Auto-Corner Detection Based on the Eigenvalues Product of Covariance Matrices over Multi-Regions of Support

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***Abstract*—In this paper we present an auto-detection corner based on eigenvalues product of covariance matrices (A-DEPCM) of boundary points over multi-region of support. The algorithm starts with extracting the contour of an object, and then computes the eigenvalues product of covariance matrices of this contour at various regions of support. Finally determine automatically peaks of the graph of eigenvalues product function. We consider that points corresponding to peaks of eigenvalues product graph are reported as corners, which avoids human judgment and curvature threshold settings. Experimental results show that the proposed method has more robustness for noise and various geometrical transform.**

***Index Terms*—Corner detection, Covariance matrix, Eigenvalues product, Region of support, Average repeatability, Localization error**

I. Introduction

Corner detection is an important task in various computer vision and image processing research. It is usually a front-end processing in a feature based image understanding system. Thus, the performance of corner detection has a great effect on the following processing and the whole system. Generally speaking, corner points should satisfy the following performances:

- Detection. All the true corners should be detected and no false corners should be detected.
- Robust. Corner detector should be robust with respect to noise.
- Localization. The corners should be well localized.
- Stability. The detected position of corner should not move when multiple images are acquired of the same scene.
- Complexity. Corner detector should be efficient.

This paper proposes a new scale-space corner detection method [1] [2] based on eigenvalues of covariance matrix. The curvature scale-space technique is suitable for extraction of curvature features from an input contour at a continuum of scales. This corner-detection method requires image edge contours. In the implementation of the new scale-space detector, a Canny edge detector [3] was used.

Much work has been carried out on corner detection, and Section 2 gives a review of corner detection methods. Section 3 briefly describes covariance matrix of data points on a digital boundary over a region of support and the relation with the curvature of contours. Section 4 presents an overview of multi-regions of support (multi-ROS) Eigenvalues product and the idea of auto-detection corners based multi-ROS eigenvalues product and then a simple analysis of proposed algorithm is presented. The performance of proposed method for corner detection is evaluated in terms of the average repeatability and localization error, the results of comparative experiments and a discussion of the results are given in section 5. The conclusions are presented in Section 6.

II. A Review of Corner Detection Methods

Considerable research has been carried out on corner detection in recent years. This section briefly reviews a number of proposed algorithms. Exiting corner detection techniques can be classified into two categories: intensity based corner detection (ICD) and contour based corner detection (CCD). We classify the intensity based corner detection technique into two types, namely template based methods and intensity gradient based methods. For the template based corner detection methods such as [4]-[6], mathematical models for corner structures are set up first, then correlations between the models and the image are used to detect the corners. As the models cannot cover all the types of corners that have different orientations and subtended angles, the performance is not satisfactory in practical applications. For the intensity gradient based corner detection methods such as [7] - [9], it is difficult to determine thresholds, and more sensitive to noise. Contour based methods have existed for a long time; some of more recent ones are presented. In this paper, we will focus on the contour-based approach.

The contour-based approaches are mainly used for shape description; curvatures of curves are the key to detect the salient points and to compute the shape descriptors. Points with high curvature on the boundaries are identified as corners. Now there are various methods such as [10]-[12] to detect corner. In recent years, the

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curvature scale space (CSS) technique has been applied successfully on planar-curve corner detection [13] [14], especially for those object shapes inherited with fractal-like features and noisy boundaries. For the CSS corner detection [14]-[20], the curve is represented as a parametric function of the arc-length. They then smooth the parameterized curve at different smoothing-scales [13]-[15] and calculate the absolute curvature on each point of the smoothed curve. Thereafter, they look for curvature maxima points as candidate corner set, from which the weak and false corners are eliminated using thresholds; the corners are detected at some specific scales, they are tracked down to the finest scale in order to improve localization [14,15]. The existing CSS corner detectors suffer from two key problems. The first problem is directly related to the curvature definition. By definition [13], the curvature means the instantaneous rate of change of tangential angle so that it is highly sensitive to the local variation and noise on the curve. In addition, the curvature estimation involves higher order derivatives of curve point-locations up to second order which cause errors and instability in results. The second problem is related to the curve smoothing using the Gaussian function. The aim of the smoothing is to reduce the effect of local variation and noise so as to remove weak and false corners. But determination of the proper Gaussian smoothing-scale is a very difficult problem. Consequently, smoothing with inappropriate Gaussian scales by the existing CSS detectors results in poor corner detection performance.

