# Decision-making for Investment Based On Option and Term Structure

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***Abstract*—Static and dynamic term structure model of interest rates are studied according to the need for using. As for the static model, exponential splines model is studied and every cash flow of the project is discounted relatively accurately by getting the model of discount rate. As for the dynamic model, a basic model is studied, and the option pricing formula under changing risk-free rate is gotten by bringing it into the option pricing formula. And then a complex model which is more approaching the fact is used to describe the moving pattern of risk-free interest rates and the former formula that have been educed is modified. Finally, Empirical research is carried on based on the data of Chinese financial market.**

***Index Terms*—real option, exponential splines, simulation, GMM(Generalized Method of Moments) .**

## I. INTRODUCTION

The traditional DCF methodology can not capture the value of the flexibility that may be present in a project (Edward H. Bowman, 2001) [1].The real options approach has been developed against the shortcomings of the conventional traditional techniques to decision making and been suggested as a capital budgeting and strategic decision-making tool because it explicitly accounts for the value of future flexibility (Dixit,Avinash K., Robert S. Pindyck, 1994, Trigeorgis 1996, Amram and Kulatilaka 1999) [2, 3, 4].

There are many scholars having studied the real option theory such as Myers (1977) [5];Ross (1978) [6];Kester(1984)[7];TRIGEORGIS, etc (1987)[8]; KULA T ILA KA, etc (1992)[9]; Edward H. Bowman (2001) [2] ;Andrew, Consultant (2002) [10] ;Niklas Kari (2002) [11];Cherif Selima [12];Miller, LukeT. and Park, Chan S. (2002)[13] etc.

A simplification is often made when using real option theory to estimate the investment value of a project, in which risk-free interest rate and discount rate are regarded as constants and not distinguished in many cases. But in many cases, the reality is that the risk-free interest rate and discount rate may change. Hu Wen-xiu, Liu Xiang-fang (2006)[14] researched on the real option pricing model on the condition of risk-free interest rate changing. They did not distinguish risk-free interest rate and discount rate, and did not give out specific parameter estimation but made a parameter assumption directly. HE Qi-zhi, HE Jian-min (2007) [15] researched on the R&D investment by combining the game theory and term structure of interest rates. But they have not applied the term structure model to the option pricing formula. Many others researched the option pricing based on the stochastic interest rate, but the results are difficult at the practical application. HE Qi-zhi, HE Jian-min (2008) [16] studied the dynamic and static term structure model of interest rates and applied them to the real option pricing. But they only researched and applied the simple term structure models which are polynomial splines model and a simple basic dynamic model.

In the paper we will study and apply the complex model which is more approaching the fact or can fit the term structure of interest rate of China more accurate.

## II. THEORY OF TERM STRUCTURE OF INTEREST RATES

According to the existing research findings, there are two kinds of construction methods of the term structure of interest rate, one is static estimation model, and the other is dynamic estimation model.

### *A. Static Term Structure Model of Interest Rates*

Static term structure model of interest rates is to fit the term structure of interest rates by the actual data acquired from the market. Polynomial spline methods are very important and widely used method among them. While there are several drawbacks to their use in fitting discount functions. The discount function is principally of an exponential shape. Splines, being piecewise polynomials, are inherently ill suited to fit an exponential type curve. Polynomials have a different curvature from exponentials, and although a polynomial spline can be forced to be arbitrarily close to an exponential curve by choosing a sufficiently large number of knot points, the local fit is not good. A practical manifestation of this phenomenon is that a polynomial spline tends to “weave” around the exponential, resulting in highly unstable forward rates (which are the derivatives of the logarithm of the discount function). Another problem with polynomial splines is their undesirable asymptotic properties. Polynomial splines cannot be forced to tail off in an exponential form with increasing maturities. (Vasicek and Frong, 1982)[18] Vasicek and Frong(1982) presented a different approach, which can be termed an exponential spline fitting. The model has desirable asymptotic properties for long maturities, and exhibits both a sufficient flexibility to fit a wide variety of shapes of the term structure, and a sufficient robustness to produce stable forward rate curves.

