CT Image De-noising Model Based on Independent Component Analysis and Curvelet Transform

Guangming Zhang[1]
[1] The Institute of Intelligent Information Processing and Application, Soochow University, Suzhou, China, 215006
Email: gmwell@gmail.com

Zhiming Cui[1,2], Jianming Chen[1] and Jian Wu[1,2]
[1] The Institute of Intelligent Information Processing and Application, Soochow University, Suzhou, China, 215006
[2] JiangSu Province Support Software Engineering R&D Center for Modern Information Technology Application in Enterprise, Suzhou, China, 215104
Email: szzmcui@suda.edu.cn

***Abstract*—CT image De-noising is an important research topic both in image processing and biomedical engineering. Independent component analysis (ICA) is a statistical technique where the goal is to represent a set of random variables as a linear transformation of statistically independent component variables. The curvelet transform as a multiscale transform has directional parameters occurs at all scales, locations, and orientations. This paper proposes a new model for CT medical image de-noising, which is using independent component analysis and curvelet transform. Firstly, a random matrix was produce to separate the CT image into a separated image for estimate. Then curvelet transform was applied to optimize the coefficients. At last, the coefficients were selected for image reconstruction by inverse of the curvelet transform. By contrast, this approach could remove more noises and reserve more details, and the efficiency of our approach is better than other traditional de-noising approaches.**

Keywords- independent component analysis; de-noising; curvelet; optimize

I. INTRODUCTION

In recent years, The independent component analysis (ICA) [1] as a kind of new signal processing method developed quickly and widely used ,especially in image processing. Meanwhile Candes and Donoho [2]develeped a new theory of multiresolution analysis called the curvelet transform. This mathematical transform differs from wavelet and related other mathematical transform. Curvelets take the form of basis elements, which exhibit a very high directional sensitivity and are highly anisotropic. In two dimensions, for instance, curvelets are localized along curves and in three dimensions along sheets. Because this new mathematic transform is based on the wavelet transform and radon transform. It has overcome some limitations of wavelet transform in medical image fusion. Because of the character of these two image processing method, we apply of independent component analysis and curvelet transform to optimize the coefficients for medical image de-noising.

As a signal analysis technique, ICA is a useful method for separating the independent signals from overlapping signals [3]. It was greatly developed as a potential statistical technique for blind source separation (BSS). It aims at finding the hidden components inside the original signals, and the components capture the essential structures of the signals. ICA is often used by the image processing. Through making full use of the high-order statistical characteristics of the source, i.e., the fourth-order central moment, ICA can effectively resolve the independent components (ICs) from the measured mixed signals without any additional information about the source signals. It had been widely applied in the signal processing fields, such as biomedical signals, image processing and financial analysis.

Curvelet transform [2]as a newly developed mathematical transform is often used as time-frequency and multiresolution analysis tool in the signal and image processing domain. It combined the anisotropic of ridgelet with the multiscale characteristic of wavelet. The prominent characteristic of curvelet is multiscale and high anisotropic, the curvelet transform is well-adapted to analyze and synthesize medical images containing edges. So in the view of the combination ICA technique and curvelet transform, this research is initial. For this reason, an image de-noising extended model based on ICA and curvelet transform are proposed in the paper.

Computed tomography (CT) [4]as a medical imaging method is widely used for diagnostic purposes. It is a method of body imaging in which a thin x-ray beam rotates around the patient. Small detectors measure the amount of x-rays that make it through the patient or particular area of interest. A computer analyzes the data to construct a cross-sectional image. These images can be stored, viewed on a monitor, or printed on film. In addition, three-dimensional models of organs can be created by stacking the individual images, or "slices". Due to its ability to provide clear images of bone, muscle, and blood vessels, CT imaging is a valuable tool for the diagnosis and treatment of musculoskeletal

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disorders and injuries. It is often used to measure bone mineral density and to detect injuries to internal organs. CT imaging is even used for the diagnosis and treatment of certain vascular diseases that undetected and untreated. By analyzing the characters of CT medical images, we find that when we denoised the medical image we should decompose the image by ICA. Because a great deal of medical image sequences are taken by the same instrument at different time, so the image decomposition signal could be optimized by curvelet transform.

