New Algorithm for Indefinite Multi-objective Decision Making Based on Multiple-valued Intuitive Fuzzy Set Theory

ZHOU Qihai
Information Technology Application Research Institute, Southwestern University Of Finance and Economics, Chengdu, Sichuan, China; School of Economic Information Engineering, Southwestern University Of Finance and Economics, Chengdu, Sichuan, China
Email: zhouqh@swufe.edu.cn

LI Yan
Information Technology Application Research Institute, Southwestern University Of Finance and Economics, Chengdu, Sichuan, China; School of Economic Information Engineering, Southwestern University Of Finance and Economics, Chengdu, Sichuan, China
Email: Liyan77@163.com

***Abstract*—There are three major difficulties in the indefinite multi-objective decision making process: 1) how to express the indefinite information because of the information about attributes being indefinite; 2) how to express the indefinite information because of the information has multi-channels; 3) how to fuse the information into synthetic information. The multiple-valued intuitive fuzzy sets is one new mathematical model, this model can process well fuzzy information gained from multi-sources. In this paper, firstly conduct the research to the multiple-valued intuitive fuzzy set's information fusion and construct some methods to fuse the information included in the degree of membership or non-degree of membership of multiple-valued intuitive fuzzy set , then use the isomorphism mind to research indefinite isomerism multi-objective decision making and construct one new algorithm for interval value and indefinite language isomerism multi-objective decision making based on isomorphism information fusion.**

***Index Terms*—Isomorphism, Multi-objective Decision, Multiple-valued intuitive fuzzy sets, Indefinite multi-objective decision-making**

I. INTRODUCTION

In decision-making process, as the decision information is unprecise, incomplete and so on, in addition the policy-maker's information-handling capacity is limit. So sometimes gain the accurate attribute value is very difficult, even was impossible. Conducting the research to this kind of multi-objective decision making containing the incomplete information is further expansion to the research of the traditional multi-objective decision making question. For the fundamental research and solving actual problems, the indefinite multi-objective decision making question gains more and more people's attention. Literature [1]-[8] has conducted the research from the different angle to the indefinite multi-objective decision making question. In this paper, for the two major difficulties in the multi-objective decision making process, introduces the multiple-valued intuitive fuzzy sets into the multi-objective decision making question, and study the heterogeneous indefinite multi-attribute decision-making.

II. MULTIPLE-VALUED INTUITIVE FUZZY SETS INFORMATION FUSION

A. Elementary Knowledge

Definition1. (Intuitive Fuzzy Sets [9]) Suppose X as the given domain, then an intuitive fuzzy set in X is:

$$A = \{< x, \mu_A(x), \gamma_A(x) > \big| x \in X\}$$

Where, $\mu_A(x): X \to [0,1]$ $\gamma_A(x): X \to [0,1]$

Represent the membership function $\mu_A(x)$ and the non-membership function

$$\gamma_A(x). \forall x \in X \text{ ,} 0 \le \mu_A(x) + \gamma_A(x) \le 1$$

is establishment.

When the given domain X is a continual space:

$$\mathrm{A} = \int_A < \mu_A(x), \gamma_A(x) > / x \text{ , } x \in X \text{ ;}$$

When given domain X is a discrete space:

$$A = \sum_{i=1}^{n} < \mu_A(x_i), \gamma_A(x_i) > / x_i \text{ , } x_i \in X \ (i = 1,2,\cdots,n)$$

Definition2. (Multiple-valued Intuitive Fuzzy Sets [10]) Suppose X as the given domain, then a multiple-valued intuitive fuzzy sets in X is:

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$$A=\{<x,[\mu_1^A(x),\mu_2^A(x),\cdots,\mu_n^A(x)],$$
$$[\gamma_1^A(x),\gamma_2^A(x),\cdots,\gamma_n^A(x)]>|x\in X\}$$

Where, $\mu_i^A(x):X\to[0,1]\quad \gamma_i^A(x):X\to[0,1]$

Represent the first "i" membership function $\mu_i^A(x)$ and the non-membership function $\gamma_i^A(x)$, and $\forall x\in X$, $0\le\mu_i^A(x)+\gamma_i^A(x)\le 1$, $(i=1,\ 2,\ \cdots,n)$ is establishment.

