# A Novel Medical Image Dynamic Fuzzy Classification Model Based on Ridgelet Transform

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***Abstract*—Medical image classification as an important research topic both in image processing and biomedical engineering. The ridgelet transform has good directional selective ability to locally and sparsely in representing the image compared with the traditional wavelet transform. This paper proposes a novel classification model for medical image, which is using ridgelet transform and dynamic fuzzy theory. Firstly, the image was decomposed by digital ridgelet transform to obtain the approximation coefficients and detailed coefficients in different sub-bands with directional parameters. Then the dynamic fuzzy theory was applied to construct a membership function to calculate coefficients from each sub-bands respectively, and a weight of sub-bands degree was adjust by precision requirement. At last similarity degrees are calculated by coefficients degree and weight. Medical images were classified by the result sort order of the degrees effectively.**

***Keywords-medical image classification; ridgelet transform; dynamic fuzzy; similarity degree***

## I. INTRODUCTION

In recent years, ridgelet transform[1] developed by Candes and Donoho as an answer to the weakness of the separable wavelet transform in sparsely representing what appears to be simple building atoms in an image, that is lines, curves and edges. Ridgelet transform [2] takes the form of basis elements which exhibit high directional sensitivity and are highly anisotropic. It has received more and more attention due to its particular characters. Many classical multiresolution ideas only address a portion of the whole range of possible multiscale phenomena, like the classical wavelet viewpoint, there are objects, e.g. images that do not exhibit isotropic scaling and, thus, call for other types of multiscale representation. This mathematical transform differs from wavelet and related other mathematical transform. The ridgelet transform can be computed by performing a wavelet analysis in the Radon domain. In two dimensions, points and lines are related via the Radon transform, thus the wavelet and ridgelet transforms are linked via the Radon transform. Because this new mathematic transform is based on the wavelet transform and radon transform. It has overcome some limitations of wavelet transform in medical image classification.

Computed tomography (CT) [3] as a medical imaging method is widely used for diagnostic purposes. It is a method of body imaging in which a thin x-ray beam rotates around the patient. Small detectors measure the amount of x-rays that make it through the patient or particular area of interest. A computer analyzes the data to construct a cross-sectional image. These images can be stored, viewed on a monitor, or printed on film. In addition, three-dimensional models of organs can be created by stacking the individual images, or "slices". Due to its ability to provide clear images of bone, muscle, and blood vessels, CT imaging is a valuable tool for the diagnosis and treatment of musculoskeletal disorders and injuries. It is often used to measure bone mineral density and to detect injuries to internal organs. CT imaging is even used for the diagnosis and treatment of certain vascular diseases that undetected and untreated. By analyzing the characters of CT medical images, we find that the background image and each images are taken by the same instrument at different time. And most of the medical images are gray pictures. Due to this character, when we classified the medical image we should analysis the similarity in a certain scope by using ridgelet transform decomposition. Because a great deal of medical image sequences are taken by the same instrument at different time, so the change trend of similarity could be used in classification process. This technique is also based on dynamic fuzzy theory.

The CT medical image sequences of different parts of human' body are shown in Figure 1 and Figure 2.

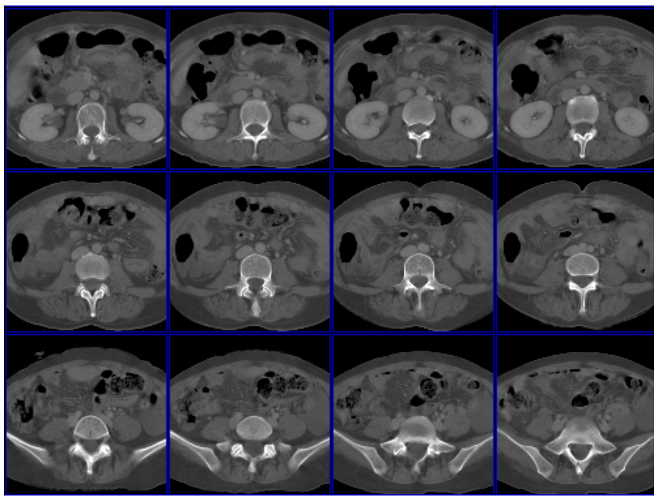

Figure 1. Human Abdomen Images


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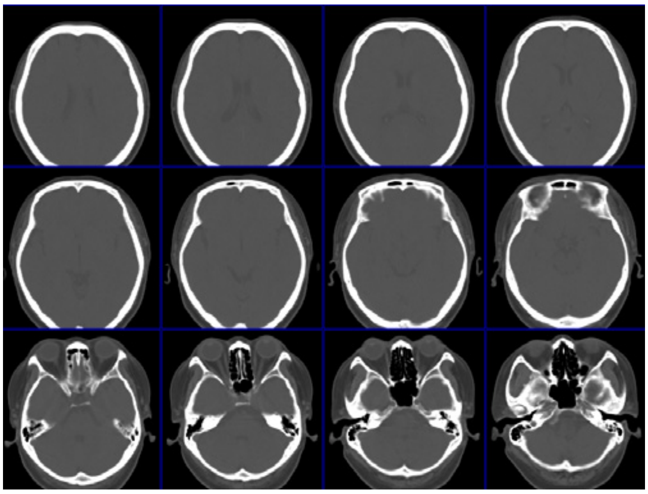