In this paper we use CT medical image of human's abdomen as the research object, which is shown in Figure 1.

The CT medical image sequences of different parts of human' abdomen are shown in Figure 2.

There is always some noise produced by the CT imaging equipments and the processing of transmission. For this reason, this paper proposes a new model which could denoise the CT image of human's abdomen by ICA and curvelet transform. By experiment, the CT image after this approach's processing can display the details much more clearly and smoothly.

The rest of this paper is organized as follows. The independent component analysis is given in Sections II. Curvelet transform analysis is given in Sections III. Section IV describes the experiment which is using ICA and curvelet transform, then discusses the results. Conclusions are presented in Section V.

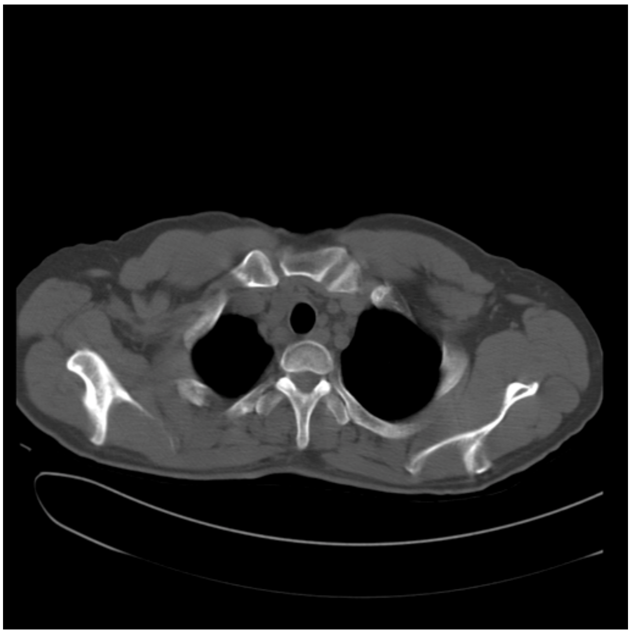


Figure 1. CT image of human's abdomen

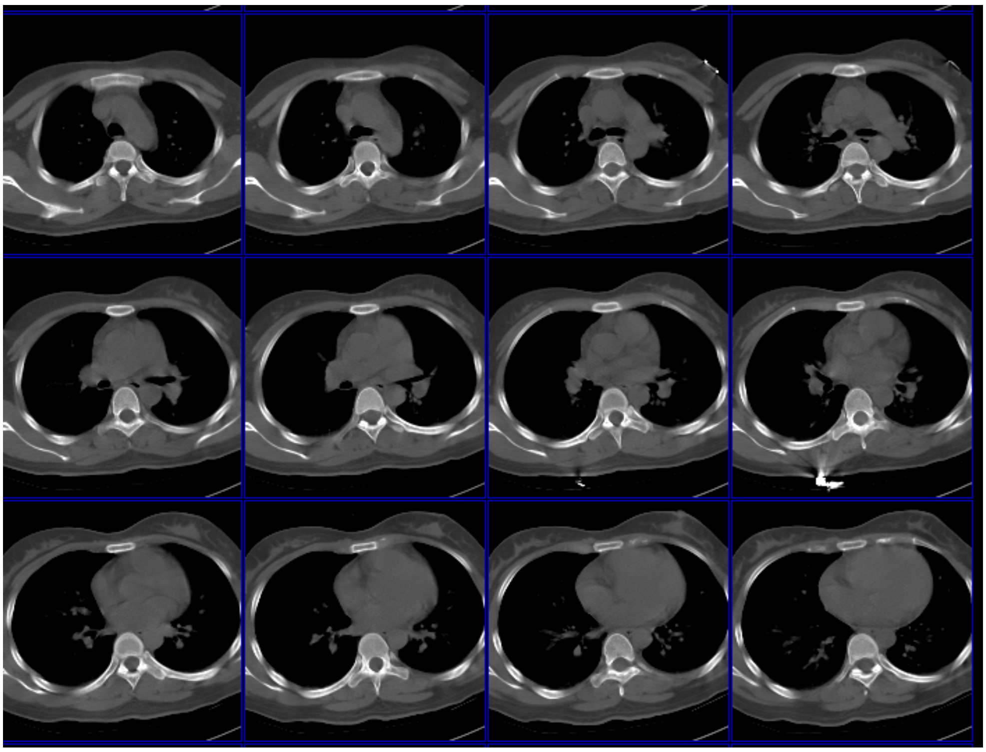


Figure 2. CT medical image sequences

II. Independent Component Analysis

Independent component analysis (ICA) [3]is a modern factor analysis tool developed in the last two decades, which researches the signal's independence relations according to higher order statistics, as opposed to the principal component analysis that researches the pertinent relation among signals based on the second order statistics. ICA can decompose the random to many mutual independent components which are the most possible independent. It is widely used for an image processing that can strengthen the signal non-Gauss. In order to get the de-noising independent components after separating, we can exchange it to the clean image. Fast ICA is a fast way of ICA which has good effect to the various noises.

The concept of ICA firstly put forward by Herault in 1988 [5]. The standard ICA model can be defined as follows:

$$\mathbf{x=As} \tag{1}$$

This model describes that the observation variables are mixed from source variable. The source variable is unknown, could not be observed, and matrix is not known yet. Only random variables can be observed. Consequently, we have to estimate matrix and the independent components according to a assumption: The source variable is statistic independent and non-gauss distributing. Through estimating matrix, we could get the contradictory of A, it also be called abruption matrix. So W estimate could be obtained from s .