Represent the multiple-valued intuitive fuzzy sets A as:

When given domain X is the continual space:

$$A=\int_A<[\mu_1^A(x),\mu_2^A(x),\cdots,\mu_n^A(x)],$$
$$[\gamma_1^A(x),\gamma_2^A(x),\cdots,\mu_n^A(x)]>/x,x\in X\ ;$$

When given domain X is the discrete space, suppose $X=\{x_1,x_2,\cdots,x_m\}$:.

$$A=\sum_{j=1}^{m}<[\mu_1^A(x_j),\mu_2^A(x_j),\cdots,\mu_n^A(x_j)],$$
$$[\gamma_1^A(x_j),\gamma_2^A(x_j),\cdots,\gamma_n^A(x_j)]>/x_j,\ x_j\in X\ ,$$
$$j=1,2,\cdots,m\ .$$

B. Degree of Membership or Non-degree of Membership of Multiple-valued Intuitive Fuzzy Set Information Fusion

The degree of membership or non-degree of membership of multiple-valued intuitive fuzzy sets information fusion refers to fusing the degree of membership or non-degree of membership into one degree of membership or non-degree of membership, thus multiple-valued intuitive fuzzy sets will be transformed into a general intuitive fuzzy sets. Suppose A as a multiple-valued intuitive fuzzy set:

$$A=\{<x,[\mu_1^A(x),\mu_2^A(x),\cdots,\mu_n^A(x)],$$
$$[\gamma_1^A(x),\gamma_2^A(x),\cdots,\gamma_n^A(x)]>|x\in X\}\ .$$

Following, construct several methods to fuse this multiple-valued intuitive fuzzy sets into a general intuitive fuzzy set:

$$B=\{<x,\mu_B(x),\gamma_B(x)>|x\in X\}\ .$$

Because of that:

(1) Median method

(1.1) The median of the material not grouped

Firstly group various degrees of membership or the non-degree of membership's value by ascending. Then, compute median:

When n is an odd number:

$$\mu_B(x)=\mu_{(n+1)/2}^A(x)\qquad \gamma_B(x)=\gamma_{(n+1)/2}^A(x)$$

When n is an even number:

$$\mu_B(x)=\frac{\mu_{n/2}^A(x)+\mu_{(n+1)/2}^A(x)}{2}$$

$$\gamma_B(x)=\frac{\gamma_{n/2}^A(x)+\gamma_{(n+1)/2}^A(x)}{2}$$

(1.2) The median of the material grouped

If the material has grouped, and establishes distribution list, then calculate the median using the distribution list, its formula is:

$$\mu_B(x)=L_\mu+\frac{i_\mu}{f_\mu}(\frac{n}{2}-c_\mu)$$

$$\gamma_B(x)=L_\gamma+\frac{i_\gamma}{f_\gamma}(\frac{n}{2}-c_\gamma)$$

In the formula:

L_μ,L_γ —lower limit;

i_μ,i_γ —interval;

f_μ,f_γ —number of times;

n—total degree;

c_μ,c_γ —number of times smaller than the median.

(2) Simple weighted arithmetic average method

$$\mu_B(x)=\sum_{i=1}^{n}\lambda_i\mu_i^A(x)$$

$$\gamma_B(x)=\sum_{i=1}^{n}\lambda_i\gamma_i^A(x)$$

(3) Harmonic mean method

(3.1) Simple harmonic mean method

$$\mu_B(x)=\frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{\mu_i^A(x)}}=\frac{n}{\sum_{i=1}^{n}\frac{1}{\mu_i^A(x)}}$$

$$\gamma_B(x)=\frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{\gamma_i^A(x)}}=\frac{n}{\sum_{i=1}^{n}\frac{1}{\gamma_i^A(x)}}$$

(3.2) weighting harmonic mean method

$$\mu_B(x)=\frac{1}{\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i^A(x)}}$$

$$\gamma_B(x)=\frac{1}{\sum_{i=1}^{n}\frac{\lambda_i}{\lambda_i^A(x)}}$$

Where $\lambda_1,\lambda_2,\cdots,\lambda_n$ satisfy the following conditions:

$$\sum_{i=1}^{n}\lambda_i=1,1\ge\lambda_i\ge 0\ (i=1,2,\cdots,n)$$

(4) Combination of mean values [11]

The combination mean value defers that many kinds of traditional mean values carry on the weighted average. Therefore, its formula is:

$$p_0 = \sum_{i=1}^{n} \omega_i p_i$$

In the formula:

P_0—combination mean value;

p_i—different type mean value, where $i = 1,2, \cdots, n$ (similarly hereinafter, omitted);

ω_i —weight of various mean values, they satisfy $\sum_{i=1}^{n} \omega_i = 1$. Combination mean value may collect each kind of mean value the superiority, reflects more accurately the information in the general level of data.