The first problem discussed above was an inherent problem with all the existing CSS corner detectors, so the improved or proposed methods aim at the second problem. Mokhtarian and Suomela[14] update the locations of the corners by the coarse-to-fine tracking. As the noise is smooth away and the sharp corners remain at high scale whereas the strong corners which might require low smoothing scales to be detected will trend to disappear, this detector still suffers from the second problem. Many improved methods (e.g., [15]), which select the smoothing-scales based on the curve-length, also fail to overcome this problem. Because even curves of the same length may require different smoothing-scales depending on the level of local variation and noise. The difference of Gaussian (DoG) detector [18] and the multi-scale curvature product (MSCP) detector [19] try to overcome the second problem to a great extent. They compensate the risk associated with possible inappropriate smoothing-scale selection by adopting different strategies. The DoG detector uses these scale evolution differences of planar curves to determine corners. The MSCP detector computes the curvature product of different smoothing-scales at each point, the curvature products of strong corners are bigger and the ones of weak corners are smaller. As a result, in terms of curvature product, the strong corners became more distinguishable from the weak corners. Consequently, both of them offered better corner detection performance than many of the aforementioned detectors.

In this section, we reviewed most of the main methods for corner detection, especially the CSS methods. Although a lot of methods are proposed or improved, the problems still exist. In this paper, we intend to propose a new algorithm that overcomes the aforementioned problems associated with the existing CSS corner detectors. We present a robust corner detection technique based on the eigenvalues of covariance matrix of data points on a curve segment for the discrete curvature estimation [1, 2]. The eigenvalues of covariance matrix discrete curvature estimation technique is less sensitive to the local variation and noise on the curve. It does not use any derivative of the curve-point locations at all. Moreover, it does not have the undesirable effect of the Gaussian smoothing. As a result, the proposed auto-detection corner based on eigenvalues product of covariance matrices corner detector in the paper greatly overcomes the problems associated with the existing CSS corner detectors and offers better performance.

III. COVARIANCE MATRICES

In this section, we present the eigenvalues of covariance matrix of data points on a digital boundary over a region of support and the relation with the prominence of a corner, which is the basis of the proposed A-DEPCM corner detector. Let n sequential digital points describe a boundary L of a P object,

$$L = \left\{ p_i = (x_i, y_i), \quad i = 1, 2, ..., n \right\}$$

Where (x_i, y_i) is the coordinate of the point p_i in the boundary and p_{i+1} is adjacent p_i on L. Let $S_k(p_i)$ denotes a small curve segment of L, which is defined by region of support (ROS) between points p_{i-k} and p_{i+k} for some integer k, and point p_i is the center of $S_k(p_i)$. That is

$$S_k(p_i) = \left\{ p_j \mid j = i-k, i-k+1, ..., i+k-1, i+k \right\}$$

Therefore, the covariance matrix C for point pi of a curve segment $S_k(p_i)$ is given as follows [1] [2]:

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \tag{1}$$

Where

$$c_{11} = \left[\frac{1}{2k+1} \sum_{j=i-k}^{i+k} x_j^2 \right] - c_x^2$$

$$c_{22} = \left[\frac{1}{2k+1} \sum_{j=i-k}^{i+k} y_j^2 \right] - c_y^2$$

$$c_{12} = c_{21} = \left[\frac{1}{2k+1} \sum_{j=i-k}^{i+k} x_j y_j \right] - c_x c_y$$

c_x and c_y are the geometrical center of $S_k(p_i)$, then

$$c_x = \frac{1}{2k+1} \sum_{j=i-k}^{i+k} x_j$$

$$c_y = \frac{1}{2k+1} \sum_{j=i-k}^{i+k} y_j$$

The covariance matrix C is symmetric and positive semidefinite. The eigenvalues λ_L and λ_S of the matrix C are as follows

$$\lambda_L = \frac{1}{2}\left[c_{11} + c_{22} + \sqrt{(c_{11} - c_{22})^2 + 4c_{12}^2} \right] \quad (2)$$

$$\lambda_s = \frac{1}{2}\left[c_{11} + c_{22} - \sqrt{(c_{11} - c_{22})^2 + 4c_{12}^2} \right] \quad (3)$$

From the formula of eigenvalues we can see $\lambda_L \geq \lambda_s$.