Figure 2. Human Brain Image

There are a great number of developments and a series of substantial achievements in the domain of fuzzy mathematics' theory research and application, since L. A. Zadeh proposed fuzzy sets in 1965 [4]. However, theses theories can only help to solve those static problems. Dynamic fuzzy logic as an effective theory to solve dynamic fuzzy problems is widely researched. In real world, dynamic fuzzy problems exist universally, especially in the domain of image fusion. For example, these images become smoother and smoother. The word "become" reflects dynamic character and "smoother" reflects fuzzy character. The whole clause is dynamic fuzzy data. Dynamic fuzzy logic (DFL) based on dynamic fuzzy data is used to solve those problems and has made a series of research achievements. Because of the fuzzy character of the image classification coefficients, we apply of DFL to analysis the decomposition information of ridgelet transform for medical image classification.

Ridgelet transform as a newly developed mathematical transform is often used as time-frequency and multiresolution analysis tool in the signal and image processing domain. It combined the wavelet analysis with the radon transform. The prominent characteristic of ridgelet is that ridgelet transform can compress the energy of the image into a smaller number of ridgelet coefficients. On the other hand, the wavelet transform produces many large wavelet coefficients on the edges on every scale of the 2D wavelet decomposition. This means that many wavelet coefficients are needed in order to reconstruct the edges in the image. In view of the combination ridgelet transform technique and DFL theory, This research is initial. For this reason, a novel medical image dynamic fuzzy classification model based on ridgelet transform is proposed in the paper.

The rest of this paper is organized as follows. The ridgelet transform analysis is given in Sections II. Dynamic fuzzy theory is given in Sections III. Section IV proposes a classification model by ridgelet fusion by ridgelet transform and DFL. Section V describes our method in the experiment and discusses the results. Conclusions are presented in last section.

## II. Ridgelet Transform Analysis

The ridgelet transform is special member of the family of multiscale orientation-selective transforms, which has recently led to an advanced research activity in the field of computational and applied harmonic analysis. It has good directional selectivity and is able to locally and sparsely represent the signal when compared with the traditional transforms such as wavelet transform. As a new multiscale representation for functions on continuous spaces it is smooth away from discontinuities along lines. Ridgelet analysis makes available representations of functions by superpositions of ridge functions or by simple elements that are in some way related to ridge functions $r(a_1x_1+\ldots+a_nx_n)$ ; these are functions of n variables, constant along hyperplanes $a_1x_1+\ldots+a_nx_n = c$ ; the graph of such a function in dimension two looks like a "ridge". The terminology "Ridge function" arose first in tomography, and ridgelet analysis makes use of a key tomographic concept, the Radon transform[5].

### A. Continuous Ridgelet Transform

Before the Ridgelet Transform, some attribute should be defined firstly as follows:

The $\Delta sf$ layer contains objects with frequencies near domain $|\xi|\in[2^{2s},2^{2s+2}]$.

We expect to find ridges with width *width* ≈ $2^{-2s}$

Windowing creates ridges of width *width* ≈ $2^{-2s}$ and *length* ≈ $2^{-2s}$ .

The renormalized ridge has an aspect ratio of $width \approx length^2$ .

By using the ridgelet transform, we would like to encode those ridges efficiently.

There are some key properties before defined ridgelet transform as follows[6]:

- Divides the frequency domain to dyadic coronae:

$$|\xi|\in[2^s,2^{s+1}]$$

- In the angular direction, samples the s-th corona at least $2^s$ times.
- In the radial direction, samples using local wavelets.

The ortho-ridgelet element has a formula in the frequency domain:

$$\hat{\rho}_\lambda(\xi)=\tfrac{1}{2}|\xi|^{-\frac{1}{2}}\left(\hat{\psi}_{j,k}(|\xi|)\cdot\omega_{i,l}(\theta)+\hat{\psi}_{j,k}(-|\xi|)\cdot\omega_{i,l}(\theta+\pi)\right) \quad (1)$$

where,

- $\omega_{il}$ are periodic wavelets for [-π, π ).

- $i$ is the angular scale and l∈[0, $2^{i-1}-1$ ] is the angular location.
- $\psi_{jk}$ are Meyer wavelets for $\theta$.
- $j$ is the ridgelet scale and $k$ is the ridgelet location.

Each normalized square is analyzed in the ridgelet system:

- The ridge fragment has an aspect ratio of $2^{-2s} \times 2^{-s}$.
- After the renormalization, it has localized frequency in band $|\xi| \in [2^{s}, 2^{s+1}]$.

A ridge fragment needs only a very few ridgelet coefficients to represent it.