$$\mathbf{u=Wx=WAs} \tag{2}$$

In order to separate a set of estimate $W=(W_1,W_2,\cdots,W_n)$ of independent statistic signal source $s=(s_1,s_2,\cdots,s_n)$ by independent component analysis from a set of observation signal $x=(x_1,x_2,\cdots,x_m)$ .The W is independent statistic. Suppose every signal is mixed linearly by independent components:

$$\begin{aligned} &x_i = a_{i1}s_1 + a_{i2}s_2 + \cdots + a_{in}s_n \\ &i = 1,2,\cdots,m. \end{aligned} \tag{3}$$

We supposed each observation variable x_i and each source variable s_j are all random variables and those mean are zero. Then Let the vector x as observation variable, and s as source variable, $A(m\times n)$ as matrix a_{ij} . Separate Mixing Image by ICA model was illustrated as follows:

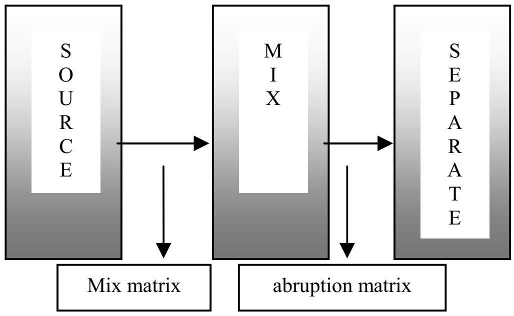


Figure 3. Separate Mixing Image by ICA

Fast ICA arithmetic is put forward by Hyvarinen [6-7]. By system learning this arithmetic find a way to let the projection $w^T x$ of the cell vector w is the most Gauss. Before running Fast ICA, assume it has pretreatment of ICA, such as get rid of mean and whiten processing. Fast ICA is to find the biggest of non-gauss $w^T x$ based on fixed-point theory. The Fast ICA separate only one independent component once from observation signal, so it is a fast way of ICA. It is useful to ICA image processing.

The goal of ICA is to restore the original sources by estimating the separating matrix. This can be achieved by optimizing contrast functions. Currently, there are a number of contrast functions in use including information maximization, maximum likelihood, high-order cumulants and negentropy. In the Fast ICA algorithm, the initial step is a preliminary whitening of the observations. By a linear transformation, the observations are made uncorrelated and unit-variance. The whitening facilitates the separation of the underlying independent signals, and it can be accomplished by classical PCA.