Exponential spline methods represent discount factor E ($t_0$, t) as piecewise spline functions (He Qi-Hi, 2007) [19]. Let

$$E(t_0,t)=\begin{cases}E_0(t_0,t)=a_0+b_0e^{-u\Delta t}+c_0e^{-2u\Delta t}+d_0e^{-3u\Delta t},\Delta t\in[0,6]\\ E_6(t_0,t)=E_0(t_0,t)+d_1(e^{-u\Delta t}-e^{-6u})^3,\Delta t\in[6,12]\\ E_{12}(t_0,t)=E_6(t_0,t)a_0+d_2(e^{-u\Delta t}-e^{-12u})^3],\Delta t\in[12,20]\end{cases} \quad (1)$$

From the above expression, we can see that E ($t_0$, t) is determined by $\alpha(b_0,c_0,d_0,d_1,d_2,u)$ , which is the solution of the following model:

$$\begin{cases}P_{t_0}=\hat{P}_{t_0}+\varepsilon,\varepsilon\in(\varepsilon_1,\varepsilon_2,\cdots,\varepsilon_i,\cdots,\varepsilon_n)^T\\ E(\varepsilon_i)=0,\\ Var(\varepsilon_i)=\sigma^2\omega_i^2,\sigma\in R,\\ Cov(\varepsilon_i,\varepsilon_j)=0,i\neq j\end{cases} \quad (2)$$

Where $P_{t_0}=(p_{t_0}^j)_{j=1,2,\cdots,n}$ and $\hat{P}_{t_0}=(\hat{p}_{t_0}^j)_{j=1,2,\cdots,n}$ denote price vector respectively. $p_{t_0}^j$ is the market price of the j-th bond at time $t_0$, $\hat{p}_{t_0}^j$ is the theoretic price of the j-th bond at time $t_0$ , namely, computational price according to the formulary

$$\hat{p}_{t_0}^j=\sum_{t_i}F_{t_i}^{(j)}E(t_i), \quad (3)$$

Where $F_{t_i}^j$ is the cash flow of the j-th bond at time $t_i$.

The model described by equation (2) is used in the estimation of the term structure. It is nonlinear in the parameters $\alpha(a_0,b_0,c_0,d_0,d_1,d_2,u)$ with residual covariance matrix proportional to

$$\Omega=\begin{pmatrix}\omega_1 & & & \\ & \omega_2 & & \\ & & \ddots & \\ & & & \omega_n\end{pmatrix}. \quad (4)$$

If we fix u [1], then the least-squares estimate of $\alpha(a_0,b_0,c_0,d_0,d_1,d_2)$ conditional on the value of u can be directly calculated by the generalized least squares regression equation

$$\hat{\alpha}=(X'\Omega^{-1}X)^{-1}X'\Omega^{-1}P_{t_0} \quad (5)$$

Where X is the coefficient matrix of equation (2). Imposing on $E(t_0,t_0)$ =1, namely, $A\alpha$ =1, we can get the amendatory estimate.

[1]The parameter u, constitutes the limiting value of the forward rates, should be in a rational range. We can then find the value of u that minimizes $S(u)=(p_{t_0}-X\hat{\alpha}_c)'\Omega^{-1}(p_{t_0}-X\hat{\alpha}_c)$ .

$$\hat{\alpha}_c=\hat{\alpha}-(X'\Omega^{-1}X)^{-1}A'(A(X'\Omega^{-1}X)^{-1}A')^{-1}(A\hat{\alpha}-1) \quad (6)$$

In fact, the key of whole optimized decision process is to determine heteroscedasticity $\omega_i$ , In reality, only one kind of pure discount rate is needed for fixing the price of the bond under one year, but over 20 kinds of discount rate for the bond of 20 years. So the pricing error of the long term bond is often greater than that of the short term bond. This is the unavoidable different variance characteristic of sample while fitting the interest rates curve. To solve this problem, Vasicek and Fong (1982), Martellini，L. and Priaulet,P(2001) pointed out that we can fetch $\omega_i=T_i^2$ , where $T_i$ is the time to maturity of the i-th bond.

After obtaining the discount factors, translate it into the annualized spot rate through the following formula:

$$r(t_0,t)=[\frac{1}{E(t_0,t)}]^{\frac{1}{t-t_0}}-1 \quad (7)$$

*B. Dynamic Term Structure Model of Interest Rates*

The dynamic estimation model of the term structure of interest rate is the modelling of random behaviour of interest rate on the base of economic hypothesis. Many term structure models—both single-factor and multifactor—imply dynamics for the short-term riskless rate r can be nested within the following stochastic differential equation (Chan, and Karolyi, 1992) [20,21,22]:

$$dr_t=(\alpha+\beta r_t)dt+\sigma r_t^{\gamma}dZ_t \quad (8)$$

The stochastic differential equation given in (8) defines a broad class of interest rate processes which includes many well-known interest rate models.