(5) Mathematics optimization method

Regarding each pair $< \mu_i^A, \gamma_i^A >$ $(i = 1,2, \cdots, n)$ in the multiple-valued is intuition fuzzy set, where each of them expresses information which obtains from the different attributes. When carry on the information fusion, a very natural idea is that: In the information fusion process, as far as possible to make the modification to the existing information to a minimum. We may establish the following mathematical programming model according to this principle:

When is X a continual space:

$$\min(\int_X [\sum_{i=1}^{n} \left(\mu_i^A(x) - \mu_B(x)\right)^2 + \sum_{i=1}^{n} \left(\gamma_i^A(x) - \gamma_B(x)\right)^2]dx$$

$$s.t. 0 \le \mu_B(x) + \gamma_B(x) \le 1$$
$$0 \le \mu_B(x) \le 1$$
$$0 \le \gamma_B(x) \le 1$$
$$x \in X$$
$$(A1)$$

Through solving the optimize question $(A1)$, may obtain the various parameters' estimated value in the function $\mu_B(x)$ and $\gamma_B(x), (x \in X)$.

When is X the discrete space:

$$\min\left(\sum_{i=1}^{n}\sum_{j=1}^{m} \left(\mu_i^A(x_j) - \mu_B(x_j)\right)^2 + \sum_{i=1}^{n}\sum_{j=1}^{m} \left(\gamma_i^A(x_j) - \gamma_B(x_j)\right)^2 \right)$$

$$s.t. 0 \le \mu_B(x_j) + \gamma_B(x_j) \le 1$$
$$0 \le \mu_B(x_j) \le 1$$
$$0 \le \gamma_B(x_j) \le 1$$
$$(j = 1,2, \cdots, m)$$
$$(A2)$$

Through solving the optimize question $(A2)$, may obtain the following values:

$$\mu_i^A(x_j), \gamma_i^A(x_j),\ x_j \in X\ (j = 1,2, \cdots, m)$$
$$(i = 1,2, \cdots, n)$$

III. CONSTRUCTING DECISION METHOD

Suppose $A = \{A_1, A_2, \cdots, A_n\}$ to the decision plan set and $U = \{u_1, u_2, \cdots, u_m\}$ to the attribute set. When carry on the qualitative measure to attributes, generally need suitable language evaluation scale. Therefore, we should establish language evaluation scale S = $\{ s_a \mid a = -t, \cdots, t \}$ where s_a express language Variables. Specially s_{-t} and s_t separately expresses the scale's low limit and the up limit. The commonly used language evaluation scale may be: Third-level evaluation scale S_1= {bad, general, good}; Seven-level evaluation scale S_2= {worst, worse, bad, general, good, better, best} or {smallest, smaller, small, general, big, bigger, biggest} and so on. The mark"$\prec$" expresses the relation in various linguistic values .Definite "bad $\prec$ general $\prec$ good", in this formal rank-number of a linguistic value setting the left of "$\prec$" is smaller "1" to that of a linguistic value setting the right of "$\prec$", and the rank-number can be accumulated. For example: In this third-level evaluation scale "bad $\prec$ general $\prec$ good" may promote the rank-number of the linguistic value "good" is bigger 2 to that of the linguistic value "bad". But it cannot promote "bad $\prec$ good", because "bad" are smaller two ranks to "good", but is not one rank. May similarly definite in seven- level of evaluation scales S_2 "worst $\prec$ worse $\prec$ bad $\prec$ general $\prec$ good $\prec$ better $\prec$ best" or "smallest $\prec$ smaller $\prec$ small $\prec$ general $\prec$ big $\prec$ bigger $\prec$ biggest". The symbol $\tilde{v}_{ki}$ represents a value by measuring the attribute u_i $(i = 1,2, \cdots, m)$ of plan A_k ($k = 1,2, \cdots, n$). In $\tilde{v}_{ki}$ various values' arrangement rule is: When u_i is cost-attribute, various linguistic values carry on sorting according to the descending sequence of rank number (or real number size), otherwise according to ascending sequence to carry on sorting. Records the right margin value of $\tilde{v}_{ki}$ is $\tilde{v}_{ki}^R$. Supposes $\tilde{v}_i = \bigcup_{k=1}^{n} \tilde{v}_{ki}$, the right margin value which records is, Records the right margin value of $\tilde{v}_i$ is $\tilde{v}_i^R$, the left margin value of $\tilde{v}_i$ is $\tilde{v}_i^L$.