We define an integrable bivariate function $f(x) \in R^2$ relative. The continuous ridgelet transform (CRT) [1, 2] in $R^2$ is defined as follows:

$$CRT_f(a,b,\theta) = \int_{R^2} \psi_{a,b,\theta}(x) f(x) dx \quad (2)$$

where the ridgelets $\psi_{a,b,\theta}(x)$ in 2-D are defined from a wavelet-type function in 1-D $\psi(x)$ as follows:

$$\psi_{a,b,\theta}(x) = a^{\frac{1}{2}} \psi\left(\frac{x_1\cos(\theta) + x_2\sin(\theta) - b}{a}\right) \quad (3)$$

A typical ridgelet are shown in Figure 3:

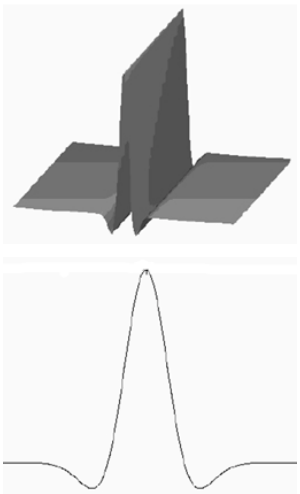


Figure 3. A typical ridgelet

A ridgelet is constant along lines $x_1\cos(\theta) + x_2\sin(\theta)$ = const. Transverse to these ridges it is a wavelet.

Given an integrable bivariate function f(x), we define its ridgelet coefficients by

$$R_f(a,b,\theta) = \int_{R^2} \psi_{a,b,\theta}(x) f(x) dx \quad (4)$$

The reconstruction formula

$$f(x) = \int_0^{2\pi} \int_{-\infty}^{\infty} \int_0^{\infty} R_f(a,b,\theta) \psi_{a,b,\theta}(x) f(x) \frac{da}{a^3} db \frac{d\theta}{4\pi} \quad (5)$$

The (separable) continuous wavelet transform (CWT) in $R^2$ of $f(x)$ can be written as follows:

$$CWT_f(a_1,a_2,b_1,b_2) = \int_{R^2} \psi_{a_1,a_2,b_1,b_2}(x) f(x) dx \quad (6)$$

where the wavelets in 2-D are tensor products

$$\psi_{a_1,a_2,b_1,b_2}(x) = \psi_{a_1,b_1}(x_1) \psi_{a2,b2}(x_2) \quad (7)$$

of 1-D wavelets, $\psi_{a,b}(t) = a^{\frac{1}{2}} \psi\left(\frac{t-b}{a}\right)$.

By comparison, we can see, the CRT is similar to the 2-D CWT except that the point parameters ( $b_1, b_2$ ) are replaced by the line parameters (b, $\theta$ ). That is to say, these 2-D multiscale transforms have the relations as follows[7]:

Wavelets $\rightarrow \psi_{\text{scale,point-position}}$

Ridgelets $\rightarrow \psi_{\text{scale,line-position}}$

Therefore, wavelets are very effective in representing objects with isolated point singularities, while ridgelets are very effective in representing objects with singularities along lines. In fact, one can think of ridgelets as a way of concatenating 1-D wavelets along lines. Hence the motivation for using ridgelets in image processing tasks is appealing since singularities are often joined together along edges or contours in images.

It is easy to extend the 1-D case to the 2-D case, points and lines are related via the Radon transform, thus the wavelet and ridgelet transforms are linked via the Radon transform. More precisely, denote the Radon transform as follows:

$$R_f(\theta,t) = \int_{R^2} f(x) \psi(x_1\cos\theta + x_2\sin\theta - t) dx \quad (8)$$

then the ridgelet transform is the application of a 1-D wavelet transform to the slices (also referred to as projections) of the Radon transform,

$$CWT_f(a,b,\theta) = \int_{R^2} \psi_{a,b}(t) R_f(\theta,t) dt \quad (9)$$

*B. Digital Ridgelet Transform*

In the Fourier domain, the implementation of the ridgelet transform can be performed quickly[8].

- Firstly, compute the two dimensional Fourier transform, $F$ (u, v) for the input image $f$ (x, y). Using an interpolation scheme, substitute the sampled values of the Fourier transform obtained on the square lattice with the sampled values on a polar lattice.

- Secondly, Cartesian-to-polar conversion is used for an image of size n×n, 2n.
- Thirdly, one-dimensional inverse Fourier transform is applied on each line, i.e., for each value of the angular parameter.
- Finally, one-dimensional wavelet transform is applied along the radial variable in Radon space.

Here, wavelet transform could be used in conjunction with nonlinear processing such as hard-thresholding of individual wavelet coefficients particularly.

## III. Dynamic Fuzzy Theory

In order to apply dynamic fuzzy theory[9] in the domain of medical image classification, we introduce the dynamic fuzzy data sets, estimate of dynamic fuzzy data, DFL proposition calculus and DFL Predicate Calculus.

### A. Dynamic Fuzzy Data Sets

**Definition 1.** Let a mapping be defined in the domain U.

$\overrightarrow{A}:\overrightarrow{a}\rightarrow[0,1]$, $\overrightarrow{a}\rightarrow\overrightarrow{A}(\overrightarrow{a})$ or

$\overleftarrow{A}:\overleftarrow{a}\rightarrow[0,1]$, $\overleftarrow{a}\rightarrow\overleftarrow{A}(\overleftarrow{a})$.

We write $(\overleftarrow{A},\overrightarrow{A}) = \overrightarrow{A}$ or $\overleftarrow{A}$, then we named $(\overrightarrow{A},\overleftarrow{A})$ the dynamic fuzzy data sets (DFDS) of U ; you can say that is the membership degree of $(\overrightarrow{A},\overleftarrow{A})$ to the membership function of ($\overleftarrow{A}(\overleftarrow{u})$ , $\overrightarrow{A}(\overrightarrow{u})$ ). Thus we provided the definition of dynamic fuzzy data sets.