Tsai and Hou etc [2] present the eigenvalues of the matrix C can be used to extract the shape information about a curve and the smaller eigenvalues λ_S can be utilized to measure the prominence of a corner for each boundary point p_i over the curve segment $S_k(p_i)$. That is to say, the sharp corners have the large λ_S and the weak corners have the small λ_S. So the smaller eigenvalues λ_S can be approximate curvature of curves, corners can be determined according to λ_S value exceeding a predetermined threshold. They determine the threshold T_{λ_s} according to number of desired corners specified by a human viewer, and use a single ROS in detection procedure. Although this algorithm has a better detection performance, the corners of eigenvalues $\lambda_s > T_{\lambda_s}$ may not be correct because of the effect of noise, human and digital process. Since the local variation and noise on the curve are unknown, the different regions ROS of the same curve may result in different corners. Therefore, choosing an appropriate ROS for a given curve is a difficult task. In this paper, we present the automatic detection corners method by multi-ROS which compensates the shortage of a human viewer and single ROS.

IV. ANALYSIS OF MULTI-ROS EIGENVALUE PRODUCT

Curves of the same length may contain different types of corners, so choosing a ROS for a given curve is difficult. For example, when a single ROS is used, the corner detector will be sensitive to noise if the ROS is set too small, but locations of corners will be not accurate if the ROS is set too high. Therefore, we can compute curvature values at each point using multi-ROS curvature product, which can not only compensate the risk associated with a single region of support, but also make the strong corners more distinguishable than the weak and false corners. For example, Fig. 1 (b)-(d) shows the curvature estimation using formula (3) at ROS *12, 16, 20*, and Fig. 1 (e) shows the product of curvatures. From Fig. 1 (b)-(e), we can see that the small features and the noise are suppressed and the responses of the corners become more salient according to product of curvatures. The corners (curvature extrema points) are easily identifiable.

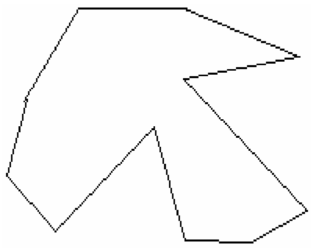
We define the concept of multi-ROS curvature product. Let $S_k(p_i)$ denotes the region of support of point p_i. According to (1)-(3), we can calculate the curvature λ_S at the jth ROS, and we define the concept of multi-ROS curvature product as follows:

$$P_n(p_i) = \prod_{j=1}^{n} \lambda_s(p_i, s_j) \quad (4)$$

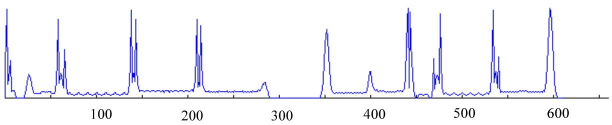
More generally the multi-ROS curvature product on an arbitrary set of different scales denoted by

$$P(p_i) = \prod_{s \in \Omega} \lambda_s(p_i, s) \quad (5)$$

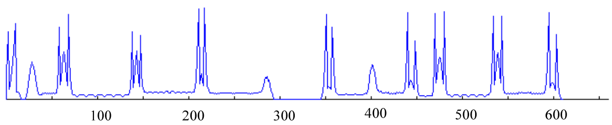
Where Ω is the set of different scales. In this paper, we calculate three curvature values at each point suing three ROS of different lengths. An example of the cross-scales product is given in Fig. 1.