Definition3. Policy-maker takes a value in an indefinite value, and supposes that attribute's value is not smaller than this value. Then calls this spot as the vacillation decision point.

Obviously as the vacillation decision point toward the right migration, the plan's performance is better in this attribute. When carring on the decision-making at this kind of suppose, policy-maker must withstand the bigger risk. Therefore the vacillation decision point's integer and policy-maker's risk manner has the relation.

A. Policy-makers Risk Preferences

People carry on the decision-making at the definite condition, because policy-maker risk preferences is different. With a program, to a certain decision-makers policy makers it is a certain optimal program, but in terms of other policy-makers it isn't necessarily optimal program. Therefore in the indefinite multi-objective decision making, considers policy-maker's risk preferences is very essential.

TABLE Ⅰ
Policy-maker Risk - income Balance Table

Risk scale	Risk-income balance
$R_2 = \{r_1^2, r_2^2\}$	$W_2 = \{\lambda_1^2, \lambda_2^2\}, where, \lambda_1^2 + \lambda_2^2 = 1$ $0 \le \lambda_1^2, \lambda_2^2 \le 1$
$R_3 = \{r_1^3, r_2^3, r_3^3\}$	$W_3 = \{\lambda_1^3, \lambda_2^3, \lambda_3^3\}, \lambda_1^3 + \lambda_2^3 + \lambda_3^3 = 1$ $0 \le \lambda_1^3, \lambda_2^3, \lambda_3^3 \le 1$
$\vdots$	$\vdots$
$R_p = \{r_1^p, r_2^p, \cdots, r_p^p\}$	$W_p = \{\lambda_1^p, \lambda_2^p, \cdots, \lambda_p^p\}, \lambda_1^p + \lambda_2^p + \cdots + \lambda_p^p = 1$ $0 \le \lambda_1^p, \lambda_2^p, \cdots, \lambda_p^p \le 1$

Note: On this table, in every risk evaluation scale, the risk degree along with the subscript increases. The value λ expresses risk-preference degree of policy-maker. The more the value of λ is big, the more policy-maker is like to the corresponding risk degree. $p = \max(p[k,i],$ $(i = 1,2,\cdots,m)$, $(k = 1,2,\cdots,n)$)

Sector value discretization: Suppose M as the most district of span in all sector value (Before asks district of span, carry on standardized processing. Approach is that the right margin value and the left margin value respectively divide maximum value of this attribute). Supposes $g = \dfrac{M}{p-2}$, divide the various standardized sectors with g , then obtain a series of break points (including the right margin value and the left margin value), separately record as $v_1^{ki}, v_2^{ki}, \cdots, v_{p[k,i]}^{ki}$, and take these break points as vacillation decision point.

B. Using the Intuitive Fuzzy Sets to Express the Indefinite Information

In 3.1, we can express the uncertainty information of the value of $\tilde{v}_{ki}$ as a series of intuitive fuzzy values owing different degrees of risk, respectively records as $(<\mu_j^{ki}, \gamma_j^{ki}>, \lambda_j^{p[k,i]})$ $j = 1,2,\cdots,p[k,i]$.

Transformation method:

$$\mu_j^{ki} = \frac{\left\|\tilde{v}_j^{ki} \dot{-} \tilde{v}_i^L\right\|}{\left\|\tilde{v}_i^R \dot{-} \tilde{v}_i^L\right\|} \qquad \gamma_j^{ki} = \frac{\left\|\tilde{v}_i^R \dot{-} \tilde{v}_{ki}^R\right\|}{\left\|\tilde{v}_i^R \dot{-} \tilde{v}_i^L\right\|} \tag{A}$$

Where Symbol $\|x \dot{-} y\|$ represents distance or grading number between value x and value y .