These models can be obtained from (8) by simply placing the appropriate restrictions on the four parameters $\alpha,\beta,\sigma$ and $\gamma$ .

The common dynamic models of interest rate are as follows:

1. Merton $dr_t=\alpha dt+\sigma dZ_t$
2. Vasicek $dr_t=k(\mu-r_t)dt+\sigma dZ_t$
3. CIR $dr_t=k(\mu-r_t)dt+\sigma r^{0.5}dZ_t$
4. Dothan $dr_t=\sigma r_t dZ_t$
5. GBM $dr_t=\beta r_t+\sigma r_t dZ_t$
6. CKLS $dr_t=k(\mu-r_t)dt+\sigma r^{1.5}dZ_t$

The paper adopts a more realistic model of interest rate as follows:

$$dr_t=k(\mu-r_t)/r_t dt+\sigma r^{1.5}dZ_t \quad (9)$$

Where $k,\mu,\sigma$ is the parameter to be estimated. dZt is the increment of Wiener Process; rt is the interest rate at the time t. The drift of the model is $k(\mu-r_t)/r_t$ , which shows that the interest rates have very strong mean-reversion characteristic. $\mu$ represents the long-term average level of interest rates. $k/r_t$ represents the speed of mean-reversion, The smaller value of $r_t$, the faster speed of mean-reversion when $r_t$ deviates from the long-term level. The diffusion is $\sigma r^{1.5} dZ_t$, which reflects the level effect of interest rate fluctuations, namely the high level of interest rates corresponds to its higher volatility, and at the same time avoids the possibility of negative interest rates.

Parameter can be estimated though the discretization of formula (7). Let the time from t to t+1 is δ. Because interest rate data to estimate the parameters are daily data, δ is 1 / 365. Following Brennan and Schwartz (1982), Dietrich-Campbell and Schwartz (1986), Sanders and Unal (1988), and others, we estimate the parameters of the continuous-time model using a discrete-time econometric specification

$$r_{t+1} - r_t = \alpha \frac{1}{r_t}(\mu - r_t)\delta + \varepsilon_{t+1} \tag{10}$$

$$E[\varepsilon_{t+1}] = 0 \qquad E[\varepsilon_{t+1}^2] = \sigma^2 r_t^3 \delta \tag{11}$$

This discrete-time model has the advantage of allowing the variance of interest rate changes to depend directly on the level of the interest rate in a way consistent with the continuous-time model.

Our econometric approach is to test (10) and (11) as a set of overidentifying restrictions on a system of moment equations using the Generalized Method of Moments (GMM) of Hansen (1982). This technique has a number of important advantages that make it an intuitive and logical choice for the estimation of the continuous-time interest rate processes. First, the GMM approach does not require that the distribution of interest rate changes be normal; the asymptotic justification for the GMM procedure requires only that the distribution of interest rate changes be stationary and ergodic and that the relevant expectations exist. This is of particular importance in testing the continuous-time term structure models since each implies a different distribution for changes in r. Second, the GMM estimators and their standard errors are consistent even if the disturbances, $\varepsilon_{t+1}$, are conditionally heteroskedastic.

Define $\theta$ to be the parameter vector with elements $\alpha, \mu, \sigma$.

Given

$$\varepsilon_{t+1} = r_{t+1} - r_t - \alpha \frac{1}{r_t}(\mu - r_t)\delta, \tag{12}$$

Let the vector $f_t(\theta)$ be

$$f_t(\theta) = \begin{pmatrix} \varepsilon_{t+1} \\ \varepsilon_{t+1} r_t \\ \varepsilon_{t+1}^2 - \frac{1}{365}\sigma^2 r_t^3 \end{pmatrix} \tag{13}$$

Under the null hypothesis that the restrictions implied by (10) and (11) are true, we can get

$$E(f_t(\theta)) = 0. \tag{14}$$

The GMM procedure consists of replacing $E(f_t(\theta))$ with its sample counterpart, using the T observations where

$$m_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} f_t(\theta) \tag{15}$$

And then choosing parameter estimates that minimize the quadratic form

$$J_T(\theta) = m_T^{'}(\theta) W_T(\theta) m_T(\theta) \tag{16}$$

Where $W_T(\theta)$ is a positive-definite symmetric weighting matrix.