**Definition 2.** Let(A,A),(B,B) ∈DF(U), if $\forall(\overleftarrow{a},\overrightarrow{a})\in$(U), $(\overleftarrow{B},\overrightarrow{B})(\overleftarrow{a},\overrightarrow{a})\leq(\overleftarrow{A},\overrightarrow{A})(\overleftarrow{a},\overrightarrow{a})$ then called $(\overleftarrow{B},\overrightarrow{B})$ is contained in $(\overleftarrow{A},\overrightarrow{A})$ marked as $(\overleftarrow{B},\overrightarrow{B})\subseteq(\overleftarrow{A},\overrightarrow{A})$ and then $(\overleftarrow{A},\overrightarrow{A})=(\overleftarrow{B},\overrightarrow{B})$.Obviously, the relation of contain "$\subseteq$" has features as follows:

(1) Reflexivity:

$\forall$(A,A)$\in$DF(U), $(\overleftarrow{A},\overrightarrow{A})\subseteq(\overleftarrow{A},\overrightarrow{A})$;

(2) Transitive Characteristic:

$(\overleftarrow{A},\overrightarrow{A})\subseteq(\overleftarrow{B},\overrightarrow{B})$; $(\overleftarrow{B},\overrightarrow{B})\subseteq(\overleftarrow{C},\overrightarrow{C})\Rightarrow(\overleftarrow{A},\overrightarrow{A})\subseteq(\overleftarrow{C},\overrightarrow{C})$

(3) Anti-symmetry:

$(\overleftarrow{A},\overrightarrow{A})\subseteq(\overleftarrow{B},\overrightarrow{B})$; $(\overleftarrow{B},\overrightarrow{B})\subseteq(\overleftarrow{A},\overrightarrow{A})\Rightarrow(\overleftarrow{A},\overrightarrow{A})=(\overleftarrow{B},\overrightarrow{B})$

**Definition 3.** Let (A,A),(B,B) $\in$ DF(U), then called $(\overleftarrow{A},\overrightarrow{A})\cup(\overleftarrow{B},\overrightarrow{B})$as the union of $(\overleftarrow{A},\overrightarrow{A})$ and $(\overleftarrow{B},\overrightarrow{B})$, and $(\overleftarrow{A},\overrightarrow{A})\cap(\overleftarrow{B},\overrightarrow{B})$ as the intersection of $(\overleftarrow{A},\overrightarrow{A})$ and $(\overleftarrow{B},\overrightarrow{B})$, still we called $(\overleftarrow{A},\overrightarrow{A})^c$ as the complementation of $(\overleftarrow{A},\overrightarrow{A})$. We provided several following membership functions to the three above calculations:

$$((\overleftarrow{A},\overrightarrow{A})\cup(\overleftarrow{B},\overrightarrow{B}))(\overleftarrow{a},\overrightarrow{a})$$
$$=(\overleftarrow{A},\overrightarrow{A})(\overleftarrow{a},\overrightarrow{a})\vee(\overleftarrow{B},\overrightarrow{B})(\overleftarrow{a},\overrightarrow{a})$$
$$\triangleq\max((\overleftarrow{A},\overrightarrow{A})(\overleftarrow{a},\overrightarrow{a}),(\overleftarrow{B},\overrightarrow{B})(\overleftarrow{a},\overrightarrow{a}));$$
$$((\overleftarrow{A},\overrightarrow{A})\cap(\overleftarrow{B},\overrightarrow{B}))(\overleftarrow{a},\overrightarrow{a})$$
$$=(\overleftarrow{A},\overrightarrow{A})(\overleftarrow{a},\overrightarrow{a})\wedge(\overleftarrow{B},\overrightarrow{B})(\overleftarrow{a},\overrightarrow{a})$$
$$\triangleq\min((\overleftarrow{A},\overrightarrow{A})(\overleftarrow{a},\overrightarrow{a}),(\overleftarrow{B},\overrightarrow{B})(\overleftarrow{a},\overrightarrow{a}));$$
$$((\overleftarrow{A},\overrightarrow{A})(a))^c=(1-(\overleftarrow{A},\overrightarrow{A})(\overleftarrow{a},\overrightarrow{a}))\triangleq((\overleftarrow{1}-\overleftarrow{A}(\overleftarrow{a}),\overrightarrow{1}-\overrightarrow{A}(\overrightarrow{a})).$$

### B. Estimate of Dynamic Fuzzy Data

Above, we have talked about the calculation of dynamic fuzzy data. In fact, it is another important content to study estimate of dynamic fuzzy data. Here is an example: "A grows faster than B." In this example, we should measure their growth speed. This is the main content in this section. we can defined the estimate of dynamic fuzzy data as:

**Definition 4.** Mapping: $\mu:\delta\rightarrow[\overleftarrow{0},\overrightarrow{0}]$, $[\overleftarrow{1},\overrightarrow{1}]$is called DF estimate, if:

(1) $\mu(\overleftarrow{\Phi},\overrightarrow{\Phi})=(\overleftarrow{0},\overrightarrow{0})$, $\mu(\overleftarrow{X},\overrightarrow{X})=(\overleftarrow{1},\overrightarrow{1})$

(2) $(\overleftarrow{A},\overrightarrow{A})\subset(\overleftarrow{B},\overrightarrow{B})\Rightarrow\mu(\overleftarrow{A},\overrightarrow{A})\leq\mu(\overleftarrow{B},\overrightarrow{B})$

(3) $(\overleftarrow{A}_n,\overrightarrow{A}_n)\uparrow(\downarrow)(\overleftarrow{A},\overrightarrow{A})\Rightarrow\mu(\overleftarrow{A}_n,\overrightarrow{A}_n)\uparrow(\downarrow)\mu(\overleftarrow{A},\overrightarrow{A})$

Then $((\overleftarrow{A},\overrightarrow{A}),\ \delta,\ \mu)$ is called the space of DF estimate.

**Definition 5.** If $g_\lambda\rightarrow[\overleftarrow{0},\overrightarrow{0}]$, $[\overleftarrow{1},\overrightarrow{1}]$ satisfies conditions:

(1) $g_\lambda(\overleftarrow{X},\overrightarrow{X})=(\overleftarrow{1},\overrightarrow{1})$;

(2) $g_\lambda((\overleftarrow{A},\overrightarrow{A})\cup(\overleftarrow{B},\overrightarrow{B}))=$

$g_\lambda(\overleftarrow{A},\overrightarrow{A})+g_\lambda(\overleftarrow{B},\overrightarrow{B})+\lambda\, g_\lambda(\overleftarrow{A},\overrightarrow{A})\, g_\lambda(\overleftarrow{B},\overrightarrow{B})$, $(\overleftarrow{A},\overrightarrow{A})\cap(\overleftarrow{B},\overrightarrow{B})=(\overleftarrow{\Phi},\overrightarrow{\Phi})$

(3) $(\overleftarrow{A}_n,\overrightarrow{A}_n)\uparrow(\downarrow)(\overleftarrow{A},\overrightarrow{A})$

$\Rightarrow\lim_{(\overleftarrow{n},\overrightarrow{n})+\infty} g_\lambda(\overleftarrow{A}_n,\overrightarrow{A}_n)=g_\lambda(\overleftarrow{A}_n,\overrightarrow{A}_n)$

Which is called $g_\lambda$ estimate

**Theorem 1.** While $(\overleftarrow{\lambda},\overrightarrow{\lambda})>(-\overleftarrow{1},-\overrightarrow{1})$ then we call $g_\lambda$ estimate DF estimate.

**Theorem 2.** $g_\lambda(\overleftarrow{\lambda},\overrightarrow{\lambda})>(-\overleftarrow{1},-\overrightarrow{1})$ has features as follows:

(1) $g_\lambda(\overleftarrow{\lambda},\overrightarrow{\lambda})=\dfrac{(\overleftarrow{1},\overrightarrow{1})-g_\lambda(\overleftarrow{A},\overrightarrow{A})}{(\overleftarrow{1},\overrightarrow{1})+\lambda g_\lambda(\overleftarrow{A},\overrightarrow{A})}$

(2) $g_\lambda(\overleftarrow{A},\overrightarrow{A})\cap(\overleftarrow{B},\overrightarrow{B})$

$$=\frac{g_\lambda(\overleftarrow{A},\overrightarrow{A})+g_\lambda(\overleftarrow{B},\overrightarrow{B})-g_\lambda(\overleftarrow{A},\overrightarrow{A})(\overleftarrow{B},\overrightarrow{B})+\lambda g_\lambda(\overleftarrow{A},\overrightarrow{A})g_\lambda(\overleftarrow{B},\overrightarrow{B})}{(\overleftarrow{1},\overrightarrow{1})+\lambda g_\lambda(\overleftarrow{A},\overrightarrow{A})(\overleftarrow{B},\overrightarrow{B})}$$

### *C. DFL Proposition Calculus*

**Definition 6.** A statement having character of dynamic fuzzy is called dynamic fuzzy proposition that is usually symbolized by capital letter A , B, C···

**Definition 7.** A dynamic fuzzy number $(\overleftarrow{a},\overrightarrow{a})\in[0,1]$ which is used to measure a dynamic fuzzy proposition's true or false degree is called dynamic fuzzy proposition's true or false. It is usually symbolized by$(\overleftarrow{a},\overrightarrow{a})$, $(\overleftarrow{b},\overrightarrow{b})$, $(\overleftarrow{c},\overrightarrow{c})$, Here $(\overleftarrow{a},\overrightarrow{a})=\overleftarrow{a}$ or $\overrightarrow{a}$, $\min(\overleftarrow{a},\overrightarrow{a})\overset{\Delta}{=}\overleftarrow{a}$, $\max(\overleftarrow{a},\overrightarrow{a})\overset{\Delta}{=}\overrightarrow{a}$, the same are as follows.

**Definition 8.** A dynamic fuzzy propositions can be regarded as a variable whose value is in the interval [0,1]. The variable is called dynamic fuzzy proposition variable that is usually symbolized by small letter.