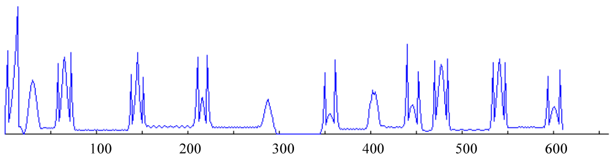
(a) Shape



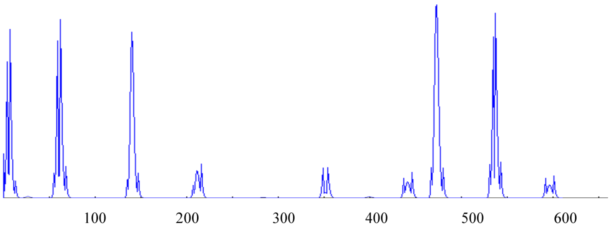
(b) ROS =12



(c) ROS =16

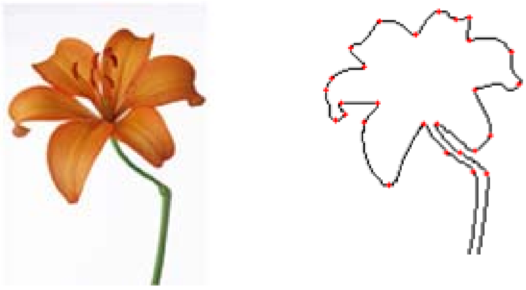


(d) ROS=20

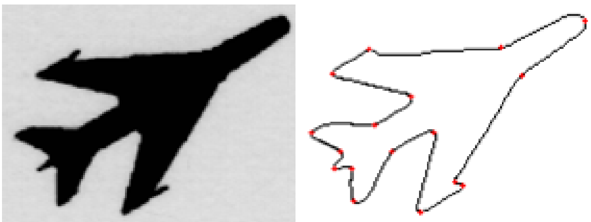


(e) Product of ROS =12,16,20

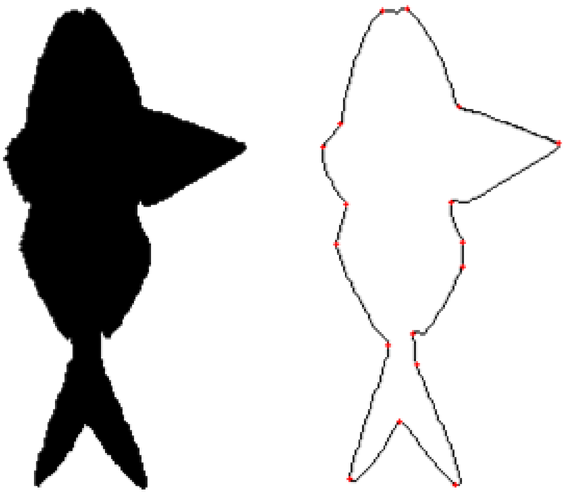
Figure 1. Eigenvalues of ROS: (a) original shape; (b)-(d) estimated curvature using different regions of support; (e) production of estimated curvatures using different regions of support.



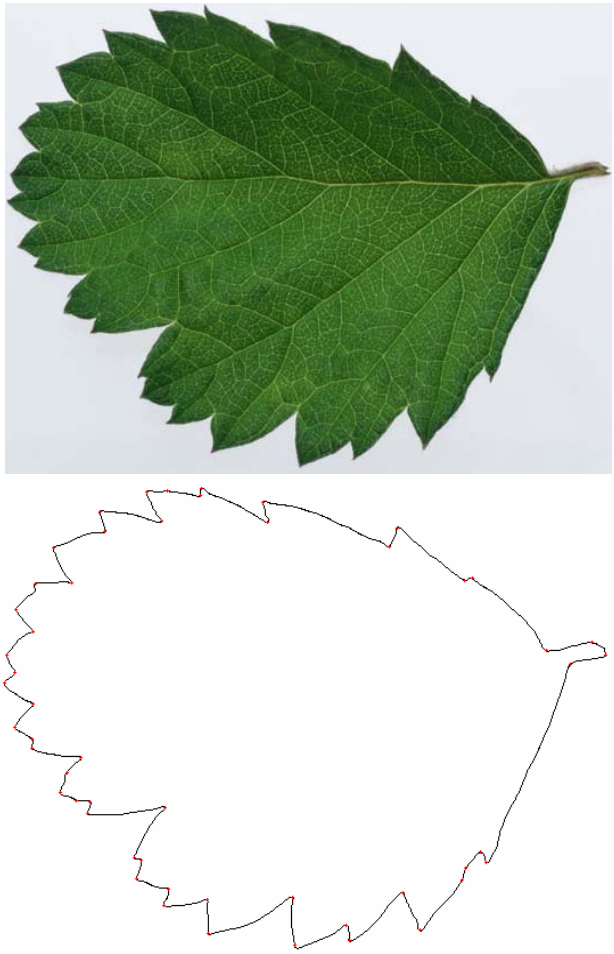
(a) Flower



(b) Airplane



(c)Shark



(d) Leaf

Figure 2. the proposed corner detector results for flower, airplane, shark and leaf respectively