III. Curvelet Transform Analysis

The curvelet transform [2], like the wavelet transform, is a multiscale transform, with frame elements indexed by scale and location parameters. Unlike the wavelet transform, it has directional parameters, and the curvelet pyramid contains elements with a very high degree of directional specificity. In addition, the curvelet transform is based on a certain anisotropic scaling principle which is quite different from the isotropic scaling of wavelets. The elements obey a special scaling law, where the length of the support of frame elements and the width of the support are linked by the relation:

$$\text{width} \approx \text{length}^2$$

Curvelet is based on combining several ideas, which are briefly reviewed:

- Ridgelets, a method of analysis suitable for objects with discontinuities across straight lines.

- Multiscale Ridgelets, a pyramid of windowed ridgelets, renormalized and transported to a wide range of scales and locations.
- Bandpass filtering, a method of separating an object out into a series of disjoint scales.

A. Ridgelet Transform

The ridgelet transform is special member of the family of multiscale orientation-selective transforms, which has recently led to an advanced research activity in the field of computational and applied harmonic analysis. It has good directional selectivity and is able to locally and sparsely represent the signal when compared with the traditional transforms such as wavelet transform. As a new multiscale representation for functions on continuous spaces it is smooth away from discontinuities along lines. Ridgelet analysis makes available representations of functions by superpositions of ridge functions or by simple elements that are in some way related to ridge functions $r(a_1x_1 + \ldots + a_nx_n)$; these are functions of n variables, constant along hyperplanes $a_1x_1 + \ldots + a_nx_n = c$; the graph of such a function in dimension two looks like a "ridge". The terminology "Ridge function" arose first in tomography, and ridgelet analysis makes use of a key tomographic concept, the Radon transform [8].

Before the ridgelet transform, some attribute should be defined firstly as follows:

The Δsf layer contains objects with frequencies near domain $|\xi| \in [2^{2s}, 2^{2s+2}]$.

We expect to find ridges with width width ≈ 2^{-2s}

Windowing creates ridges of width width ≈ 2^{-2s} and length ≈ 2^{-2s}.

The renormalized ridge has an aspect ratio of $width \approx length^2$.

By using the ridgelet transform, we would like to encode those ridges efficiently.

There are some key properties before defined ridgelet transform as follows [9]:

- Divides the frequency domain to dyadic coronae:

$$|\xi| \in [2^s, 2^{s+1}]$$

- In the angular direction, samples the s-th corona at least 2^s times.
- In the radial direction, samples using local wavelets.

The ortho-ridgelet element has a formula in the frequency domain:

$$\hat{\rho}_\lambda(\xi) = \tfrac{1}{2}|\xi|^{-\frac{1}{2}}\left(\hat{\psi}_{j,k}(|\xi|)\cdot \omega_{i,l}(\theta) + \hat{\psi}_{j,k}(-|\xi|)\cdot \omega_{i,l}(\theta+\pi)\right) \quad (4)$$

where,

- ω_{il} are periodic wavelets for [-π, π).
- i is the angular scale and l∈[0, $2^{i-1}-1$] is the angular location.
- ψ_{jk} are Meyer wavelets for θ.
- j is the ridgelet scale and k is the ridgelet location.

Each normalized square is analyzed in the ridgelet system:

- The ridge fragment has an aspect ratio of $2^{-2s} \times 2^{-s}$.
- After the renormalization, it has localized frequency in band $|\xi| \in [2^s, 2^{s+1}]$.

A ridge fragment needs only a very few ridgelet coefficients to represent it.

We define an integrable bivariate function $f(x) \in R^2$ relative. The continuous ridgelet transform (CRT) [10]in R^2 is defined as follows:

$$CRT_f(a,b,\theta) = \int_{R^2} \psi_{a,b,\theta}(x) f(x) dx \quad (5)$$

where the ridgelets $\psi_{a,b,\theta}(x)$ in 2-D are defined from a wavelet-type function in 1-D $\psi(x)$ as follows:

$$\psi_{a,b,\theta}(x) = a^{\frac{1}{2}}\psi\left(\frac{x_1\cos(\theta) + x_2\sin(\theta) - b}{a}\right) \quad (6)$$

A ridgelet is constant along lines: $x_1\cos(\theta) + x_2\sin(\theta)$ = const. Transverse to these ridges it is a wavelet.