## III. Theory of Discount Rate

The discount rate reflects the time value of the money. The essence of discount rate is a kind of investment return rate. Discount rate should higher than the risk-free rate. In the normal capital market and property rights market, interest rates of Government Bond and deposit rates is treated as the return on risk free investment by investors. If the discount rate lowers than return on risk free investment, investors will deposit their money or buy risk-free bonds instead of risk investment. Discount rate is the sum of risk-free interest rate and risk premium rate. The formula is (Chen Jing-feng,Ji Li,2005)[23]:

Discount rate = risk-free interest rate+ return on risk investment + inflation rate. (17)

The risk-free interest rate can be estimated by the static term structure model of Interest rates mentioned above using data from the shanghai stock exchange. Return on risk investment and inflation rate can be estimated by experience judgment concerning macroeconomics environment, the developing prospect of industry, market and competition of similar enterprises.

## IV. Valuing a Strategic Real Option

An option is a security giving the right to buy or sell an asset, subject to certain conditions, within a specified period of time.

### *A. The Valuation Formula of Option[24]*

By assuming some “ideal conditions” in the market for the stock and for the option, Black and Scholes (1973) [24] have derived the formula for the value of an option in terms of the price of the stock. The deduction is as follows [24]:

Creating a hedged position, consisting of a long position in the stock and a short position in the option.

In general, since the hedged position contains one share of stock long and $1/\omega_1$ options short, the value of the equity in the position is:

$$S(t) - \omega / \omega_1 \tag{18}$$

The change in the value of the equity in a short interval $\Delta t$ is:

$$\Delta S(t) - \Delta\omega / \omega_1 \tag{19}$$

Where

$$\Delta\omega = \omega(S(t)+\Delta S(t), t+\Delta t) - \omega(S(t),t)$$
$$= \omega_1 \Delta S(t) + \frac{1}{2}\omega_{11}\sigma^2 S(t)^2 \Delta t + \omega_2 \Delta t \quad (20)$$

Substituting from equation (20) into expression (19), the change in the value of the equity in the hedged position is:

$$-(\frac{1}{2}\omega_{11}\sigma^2 S(t)^2 + \omega_2)\Delta t / \omega_1 \quad (21)$$

The change in the equity (24) must equal the value of the equity (18) times $r\Delta t$.

$$-(\frac{1}{2}\omega_{11}\sigma^2 S(t)^2 + \omega_2)\Delta t / \omega_1 = (S(t) - \omega / \omega_1) r\Delta t \quad (22)$$

Boundary conditions is as follows:

$$\omega(S(T),T) = S(T) - X, S(T) \ge X = 0, S(T) < X \quad (23)$$

Using the knowledge of stochastic differential equation, the famous Black-Scholes formula can be gotten (detailed derivation can be seen in [24]):

$$\omega(x,t) = S(t)N(d_1) - Xe^{r(t-T)}N(d_2)$$
$$d_1 = \frac{\ln(S/X) + (r+\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \quad (24)$$
$$d_2 = d_1 - \sigma\sqrt{T-t}$$

In equation (24), N (d) is the cumulative normal density function.

The Black-Scholes formula (24) requires the risk-free interest rate is constant over the life of option. If the risk-free interest rates are changing during the life cycle of option, and then $rT$ can be replaced by cumulative risk-free interest rate (Song Peng-ming, 1999) [25].

*B. Option Pricing based on term structure of Interest Rate*

Using a basic dynamic model of interest rate:

$$dr_t = (\alpha + \beta r_t)dt + \sigma dW_t \quad (25)$$

In this model, the change of interest rate is a random process. At an arbitrary time point t, the value of $r_t$ is a random variable, which means that $r_t$ is uncertain at an arbitrary time point t over the life of option. Instantaneous expected value $E(r_t)$ can be estimated as instantaneous interest rate at an arbitrary time point t. From equation (25), we can get

$$\frac{1}{\sigma\beta}d(\alpha+\beta r_t) - \frac{1}{\sigma}(\alpha+\beta r_t)dt = dW_t \quad (26)$$