With the information fusion methods which are constructed in the previous section, can transform these multiple-valued intuitive fuzzy sets into an ordinary intuitive fuzzy sets, record as:

$$B = \{<\mu_1, \gamma_1> / A_1, <\mu_2, \gamma_2> / A_2, \cdots$$
$$, <\mu_n, \gamma_n> / A_n\}$$

According to the indefinite multi-objective decision making's characteristic, the weighted average method is a good fusion method:

$$\mu_k(A_k) = \sum_{j=1}^{p[k,i]} \lambda_j^{p[k,i]} \mu_j^{ki}(A_k)$$
$$\gamma_k(A_k) = \sum_{j=1}^{p[k,i]} \lambda_j^{p[k,i]} \gamma_j^{ki}(A_k) \tag{B}$$

Speaking of each decision scheme, the most ideal result is $< 1,0 >$

C. Constructing Model for Transforming the Vague Set to the Fuzzy Set

(1) Introduce to the Existing Transforming Model

Each $V \in V(X)$ ($V(X)$ expresses the set Vague set on the given universe X), each random element x in universe of X , whose degree of membership is in sector $[t_V(x), 1 - f_V(x)]$, then record the degree of membership of x to the Fuzzy set F_V (F_V expresses a Fuzzy set which has been transformed Vague set V) as μ_V^F .

Model 1[12]

$$\mu_V^F(x) = t_V(x) + \frac{1 - t_V(x) - f_V(x)}{2}$$

This method's basic philosophy: When makes the tendentious analysis to ambiguity $\pi_V(x) = 1 - t_V(x) - f_V(x)$ of the vague collection V , makes the equal distribution between the support and the opposition. This method's deficiency: Have not considered that the determination degree's power of support and the opposition influence the tendentious of the hesitater.

Model 2[12]

$$\mu_V^F(x) = t_V(x) + (1 - t_V(x) - f_V(x)) \frac{t_V(x)}{t_V(x) + f_V(x)}$$

This transformed method's basic philosophy: calculate the tendency that hesitater will cast the affirmative vote to by a proportion of $\frac{t_V(x)}{t_V(x)+f_V(x)}$. This method compared to Method 1 of the improvements: Considered influence of the determination degree of support to the tendency of the hesitater. This method's deficiency: Have not considered the influencement of the determination degree of opposition to the tendency of the hesitater.

Model 3[13]

$$\mu_V^F(x)=\begin{cases} t_V(x)+(1-t_V(x)-f_V(x))\dfrac{1-f_V(x)}{t_V(x)+f_V(x)} & t_V(x)=0 \\ t_V(x)+(1-t_V(x)-f_V(x))\dfrac{t_V(x)}{t_V(x)+f_V(x)} & 0<t_V(x)\le 0.5 \\ t_V(x)+(1-t_V(x)-f_V(x))(0.5+\dfrac{t_V(x)-0.5}{t_V(x)+f_V(x)}) & 0.5<t_V(x)\le 1 \end{cases}$$

In view of the deficiency existing in model 2, this transformed method has made some improvement. The analysis tendency of the hesitater is more exquisite. But this method's deficiency is that the value $t_V(x)+(1-t_V(x)-f_V(x))\frac{1-f_V(x)}{t_V(x)+f_V(x)}$ is possibly bigger than 1, this does not tally with the reality obviously.

Model 4[14]

$$\mu_V^F(x)=\begin{cases} t_V(x)+(1-t_V(x)-f_V(x))\dfrac{1-f_V(x)}{2} & t_V(x)=0 \\ t_V(x)+(1-t_V(x)-f_V(x))\dfrac{1+t_V(x)}{2} & f_V(x)=0 \\ t_V(x)+(1-t_V(x)-f_V(x))\dfrac{t_V(x)}{t_V(x)+f_V(x)} & t_V(x)\neq 0, f_V(x)\neq 0 \end{cases}$$

In extreme cases $t_V(x)=0$ or $f_V(x)=0$, this method deals with the tendency of hesitater is superior to the above three methods; but under the cases $t_V(x)\neq 0$ and $f_V(x)\neq 0$, this model does not reflect the influencement of the determination degree of opposition to the tendency of the hesitater.

The above four methods deal with the tendency of the hesitater, in essence using the average thoughts. However, when the variable value uncertainty, the mean value represents the average trend of variable value and the actual value of variable value may deviate from the mean value. In the process of the Vague sets transforming into the Fuzzy Sets, the tendency of the hesitater is an uncertain variable, so the issue of the Vague sets transforming into the Fuzzy Sets can be as a risk decision-making. In risk decision-making, the risk preference of decision makers determined by the subjective and objective conditions is a key factor in the decision-making process. Below will introduce the risk preference of decision makers into the question of the Vague sets transforming into the Fuzzy Sets.