Given an integrable bivariate function f(x), we define its ridgelet coefficients by

$$R_f(a,b,\theta) = \int_{R^2} \psi_{a,b,\theta}(x) f(x) dx \quad (7)$$

The reconstruction formula

$$f(x) = \int_0^{2\pi}\int_{-\infty}^{\infty}\int_0^{\infty} R_f(a,b,\theta)\psi_{a,b,\theta}(x) f(x)\frac{da}{a^3}db\frac{d\theta}{4\pi} \quad (8)$$

The (separable) continuous wavelet transform (CWT) in R^2 of $f(x)$ can be written as follows:

$$CWT_f(a_1,a_2,b_1,b_2) = \int_{R^2} \psi_{a_1,a_2,b_1,b_2}(x) f(x) dx \quad (9)$$

where the wavelets in 2-D are tensor products

$$\psi_{a_1,a_2,b_1,b_2}(x) = \psi_{a_1,b_1}(x_1)\psi_{a2,b2}(x_2) \quad (10)$$

of 1-D wavelets, $\psi_{a,b}(t) = a^{\frac{1}{2}}\psi\left(\frac{t-b}{a}\right)$.

By comparison, we can see, the CRT is similar to the 2-D CWT except that the point parameters (b_1, b_2) are replaced by the line parameters (b, θ). That is to say, these 2-D multiscale transforms have the relations as follows [11]:

$$\text{Wavelets} \rightarrow \psi_{\text{scale,point-position}}$$

$$\text{Ridgelets} \rightarrow \psi_{\text{scale,line-position}}$$

Therefore, wavelets are very effective in representing objects with isolated point singularities, while ridgelets are very effective in representing objects with singularities along lines. In fact, one can think of ridgelets as a way of concatenating 1-D wavelets along lines. Hence the motivation for using ridgelets in image processing tasks is appealing since singularities are often joined together along edges or contours in images.

It is easy to extend the 1-D case to the 2-D case, points and lines are related via the Radon transform, thus the wavelet and ridgelet transforms are linked via the Radon transform. More precisely, denote the Radon transform as follows:

$$R_f(\theta,t) = \int_{R^2} f(x)\psi(x_1\cos\theta + x_2\sin\theta - t)dx \qquad (11)$$

then the ridgelet transform is the application of a 1-D wavelet transform to the slices (also referred to as projections) of the Radon transform,

$$CWT_f(a,b,\theta) = \int_{R^2} \psi_{a,b}(t) R_f(\theta,t)dt \qquad (12)$$

In the Fourier domain, the implementation of the ridgelet transform can be performed quickly [10].

- Firstly, compute the two dimensional Fourier transform, F (u, v) for the input image f (x, y). Using an interpolation scheme, substitute the sampled values of the Fourier transform obtained on the square lattice with the sampled values on a polar lattice.
- Secondly, Cartesian-to-polar conversion is used for an image of size n$\times$n, 2n.
- Thirdly, one-dimensional inverse Fourier transform is applied on each line, i.e., for each value of the angular parameter.
- Finally, one-dimensional wavelet transform is applied along the radial variable in Radon space.

Here, wavelet transform could be used in conjunction with nonlinear processing such as hard-thresholding of individual wavelet coefficients particularly.

B. Curvelet Transform

Generally speaking, curvelet transform extends the ridgelet transform to multiple scale analysis. This means that ridgelet can be tuned to different orientations and different scales to create the curvelets, It is in the similar to Gabor filters. But different from Gabor filters which only cover part of the spectrum in the frequency domain [12], curvelets have a complete cover of the spectrum in frequency domain. That means, there is no loss of information in curvelet transform in terms of fusing the frequency information from images.

The curvelet transform opens us the possibility to analyse an image with different block sizes, but with a single transform. The idea is to first decompose the image into a set of wavelet bands, and to analyze each band by a ridgelet transform. The block size can be changed at each scale level.

The Curvelet Transform includes four stages:

(1) Sub-band decomposition:

$$f \mapsto (P_0 f, \Delta_1 f, \Delta_2 f, \ldots) \qquad (13)$$

(2) Smooth partitioning:

$$h_Q = w_Q \cdot \Delta_s f \qquad (14)$$

A grid of dyadic squares is defined as follows:

$$Q_{(s,k_1,k_2)} = \left[\frac{k_1}{2^s}, \frac{k_1+1}{2^s}\right] \times \left[\frac{k_2}{2^s}, \frac{k_2+1}{2^s}\right] \in \mathbf{Q}_s \qquad (15)$$

$\mathbf{Q}s$ – all the dyadic squares of the grid.