(1) Negation "-":

The negation of variable $(\overleftarrow{x},\overrightarrow{x})$is symbolized by $\overline{(\overleftarrow{x},\overrightarrow{x})}$, and $\overline{(\overleftarrow{x},\overrightarrow{x})}=((\overleftarrow{1}-\overleftarrow{x},\overrightarrow{1}-\overrightarrow{x}))$

(2) Disjunction "$\vee$":

$(\overleftarrow{x},\overrightarrow{x})\vee(\overleftarrow{y},\overrightarrow{y})=\mathrm{Max}((\overleftarrow{x},\overrightarrow{x}),(\overleftarrow{y},\overrightarrow{y}))$

(3) Conjunction "$\wedge$":

$(\overleftarrow{x},\overrightarrow{x})\wedge(\overleftarrow{y},\overrightarrow{y})=\mathrm{Min}((\overleftarrow{x},\overrightarrow{x}),(\overleftarrow{y},\overrightarrow{y}))$

(4) Condition "$\rightarrow$":

$(\overleftarrow{x},\overrightarrow{x})\rightarrow(\overleftarrow{y},\overrightarrow{y})$

$\Leftrightarrow\ \overline{(\overleftarrow{x},\overrightarrow{x})}\vee(\overleftarrow{y},\overrightarrow{y})$

$=\mathrm{Max}(\overline{(\overleftarrow{x},\overrightarrow{x})},(\overleftarrow{y},\overrightarrow{y}))$

(5) Bicondition "$\leftrightarrow$":

$(\overleftarrow{x},\overrightarrow{x})\leftrightarrow(\overleftarrow{y},\overrightarrow{y})$

$\Leftrightarrow \mathrm{Min}(\mathrm{Max}\ \overline{(\overleftarrow{x},\overrightarrow{x})}(\overleftarrow{y},\overrightarrow{y}),\mathrm{Max}\ \overline{(\overleftarrow{y},\overrightarrow{y})}(\overleftarrow{x},\overrightarrow{x}))$

**Definition 9.** Dynamic fuzzy calculus formation can be defined as follows:

(1) A simple dynamic fuzzy variable itself is a well formed formula.

(2) If $(\overleftarrow{x},\overrightarrow{x})$p is a well formed formula, then $\overline{(\overleftarrow{x},\overrightarrow{x})p}$ is a well formed formula, too.

(3) If $(\overleftarrow{x},\overrightarrow{x})$P and $(\overleftarrow{y},\overrightarrow{y})$Q are well formed fomulas, then $(\overleftarrow{x},\overrightarrow{x})$P $\vee$ $(\overleftarrow{y},\overrightarrow{y})$Q, $(\overleftarrow{x},\overrightarrow{x})$P $\wedge$ $(\overleftarrow{y},\overrightarrow{y})$Q, $(\overleftarrow{x},\overrightarrow{x})$P$\rightarrow$ $(\overleftarrow{y},\overrightarrow{y})$Q, $(\overleftarrow{x},\overrightarrow{x})$P $\leftrightarrow$ $(\overleftarrow{y},\overrightarrow{y})$Q are also well formed formulas.

(4) A sting of symbols including proposition variable connective and brackets is well formed formula if and only if the strings can be obtained in a finite number of steps, each of which only applies the earlier rules(1), (2) and (3).

### *D. DFL Predicate Calculus*

**Definition 10.** Recursive definition of DFL predicate is as follows:

(1) An atom (first order logical symbols) is a formula.

(2) If G and H are formulas, T is a dynamic fuzzy truth value of assignment, $(\overleftarrow{x},\overrightarrow{x})$is a free variable in DFL , then G$\vee$H, G$\wedge$H, G$\rightarrow$H, G$\leftrightarrow$ H, $\overline{G}$, $(\forall(\overleftarrow{x},\overrightarrow{x})G)$, $(\exists(\overleftarrow{x},\overrightarrow{x})G)$ are all formulas.

(3) Any string of symbol is a formula of DFL, if and only if the string can be obtained in a finite of steps, each of which only applies the earlier rules(1) and (2).

**Definition 11.** An explanation of a DFL formula G consists of a non-empty domain and some rules as follows:

For every variable symbol in G, a dynamic fuzzy element is assigned,

For every n-array function symbol in G, a mapping $U\xrightarrow{T}D$ is assigned,

For every n-array predicate symbol in G, a mapping $D\xrightarrow{T}B$ is assigned.

where B is a dynamic fuzzy logic predicate system.

## IV. Classification by Ridgelet Transform and DFL

The goal of using ridgelet transform for medical image classification is to identify a medical image's attribute, which could be extracted by ridgelet transform decomposition and calculated by DFL.

In this paper, a new model which is quadruple in order to express medical image's attribute is proposed to classification process.

$$AM=(R, E, D, S) \qquad (10)$$

Where R is a series of CT images that needs to be classified, $R=\{R_1,R_2,\ldots,R_n\}$. Where E is the sub-bands degree of the certain levels of image after ridgelet transform; D is a set of the direction weight of each factor of R after ridgelet transform; $S$ is similarity scores set of all the medical images within the whole sequences, $S = \{ S_1, S_2, \ldots, S_n \}$. The final classification result is decided by the order of $S$.