Fig. 1 (e) shows an example of cross-scales product. The use of above curvature product has an additional advantage. The curvature product not only smoothes the noise and curve details, make the real corners salient, but also corners are well located. According to the above analysis, in this paper, we think peaks of curvature product graph corresponding points as corner set. That is to say, curvature product local maxima points are corners. Therefore, the method avoids not only the impact of human but also threshold settings. We show examples (see Fig. 2) by curvature product of three ROS of different lengths.

From the Fig. 2, the proposed detector can obtain satisfactory results. Expressly Fig. 2 (b) and (c), all true corners of airplane and shark can be detected and no false corner occurs, and that the location accuracy and reliability is apparent. Fig. 2 (d) shows that all the corner of leaf can be also detected, but there is one superfluous corner; Fig. 2 (a) shows that the few corners of flower are missed. Comparing Fig. 2 (b), (c) and (d) with (a), the proposed detector is more suitable for polygonal curve.

V. PERFORMANCE EVALUATION AND EXPERIMENTATIONS

In this section, we compare the performance of the proposed A-DEPCM corner detector with the existing MSCP detectors [19] and eigenvalues of covariance matrices [2] by using two criteria-average repeatability and localization error [12]. In the comparative study, the values of all their parameters are set to the values giving the lowest possible total error for all detectors.

A. Evaluation criteria

The traditional technique of robustness evaluation involves human visual inspection which is hard to implement for proper robustness tests due to following reasons [21].To measure automatically the performance of a corner detector in terms of the robustness of the detected corners and involve no human involvement. We introduce average repeatability and localization error [12] criteria for measuring the localization accuracy and stability of corner detectors, respectively.

The criterion of average repeatability makes use of the number of corners in original image, the number of corners in each of the transformed images as well as the number of matched corners between original and transformed images. It is defined as:

$$R_{avg} = \frac{N_a}{2}\left(\frac{1}{N_o} + \frac{1}{N_t}\right) \times {}^{100}\!/_{100}$$

Where N_o and N_t are numbers of corners detected in the original and transformed images respectively and N_a is the number of matched corners between them.

The localization error is defined as the amount of pixel deviation of a matched corner. It is measured as the root-mean-square-error (RMSE) of the matched corner locations in the original and transformed images:

$$L_e = \sqrt{\frac{1}{N_a}\sum_{i=1}^{N_a}(x_{oi} - x_{ti})^2 + (y_{oi} - y_{ti})^2}$$

Where (x_{oi}, y_{oi}) and (x_{ti}, y_{ti}) are the positions of i-th matched corner in the original and transformed images respectively. An RMSE value of maximum *3* pixels is allowed to find a matched corner.

Remark 1. We use R_{avg} to describe the stability of corner detectors under the rotation, scaling, affine transforms and noise disturbing. The value of R_{avg} for stable corner detectors should be close to 100%.

Remark 2. In the tests, RMSE is used to describe the error rate of corner detectors. The value of RMSE for stable and accuracy corner detectors should be close to zero.

B.Experimental Results

According to average repeatability and localization error criteria the comparative experiments are carried out for MSCP detector, the proposed auto-detection corner based on eigenvalues product of covariance matrices (A-DEPCM) corner detector, and eigenvalues of covariance matrices under rotation, scaling, affine transforms and noise disturbing. We use the twenty different original images including some artificial images like polygon and real world images like Airplane, Flower, Leaf, Fish, Shark, etc. Many of the above original images are collected from [22, 23] and other webs. In the comparative experiment, the number and the position of corners are extracted firstly from the original image. And then we have transformed images as test images, which are obtained by applying the five different types of experiments on each original image as follows:

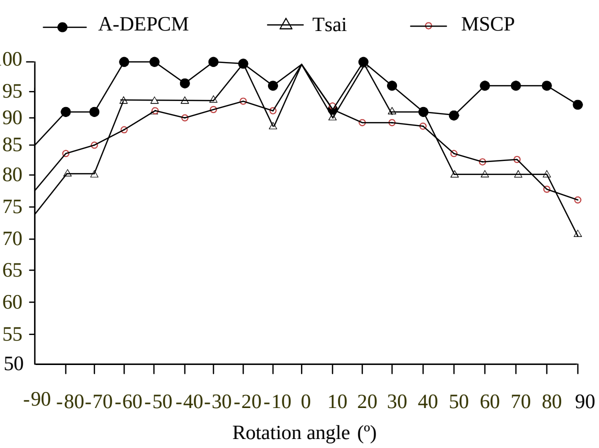
Experiment 1. Rotation: The original image is rotated with rotation angle chosen by uniform steps of the interval $[-90°, +90°]$, excluding $0°$. Distance between consecutive steps was $10°$. The number and the positions of corners in each rotated image are extracted.

Experiment 2. Uniform scale: The original image is zoomed with scale factor chosen by uniform scaling of the interval $[0.5, 1.5]$, excluding *1.0*. Distance between consecutive samples was *0.1*.

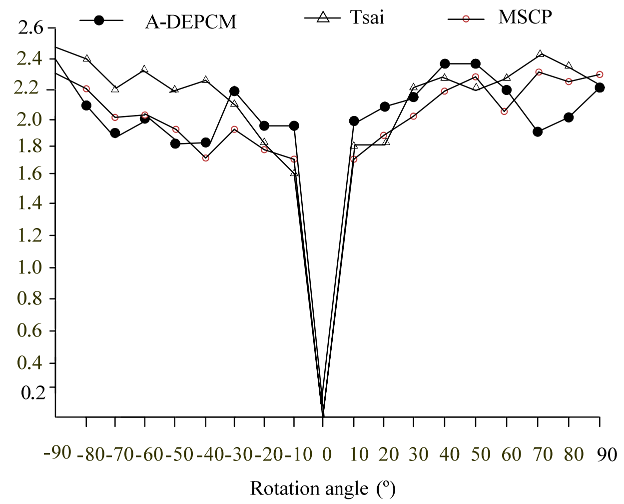
Experiment 3. Non-uniform scale: In this experiment, the original image is zoomed with scale factor by non-form scaling of the interval $[0.5, 1.5]$. Namely, Scale factors $s_x \neq s_y$ excluding the cases $s_x = s_y$.

Experiment 4. Affine transform (rot.-scale): Affine transform is applied to the original image. Here we applied rotation angles $10°$ uniform steps in $[-30°, +30°]$, excluding $0°$, followed by uniform or non-uniform scale factors s_x and s_y in $[0.5, 1.5]$ at *0.1*.

Experiment 5. Noise: In this experiment Gaussian white noise was added to the original image with zero-mean and variances chosen by uniform sampling of the intervals $[0.005, 0.05]$. Distance between consecutive samples was *0.005*.

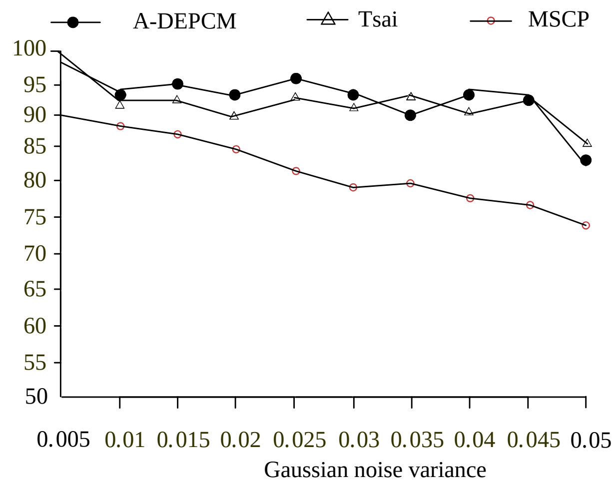


(a)

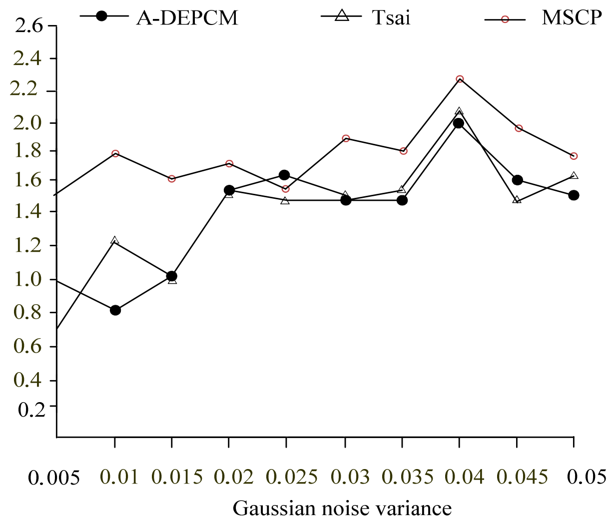


(b)