(3) Renormalization:

$$(T_Q f)(x_1, x_2) = 2^s f(2^s x_1 - k_1, 2^s x_2 - k_2) \qquad (16)$$

$$g_Q = T_Q^{-1} h_Q \qquad (17)$$

(4) Ridgelet analysis:

$$\alpha_{(Q,\lambda)} = \langle g_Q, \rho_\lambda \rangle \qquad (18)$$

There is also procedural definition of the reconstruction algorithm. The Inverse of the Curvelet Transform:

(1) Ridgelet Synthesis:

$$g_Q = \sum_\lambda \alpha_{(Q,\lambda)} \cdot \rho_\lambda \qquad (19)$$

(2) Renormalization:

$$h_Q = T_Q g_Q \qquad (20)$$

(3) Smooth Integration:

$$\Delta_s f = \sum_{Q \in \mathbf{Q}_s} w_Q \cdot h_Q \qquad (21)$$

(4) Sub-band Recomposition:

$$f = P_0(P_0 f) + \sum_s \Delta_s(\Delta_s f) \qquad (22)$$

Curvelet transform is defined via above concepts.

IV. Experimental Results and Discussion

By using independent component analysis for image de-noising,it could separate a set of estimate of independent statistic signal source from a set of observation signal.

The key of ICA is to produce a appropriate matrix. This a random matrix (3×3):

$$A=\begin{bmatrix}0.4 & 0.2 & 0.4\\ 0.3 & 0.6 & 0.1\\ 0.5 & 0.2 & 0.3\end{bmatrix} \tag{23}$$

By using this matrix we could effectively resolve the independent components from the measured mixed signals without any additional information about the source signals.

Before the experiment, we get a lot of CT image from the First Affiliated Hospital of Soochow University. In our experiment, we adds Gaussian white noise of zero mean noise with 0.01variance and salt & pepper noise with 0.015 density to the image. Then we de-noise these CT medical images separately. The steps of this processing method are as follows:

- We separated the CT image by a random matrix into independent component image.
- Then we use curvelet transform to optimize the coefficients.
- At last, we applied inverse of the curvelet transform for image reconstruction a new denoised CT medical image.

The steps we presented are illustrated as follows:

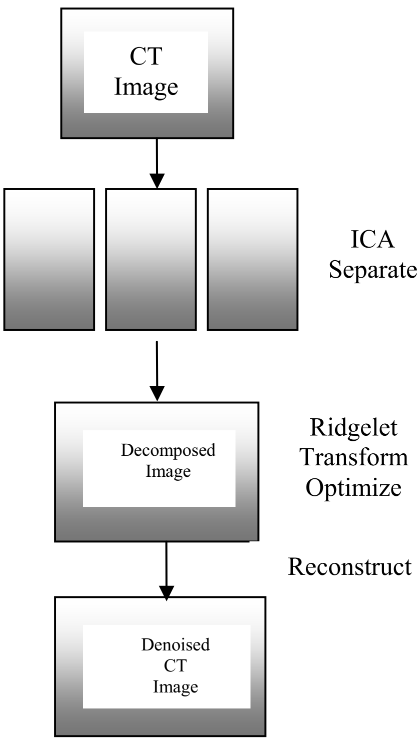
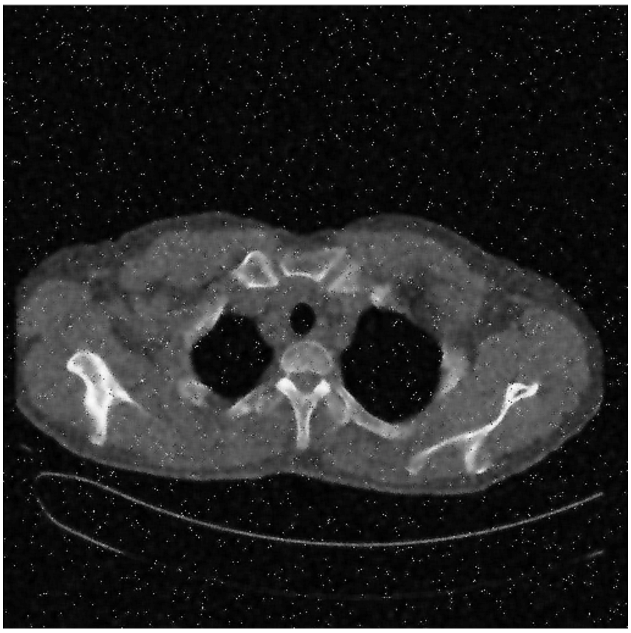