The model has some attributes as follows:

- The number of levels of ridgelet transform could be chosen by the precision of classification, the large number means high precision but cost more time.
- The approximation coefficients and detailed coefficients in different sub-bands with directional parameters of image after ridgelet transformation is a objective parameter of this similarity degree compution.
- We adjust the value of $D$ for the purpose of computing the similarity after ridgelet transform. Because the value of $D$ is in the interval [0,1], so we could use DFL to construct a membership function to adjust the weight for image classification.

We construct the membership function as follows:

$$\bar{X}(\bar{a}) = \begin{cases} 0 & \text{if } 0 \le \bar{a} \le \overleftarrow{0.3} \\ 1 - \dfrac{0.05^2}{(\bar{a} - 0.3)^2 - 0.05^2} & \text{if } \overleftarrow{0.3} \le \bar{a} \le \bar{1} \end{cases}$$

$$\vec{X}(\vec{a}) = \begin{cases} 0 & \text{if } 0 \le \vec{a} \le \overrightarrow{0.3} \\ 1 - \dfrac{0.05^2}{(\vec{a} - 0.3)^2 - 0.05^2} & \text{if } \overrightarrow{0.3} \le \vec{a} \le \vec{1} \end{cases} \quad (11)$$

By using the constructed membership function of DFL, we could find the trend of similarity for classification.

## V. Experimental Results and Discussion

Before the experiment, we get a lot of CT images from the First Affiliated Hospital of Soochow University, including different parts of human's body such as brain and abdomen. All images are follow DICOM standard.

### *A. The experimental using ridgelet transform and DFL*

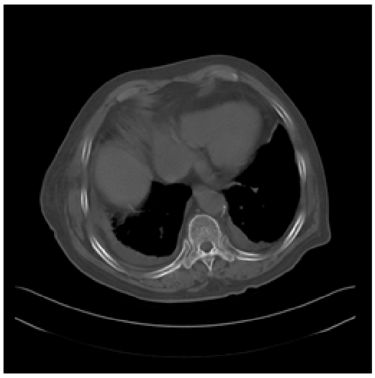
In order to classify the medical images, we proposed a new model combine ridgelet transform and DFL. The steps of this processing method as follows:

- All medical images are decomposed into the approximation coefficients and detailed coefficients in different sub-bands with directional parameters.
- Calculate the sub-bands degree of the certain sub-band coefficients after ridgelet transform for medical image.
- Construct a model based on DFL to obtain the similarity scores in order.

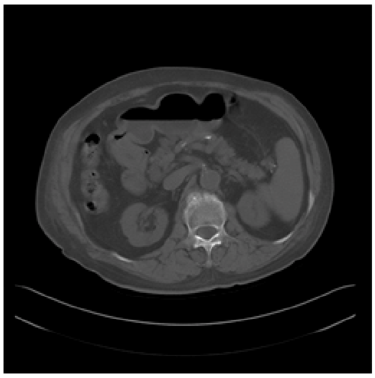
### *B. The Effect Analysis*

Due to the disordered distributing of the medical image library with a mass of information, most medical information could not be utilized and accessed. In order to classify certain image quickly and accurately and make best use of these data, the medical images classification technology are applied to adjuvant therapy and surgical planning purposes.

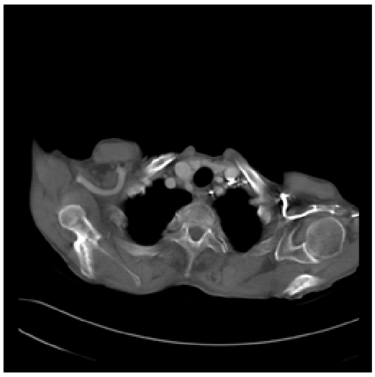
In the experiment, the medical images classification objects are shown in Figure. 4. and congeneric medical image sequences are shown in Figure. 5 respectively.



(a)

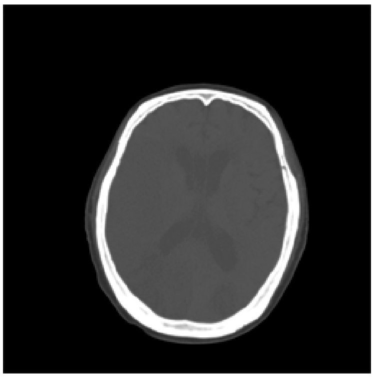


(b)

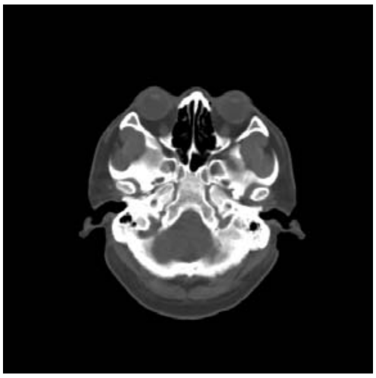


(c)

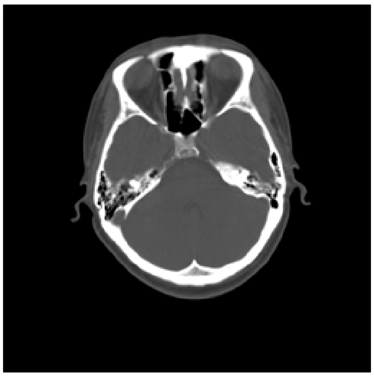
(d)