Let

$$X(t) = (\alpha + \beta r_t)$$

Then

$$\frac{1}{\sigma\beta}dX(t) - \frac{1}{\sigma}X(t)dt = dW_t \quad (27)$$

Integrate (27) from $t_0$ to $t$

$$\frac{1}{\sigma\beta}(X(t) - X(t_0)) - \frac{1}{\sigma}\int_{t_0}^{t} X(s)ds = W(t) - W(t_0) \quad (28)$$

Let

$$X(t) = c(t)e^{\beta t} \quad (29)$$

Then

$$X'(t) = (c'(t) + \beta c(t))e^{\beta t} \quad (30)$$

Substitute (30) into (27)

$$\frac{1}{\sigma\beta}(c'(t) + \beta c(t))e^{\beta t} - \frac{1}{\sigma}c(t)e^{\beta t} = W'(t) \quad (31)$$

$$\frac{c'(t)}{\sigma\beta}e^{\beta t} = W'(t) \quad (32)$$

$$c'(t) = \sigma\beta e^{-\beta t}W'(t) \quad (33)$$

Integrate (33) from $t_0$ to $t$

$$c(t) - c(t_0) = \sigma\beta\int_{t_0}^{t} e^{-\beta s}dW(s) =$$
$$\sigma\beta[e^{-\beta t}W(t) - e^{-\beta t_0}W(t) + \int_{t_0}^{t}\beta e^{-\beta s}W(s)ds] \quad (34)$$

We can get $c(t_0) = e^{-\beta t_0}X(t_0)$ from $X(t_0) = c(t_0)e^{\beta t_0}$, substituting it into (29, 34), and we can get:

$$X(t) = c(t)e^{\beta t} = \{c(t_0) + \sigma\beta[e^{-\beta t}W(t) - e^{-\beta t_0}W(t_0)$$
$$+\int_{t_0}^{t}\beta e^{-\beta s}W(s)ds]\}e^{\beta t}$$
$$= [e^{-\beta t_0}X(t_0) + \sigma\beta e^{-\beta t}W(t) - \sigma\beta e^{-\beta t_0}W(t_0)$$
$$+\sigma\beta\int_{t_0}^{t}\beta e^{-\beta s}W(s)ds]e^{\beta t}$$
$$= e^{\beta(t-t_0)}X(t_0) + \sigma\beta W(t) - \sigma\beta e^{\beta(t-t_0)}W(t_0)$$
$$+\sigma\beta^2\int_{t_0}^{t}e^{\beta(t-s)}W(s)ds$$
$$= (X(t_0) - \sigma\beta W(t_0))e^{\beta(t-t_0)} +$$
$$\sigma\beta W(t) - \sigma\beta\int_{t_0}^{t}W(s)de^{\beta(t-s)}$$
$$= (X(t_0) - \sigma\beta W(t_0))e^{\beta(t-t_0)} + \sigma\beta W(t)$$
$$-\sigma\beta W(s)e^{\beta(t-s)}/_{t_0}^{t} + \sigma\beta\int_{t_0}^{t}e^{\beta(t-s)}dW(s)$$
$$= (X(t_0) - \sigma\beta W(t_0))e^{\beta(t-t_0)} + \sigma\beta W(t) - \sigma\beta W(t)$$
$$+\sigma\beta W(t_0)e^{\beta(t-t_0)} + \sigma\beta\int_{t_0}^{t}e^{\beta(t-s)}dW(s)$$
$$= X(t_0)e^{\beta(t-t_0)} + \sigma\beta\int_{t_0}^{t}e^{\beta(t-s)}dW(s) \quad (35)$$

For simplification, we can let $t_0 = 0$, then

$$X(t) = X(0)e^{\beta t} + \sigma\beta\int_0^t e^{\beta(t-s)}dW(s) \quad (36)$$

Commutates mathematical expectation of both sides of equation (36) [26]

$$EX(t) = X(0)e^{\beta t} + \sigma\beta\int_0^t e^{\beta(t-s)}E[dW(s)] = X(0)e^{\beta t} \quad (37)$$

Substituting $X(t) = (\alpha + \beta r_t)$ into (39), we can obtain:

$$E\ (\alpha+\beta \mathrm{r_t}) = (\alpha+\beta \mathrm{r_0})e^{\beta t} \quad (38)$$