(2) Constructing new model

To each $V\in V(X)$ ($V(X)$ expresses the set Vague set on the given universe X), each random element x in universe of X , whose degree of membership is in sector $[t_V(x),1-f_V(x)]$, then record the degree of membership of x to the Fuzzy set F_V (F_V expresses a Fuzzy set which has been transformed Vague set V) as μ_V^F . From the preceding analysis we can see the value of B together with the risk of μ_V^F is equal the weighted sum of the risk-value, the tendency value and t_V .

TABLE Ⅱ
Policy-Maker Risk - income Balance Table

Risk scale	Risk-income balance
$R_2=\{r_1^2,r_2^2\}$	$W_2=\{\lambda_1^2,\ \lambda_2^2\}, where, \lambda_1^2+\lambda_2^2=1$ $0\le\lambda_1^2,\lambda_2^2\le 1$
$R_3=\{r_1^3,r_2^3,r_3^3\}$	$W_3=\{\lambda_1^3,\lambda_2^3,\lambda_3^3\}, \lambda_1^3+\lambda_2^3+\lambda_3^3=1$ $0\le\lambda_1^3,\lambda_2^3,\lambda_3^3\le 1$
⋮	⋮
$R_p=\{r_1^p,r_2^p,\cdots,r_p^p\}$	$W_P=\{\lambda_1^P,\lambda_2^P,\cdots,\lambda_P^P\}, \lambda_1^p+\lambda_2^p+\cdots+\lambda_p^p=1$ $0\le\lambda_1^p,\lambda_2^p,\cdots,\lambda_p^p\le 1$

To a Vague value $[t_V(x),1-f_V(x)]$, the degree of membership of element x to vague set V is in sector $[t_V(x),1-f_V(x)]$, the specific point is unable to determine according to the available information about the membership relations between the object x and the set V . In order to facilitate the following description, we make the following provisions: the value of μ_V^F as the policy-maker's income, the risk event as the fixed variates actual value being smaller than target value, the degree of risk as the uncertain variables is less than the actual value of the target the possibility of the size of the incident. In the ordinary circumstances the income is bigger, the risk is higher. Order $M=\max(1-t_V(x)-f_V(x),\ x\in X)$ the policy-maker, according to the subjective and objective condition, carries on the balance between risk and income, obtains the following policy-maker risk - income balance table (Table 2).

Note: On this table, in every risk evaluation scale, the risk degree along with the subscript increases. The value λ expresses risk-preference degree of policy-maker. The more the value of λ is big, the more policy-maker is like to the corresponding risk degree. The value of p is decided by the policy-maker according to the subjective and objective condition, the B value is bigger, is more exquisite to the description of risk degree.

Following use this table to quantitatively portray the policy-making risk in the process of gather the B transformating the Vague set V into Fuzzy set F, because the the tendency of support and oppose is indefinite. $\forall x \in X$, The degree of membership of element x to vague set V is in sector $[t_V(x), 1-f_V(x)]$, the degree of membership of element x to Fuzzy set F_V is also in sector $[t_V(x), 1-f_V(x)]$. Use the line segment whose the length is equal $\frac{M}{P}$ to divide sector $[0, 1-t_V(x)-f_V(x)]$, obtain a series of break points (including sector right endpoint), and record the number of the break point number as $k[x]$, respectively record these break points as $y_i, i=1,2,\cdots,k[x]$. The formula of risk value in the value of $\mu_V^F(x)$ is:

$$R(x)=\sum_{i=1}^{k[x]}\lambda_i^{k[x]}y_i$$

Presently conduct the research to the tendency value in the value of $\mu_V^F(x)$ and construct the formula to compute tendency value, the tendency supportation degree in the hesitation about the membership degree of element x to the Vague set V is decided by the following two factors: First, the promoter action of the degree of certainty supportation $t_V(x)$, Second, the resistance function of the degree of certainty opposition $f_V(x)$. The formula of tendency value in the value of $\mu_V^F(x)$ is:

$$Q(x)=(w_1\frac{t_V(x)}{2}+w_2\frac{1-f_V(x)}{2})(1-t_V(x)-f_V(x))$$

Where, the weight value w_1 and w_2 separately expresses the policy-maker preferences to these two kinds of function, $w_1+w_2=1$, $0\le w_1, w_2\le 1$.