(e)

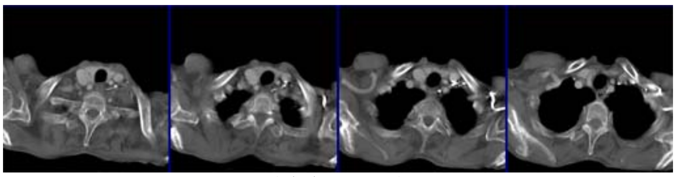


(f)

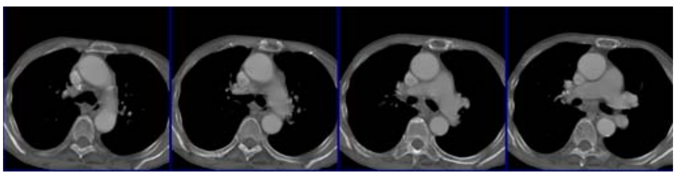


(g)

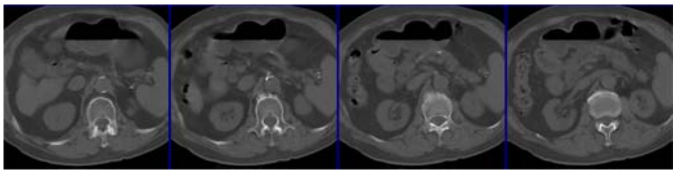
Figure 4. Medical Image Classification Objects



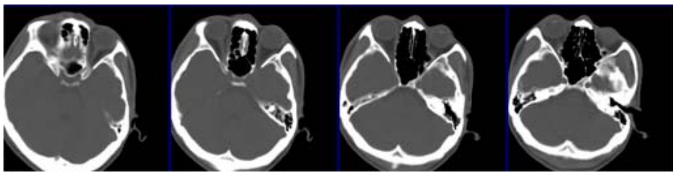
(abdomen-1)



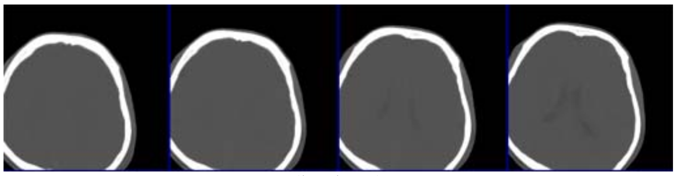
(abdomen-2)



(abdomen-3)



(brain-1)



(brain-2)

Figure 5. Congeneric Medical Image sequences

TABLE I. COEFFICIENTS DEGREE AND WEIGHT COMPUTE

| Sequence | approximation coefficients degree | detailed coefficients degree | weight | Average degree |
|---|---|---|---|---|
| abdomen-1 | 6.053 | 8.025 | 0.3 | 6.645 |
| abdomen-2 | 6.230 | 7.943 | 0.5 | 7.087 |
| abdomen-3 | 6.172 | 6.824 | 0.2 | 6.3064 |
| brain-1 | 5.983 | 8.561 | 0.75 | 7.917 |
| brain-2 | 5.242 | 7.253 | 0.25 | 5.745 |

TABLE II. SIMILARITY COMPUTE

| No. | abdomen-1 | abdomen-2 | abdomen-3 | brain-1 | brain-2 | other |
|---|---|---|---|---|---|---|
| a | 62.08 | 92.57 | 75.33 | 47.12 | 37.54 | -1 |
| b | 63.05 | 77.12 | 93.41 | 38.26 | 34.32 | -1 |
| c | 95.63 | 73.10 | 86.08 | 42.18 | 39.08 | -1 |
| d | 5.22 | 3.92 | 7.32 | 2.36 | 3.53 | 0 |
| e | 38.17 | 44.68 | 51.29 | 82.33 | 98.26 | -1 |
| f | 35.19 | 34.26 | 47.18 | 88.27 | 69.35 | -1 |
| g | 41.68 | 30.29 | 45.89 | 92.38 | 73.55 | -1 |

Use this paper's arithmetic, the coefficients degrees and average degrees are shown in the TABLE I. Different image to the average coefficients degree with congeneric medical image sequences are calculated and show in the TABLE II above.

By analysis we can see that our method could classify medical image via compute similary degree which has highest similarity degree. Otherwise, the image like image (d) is classified into "other" sort.

Our method is an image classification based content at primary level, it need further research for the aim of complete content based image retrieval in the future. It is useful for medical treatment informationization in hospital and convenient for doctor managing medical images.

## VI. CONCLUSIONS

In order to classify medical image, a new model which is using ridgelet transform and dynamic fuzzy theory was proposed. Firstly, the image was decomposed by digital ridgelet transform to obtain the approximation coefficients and detailed coefficients in different sub-bands with directional parameters. Then the dynamic fuzzy theory was applied to construct a membership function to calculate coefficients from each sub-bands respectively, and a weight of sub-bands degree was adjust by precision requirement. At last similarity degrees are calculated by coefficients degree and weight. According to the degrees, medical images were classified by the result sort order of that effectively. Further investigations on the medical image retrieval based on content completely are left for future work.

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