation loop problem solving linear equations, provides an effective method.

VI CONCLUSIONS AND FUTURE WORKS

The web is a vast collection of completely uncontrolled heterogeneous documents. Fuzzy sets are suitable for handling the issues related to understandability of patterns, incomplete/noisy data, mixed media information and human interaction, and can provide approximate solutions faster. Genetic algorithms provide efficient search algorithms to select a model, from mixed media data, based on some preference criterion/objective function. To prevent the user from being overwhelmed by a large number of uninteresting patterns. A hybridization of fuzzy sets with genetic algorithms is described for Web mining in this paper. It is

based on a hybrid technique that combines the strengths of rough set theory and genetic algorithm. In future, we will research on merging simulation annealing and particle swarm optimization in data mining fields.

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Yang Aimin: lecturer, now acting as the Math teacher in Hebei Polytechnic University. He graduated from Yanshan University, majoring in calculus mathematics. Now he is the secretary of Parallel Computing Laboratory and Scientific Computing Laboratory. He has published 4 scientific computing books, over 20 papers and presided over 6 research projects. His research directions include the design and analysis of parallel computation, elastic problems numerical simulation, and etc.