Figure 3. Average repeatability and localization error under rotation: (a) Average repeatability, (b) Localization error

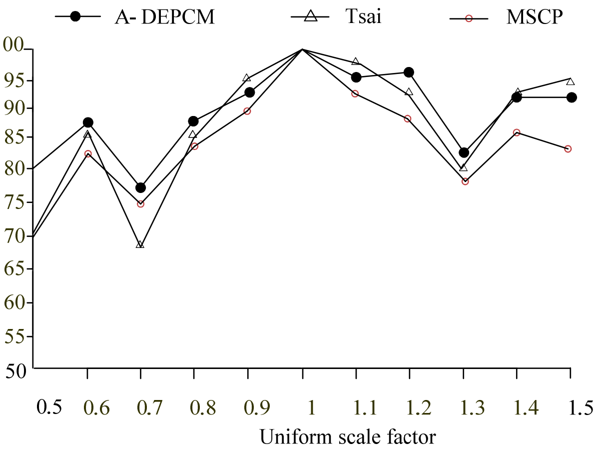


(a)

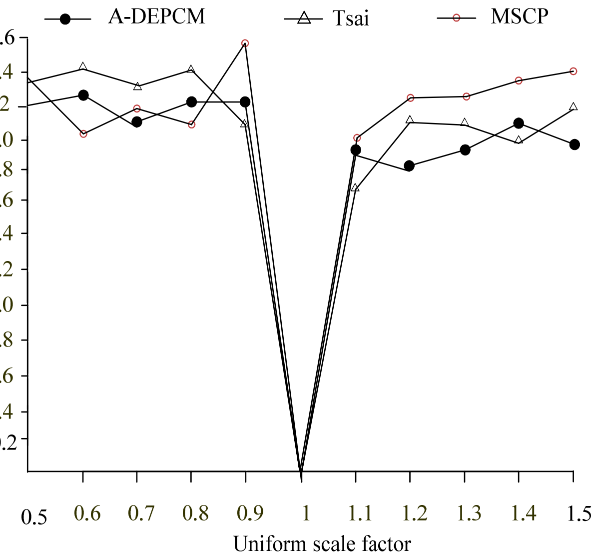


(b)

Figure 4. Average repeatability and localization error under Gaussian noise (a) Average repeatability, (b) Localization error

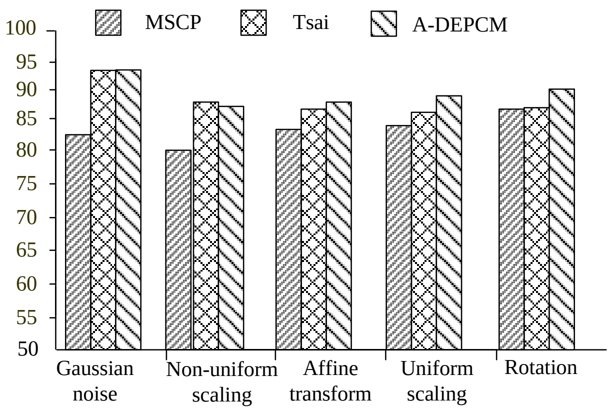


(a)

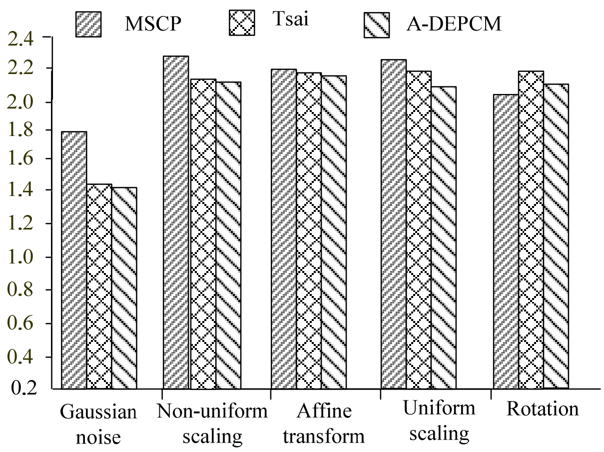


(b)