Figure 4. Image de-noising process

For the sake of testifying this paper's method is superior, we use other filter to process the source noise image which is shown in Figure 5(a). By contrast, the efficiency of our method which is shown in Figure 5(d) is better than Median and wiener approaches which are shown in Figure 5(b) and Figure 5(c) respectively. It removed Gaussian (white) noise and salt & pepper noise effectively, and many details were reserved. The results of different CT medical image de-noising methods are shown as follows:

(a)

(b)



(c)

(d)

Figure 5. Contrast Several Method of De-noising CT Medical Image

For the sake of evaluating the image quality objectively, we use reconstruction error as a standard objective measure of image quality. There are two of the error metrics used to compare the different image de-noising techniques, one is the mean square error (MSE) which is the cumulative squared error between the de-noising and the original image, and another is signal to noise ratio (SNR) which is a measure of the image error. The mathematical formulae can be expressed as follows:

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(I_{ij} - I_{ij}')^2 \tag{24}$$

$$\mathrm{SNR} = 10\lg[\sum_{\mathrm{i=1}}^{\mathrm{M}}\sum_{\mathrm{j=1}}^{\mathrm{N}}(I - I^{''})^2 / \sum_{\mathrm{i=1}}^{\mathrm{M}}\sum_{\mathrm{j=1}}^{\mathrm{N}}(I - I^{'})^2] \tag{25}$$

To contrast the effect of various de-noising methods, we define I(x, y) is the original image, I'(x, y) is the de-noising image, I''(x, y) is the mean value and M, N are the dimensions of the images. If a value for MSE is lower, that is to say the de-noising image has lesser error, and there is the inverse relation between the MSE and SNR, this create a high value of SNR. It means effect of de-noising is good because it show that the signal is more to noise in the image.

TABLE I. EVALUATION OF VARIOUS DE-NOISING METHODS

Processing Method	SNR	MSE
original	17.72	685
wiener	21.65	246
median	26.17	163
our method	37.35	97

From the table above we can see that our method have a higher value of SNR and a lower value of MSE. So it is superior to median and wiener filtering methods for CT medical image de-noising.

V. CONCLUSIONS

This paper proposes a new model for CT medical image de-noising, which is using independent component analysis and curvelet transform. Firstly, a random matrix was produce to separate the CT image into a separated image for estimate. Then curvelet transform was applied to optimize the coefficients. At last, the inverse of the curvelet transform was applied for image reconstruction. The de-noising image has a higher value of SNR and a lower value of MSE. This approach could remove more noises and reserve more details. Further investigations on the use of independent component analysis and curvelet transform to optimize 3-D medical image processing are left for future work.

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Guangming Zhang was born in Suzhou, China, in Feb. 1981. He is now a Ph.D. candidate in the School of Computer Science and Technology, Soochow University, China. His research interests include image processing, machine vision, and pattern recognition.

Zhiming Cui is a professor and doctoral advisor at the Institute of Computer Science and technology, Soochow University and a CCF senior member. His researches areas are intelligent information processing, image processing, distributing computing, DeepWeb data mining etc.

Jianming Chen is a associate professor and master advisor at the Institute of Computer Science and technology, Soochow University. His researches areas are intelligent information processing, multimedia processing.

Jian Wu was born in Nantong, 1979, and got master degree from Soochow university, Suzhou, China, in 2004. The main research direction is computer vision, image processing and pattern recognition. Now he is pursuing the doctoral degree.