By the above analysis, we can use the following double-weighted conversion operator to convert a Vague set into a Fuzzy set.

$$\mu_V^F(x)=t_V(x)+\theta_1 R(x)+\theta_2 Q(x)$$
$$=t_V(x)+\theta_1\sum_{i=1}^{k[x]}\lambda_i^{k[x]}y_i+\theta_2\left(w_1\frac{t_V(x)}{2}+ w_2\frac{1-f_V(x)}{2}\right)(1-t_V(x)-f_V(x)) \quad \text{(C)}$$

Where, the weight value θ_1, θ_2 separately expresses the policy-maker preferences to risk value and tendency value.

Policy-making algorithm:

Step 1: Respectively appraisal each decision scheme according to each attributes and carries on standardized processing to each appraisal result.

Step 2: The policy-maker determines the policy-maker risk-income balance table according to the subjective and objective condition. Based on this carries on discretization processing to each sector value, and determines vacillation decision point of each indefinite value.

Step 3: With formula (A), can express the uncertainty information of the value of $\tilde{v}_{ki}$ as a series of intuitive fuzzy values owing different degrees of risk.

Step 4: With the information fusion methods which are constructed in the Part 2, can transform these multiple-valued intuitive fuzzy sets into an ordinary intuitive fuzzy sets.

Step 5: With formula (C), separately converte this vague value $< A_i, \mu_i, \gamma_i > (i=1,2,\cdots,n)$ into Fuzzy value, record as ℓ_i.

Step 6: A_k $(k=1,2,\cdots,n)$ carry on sorting according to the corresponding the descending sequence of value ℓ_k. The first plan is the best plan.

IV. EXAMPLE

Consider one venture capital company which carries on the high tech project investment. Five alternative enterprises (plan) A_k ($k=1,2,\cdots,5$) can be chosen. Carry on the appraisal from the angle of those enterprises ability's, firstly formulate seven appraisal targets (attribute) [15]: The marketing capacity (u_1), the managed capacity (u_2), productivity (u_3), technical ability (u_4), fund ability (u_5), risk exposure ability (u_6). Obviously, these seven attributes are the efficient attribute. Use the seven-level evaluation scale to measure these seven attributes, which is "worst ≺ worse ≺ bad ≺ general ≺ good ≺ better ≺ best" or "smallest ≺ smaller ≺ small ≺ general ≺ big ≺ bigger ≺ biggest". Might as well use the mark "$s_1 \prec s_2 \prec s_3 \prec s_4 \prec s_5 \prec s_6 \prec s_6$" to express the corresponding linguistic value. Then obtain the policy-making matrix (shown as Table 3). Try to determine the best enterprise.

TABLE III
Policy-Making Table

	u_1	u_2	u_3	u_4	u_5	u_6
A_1	[6,8]	[6,8]	[7,8]	[8,10]	$[s_3,s_4,s_5]$	$[s_4,s_5]$
A_2	[8,9]	[8,10]	[8,9]	[7,8]	$[s_5,s_6]$	$[s_4,s_5]$
A_3	[7,8]	[8,9]	[7,9]	[9,10]	$[s_5,s_6]$	$[s_5,s_6]$
A_4	[8,9]	[7,8]	[6,10]	[6,8]	$[,s_4,s_5]$	$[s_6,s_7]$
A_5	*[7,8]*	*[6,9]*	*[5,9]*	*[7,8]*	$[s_3,s_4]$	$[s_5,s_6]$

Obviously, the above attribute speaking of the policy-making goal is the efficient attribute. Order $\tilde{v}_i = \bigcup_{k=1}^{n} \tilde{v}_{kj}$, extract right margin value of $\tilde{v}_j$, record as $\tilde{v}_j^R$, extract left margin value of $\tilde{v}_j$, record as $\tilde{v}_j^L$, $i = 1,2,\cdots,5$, $j = 1,2,\cdots,6$. Result as shown in Table 4.