Figure 5. Average repeatability and localization error under uniform scale (a) Average repeatability, (b) Localization error



(a)



(b)

Figure 6. The overall performance under various geometric transformations and the noise disturbing: (a) Average repeatability, (b) Localization error (a) Average repeatability (b) Localization error

Figs. 3-5 illustrate the values of average repeatability and localization error under rotation, uniform scaling and noise disturbing respectively. Note that these values are averaged across the all different images. Fig. 6 shows the average repeatability and localization error under different geometric transformations and Gaussian noise. These results indicate that all detectors have the better performance. From Fig. 3-6, we know that all the detectors offered the high average repeatability, but the large localization error. The proposed A-DEPCM corner detector offered higher average repeatability and better localization error than Tsai and MCSP detectors. The MSCP detector offered the higher average repeatability, however, it suffered from localization error; the Tsai detector offered better localization error than MCSP detector. This is mainly because the curvature estimation of the A-DEPCM and Tsai detectors does not directly implement the "curvature definition" and, therefore, they don't use any derivative of curve-point locations at all. The difference in curvature estimation makes the A-DEPCM and Tsai detectors to be less sensitive to the local variation as well as the noise on the curve than MCSP detector. As the proposed detector makes use of the product of multi-ROS curvature to suppress the weak and spurious corners as well as to strengthen the true corners, it has the best stability and accuracy with respect to geometrical transform and noise disturbing and detects more true corners and less false ones.

C. The analysis of time complexity

In experiments, there are two main steps for the MCSP, A-DEPCM and Tsai corner detectors, namely, extracting edges from original images and detecting corners on edges. The MCSP, A-DEPCM and Tsai corner detectors utilize the Canny edge detector to extracting edges from original images, so their time complexity difference is determined by corner detector. Comparing A-DEPCM with Tsai corner detector, computing curvature time of Tsai corner detector is about $O(n*k)$, where *n* is the number of points on a curve and *k* is the length of a region of support. The A-DEPCM corner detector needs $O(m*n*k)$ (*m* different regions of support) for computing curvature of a curve, *m* equals to *3* in the paper. Corners are detected according the curvature of curves. Tsai corner detector utilize bubble sort to determining the maximum curvature of *c* (*c* is the number of corners by human judgment), which takes time about $O(c*n)$. Therefore, the total computational complexity of the Tsai corner detector is $O((c+k)*n)$. With respect to the proposed A-DEPCM method determining corners according to the curvature product graph, it needs about *n* times. So the computational complexity of the proposed method is about $O((m+k+1)*n)$. The time complexity of the MSCP method is mainly determined by the convolution, which is about $O((w+i+1)*n)$, where *w* is the size of sliding window, *i* is the number of different σ, and *1* is the times of comparing curvature of the curve with curvature threshold. According to the above analysis, the time complexities of the MCSP, A-DEPCM and Tsai corner detectors to detect the corners are all the polynomial, so they have basically the same as the time complexity.

VI. CONCLUSIONS

In this paper, we present a robust corner detection based on the smaller eigenvalues of the covariance matrix of boundary points over a small region of support. As the smaller eigenvalues of covariance matrix associates with the curvature of a point on the curve. Strong points have larger eigenvalues than weak corners. So we use the product of smaller eigenvalues of covariance matrix of three different regions of support to make sure corners, which makes strong corners more distinguishable from the weak corners. We consider that points corresponding to peaks of the eigenvalues product graph are constructed as corner set. Therefore, the proposed method avoids human judgment and curvature threshold settings. The detection and localization performance analysis are performed by two theoretical criteria: average repeatability and localization error. The experiments illustrate that the proposed A-DEPCM detector has generated good detection and localization under various geometrical transform and noise disturbing.

Corner is the representative feature and can depict shape of objects. So a robust as well as stable corner detector preprocesses images successfully and effectively for pattern classification, shape recognition, and computer vision. Our future work does point matching to determine the location of three-dimensional space.

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