TABLE Ⅳ

	u_1	u_2	u_3	u_4	u_5	u_6
$\tilde{v}_j^R$	9	10	10	10	s_6	s_7
$\tilde{v}_j^L$	6	6	5	6	s_3	s_4

According to transformation formula (A), use a multiple-valued intuition fuzzy set A to express the fuzzy evaluation information which respectively included in each indefinite linguistic value or real number sector value in the policy-making table 3.

$$A=\{ <B_1,[0,1/4,2/5,1/2,0,0],[1/3,1/2,2/5,1/3,2/3]>,$$
$$<B_2,[2/3,1/2,3/5,1/4,2/3,0],[0,0,1/5,1/2,0,2/3]>,$$
$$<B_3,[1/3,1/2,2/5,3/4,2/3,1/3],[1/3,1/4,1/5,0,0,1/3]>,$$
$$<B_4,[2/3,1/4,1/5,0,1/3,2/3],[0,1/2,0,1/2,1/3,0]>,$$
$$<B_5,[1/3,0,0,1/4,0,1/3],[1/3,1/4,1/5,1/2,2/3,1/3]>$$
$$\}$$

Use the simple weighting arithmetic mean value method to separately carry on the fusion to degrees of membership and the non-degree of membership multiple-valued intuition, obtains following intuition fuzzy set C:

$$C=\{<B_1,0.2175,0.355>,<B_2,0.445,0.24>,$$
$$<B_3,0.4883,0.1974>,<B_4,0.3442,0.2083>,$$
$$<B_5,0.1667,0.3608>\}$$

Policy-makers carry on measuring their risk preference, Obtain the following policy-maker risk-preference degree table (shown as Table 5).

TABLE Ⅴ

Risk evaluation scale	Risk-preference degree
$R_2=\{r_1^2,r_2^2\}$	$W=(\lambda_1^2,\lambda_2^2\}$ =（0.3，0.7）
$R_3=\{r_1^3,r_2^3,r_3^3\}$	$W=(\lambda_1^3,\lambda_2^3,\lambda_3^3)$ =（0.1，0.3，0.6）
$R_4=\{r_1^4,r_2^4,r_3^4,r_4^4\}$	$W=(\lambda_1^4,\lambda_2^4,\lambda_3^4,\lambda_4^4)=$ (0.05,0.15,0.25,0.55)

Use formula (C) separately convert these Vague value $<B_i,\mu_i,\gamma_i>(i=1,2,\cdots,5)$ into Fuzzy value, record as ℓ_i

$$\ell_1=0.52671, \ell_2=0.60185, \ell_3=0.62669,$$
$$\ell_4=0.58785, \ell_5=0.52013$$

Obviously, $\ell_3>\ell_2>\ell_4>\ell_5>\ell_1$. Therefore, the enterprise A_3 is the best enterprise.

Ⅴ. CONCLUSION

Innovation in this paper: introduce the policy-maker to risk preferences to the decision model. So that decision-makers independently decide some parameters in the decision-making model according to their own characteristics and decision-making environmental changes. The dialogue between decision-making model and decision-makers can make the result of decision-making meeting with specific decision-making environment. On the other hand, the existing multi-attribute decision-making about heterogeneous multi-attribute decision-making is less. In the paper, construct algorithm for the heterogeneous multi-attribute decision-making containing the interval value and indefinite language value.

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Zhou Qihai (1947-) is a Full Professor (from 1995), Doctor's (and Master's) tutor and a head of Information Technology Application Research Institute, School of Economic Information Engineering, Southwestern University of Finance and Economics (SWUFE), China. He graduated in 1982 from Lanzhou University, China; has been working in SWUFE since 1982, successively hold posts from teaching assistant (1982-1987), lecturer (1987-1991), vice professor (1991-1995, promoted anomaly in 1991), professor (1995-today, promoted anomaly in 1995); and got the titles of both "Outstanding experts (enjoyed government subsidies) with outstanding contributions of Sichuan province, China" (summa cum laude of Sichuan province government, 1996) and "One hundred academic and managerial leading heads of China informationalization" (summa cum laude about this domain in China, 2006). He has published 46 academic books and over 212 academic papers; and is President of **IITAA** (**I**nternational **I**nformation **T**echnology & **A**pplications **A**ssociation), Chair or Organizing Chair of some important international conferences. His research interests are in algorithm research, computational geometry, isomorphic information processing, economics & management computation, eBusiness, and so on. More (in Chinese) about Prof. Zhou Qihai is shown here:

LI Yan (1983-) is studying in School of Economic Information Engineering, Southwestern University Of Finance and Economics，Chengdu, Sichuan, China. He has published 13 papers. His research areas are in Non-definite decision-making, intelligence information processing.