Thus, Instantaneous expected value is

$$E(r_t) = \frac{(\alpha + \beta r_0)e^{\beta t} - \alpha}{\beta} \quad (39)$$

According to Song Peng-ming(1999)[23], replacing $r_t$ by cumulative risk-free interest rate

$$\int_0^t E(r_t)dt = \int_0^t \frac{(\alpha + \beta r_0)e^{\beta t} - \alpha}{\beta}dt = \frac{(\alpha + \beta r_0)e^{\beta t}}{\beta^2} - \frac{\alpha}{\beta}t \quad (40)$$

We can derive an option pricing formula whose change of interest rate obeys basic dynamic model (25). The option pricing formula is

$$c(S_t, t) = S(t)N(d_1) - Xe^{\frac{\alpha}{\beta}t - \frac{(\alpha+\beta r_0)e^{\beta t}}{\beta^2}}N(d_2)$$

$$d_1 = \frac{\ln(S/X) + (\frac{\alpha + \beta r_0}{\beta^2}e^{\beta t} - \frac{\alpha}{\beta}t + \frac{\sigma^2}{2}t)}{\sigma\sqrt{t}} \quad (41)$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

In the above term structure model of interest rates, there is no level effect of interest rates and there is the possibility of interest rates being negative. In the next, we study the option pricing formula based on model (9), the term structure model of interest rate closer to the reality.

Similarly, we should obtain the instantaneous expected value of interest rates characterized by model (9). There are some difficulties to solve directly model (9) and we can get the instantaneous expected value Using computer simulation. Calculation steps are as follows[14]:

Step 1: Discretizing model (9) to the following model:

$$r_{t+1} - r_t = \alpha\frac{1}{r_t}(\mu - r_t)\delta + \sigma r_t^{1.5}\delta^{0.5}\xi_t \quad (42)$$

And then

$$r_{t+1} = r_t + \alpha\frac{1}{r_t}(\mu - r_t)\delta + \sigma r_t^{1.5}\delta^{0.5}\xi_t \quad (43)$$

Step 2: Determining the time interval $\delta$, initial interest rates $r_0$, the value of parameters: $\alpha$, $\mu$ and $\sigma$, simulation times: N, and get a Random variable ξ obeying to N(0,1) using computer simulation.

Step 3: Geting n pieces of simulated value of $r_1$: $r_{11}$, $r_{12}$,⋯ $r_{1n}$ by substituting $r_0$ into equation (43) with circulation N times. Take the average of $r_{11}$, $r_{12}$,⋯ $r_{1n}$ as an approximation of E ($r_1$).

Similarly we can get the approximation of E ($r_2$), E ($r_3$), ⋯E($r_k$).

Step 4: Replacing rt with $\sum_{i=1}^{k} E(r_i)\Delta t$, and we can get the pricing formula based on term structure model (9).

$$c(S_t, t) = S(t)N(d_1) - Xe^{-\sum_{i=1}^{k} E(r_i)\Delta t}N(d_2)$$

$$d_1 = \frac{\ln(S/X) + (\sum_{i=1}^{k} E(r_i)\Delta t + \frac{\sigma^2}{2}t)}{\sigma\sqrt{t}} \quad (44)$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

### *C. Real Option Pricing Based on term structure of interest rate and discount rate*

The competitive and uncertain environment in which companies operate created a major need for them to be able to adapt to changes rapidly. For this reason, these companies have to invest in projects which generates options instead of leaving them disappear. A corporate investment opportunity is like a call option because the corporation has the right but not the obligation to acquire an asset (Cherif Selima). A real option is the right to undertake some business decision and is analogous to financial option contract; it is limited-commitment investment in an asset with an uncertain payoff that conveys the right, but not the obligation, to make further investments should the payoff look attractive (Andrew, Consultant, 2002).

Real option differs from financial one in several important respects: Its underlying assets are no longer stock, bonds, options or money, but typically some certain investment project. Real option is a tool of decision-making on the non-financial assets investment with uncertain results. They typically less liquid and the real value of an investment to one firm may differ a lot from its value to another firm. This creates a substantial challenge to evaluating a real options implication. But in theory the pricing model of real options is similar to pricing model of financial options.

The general pricing model of real option can be derived through comparing different input variables with stock option as is shown in TABLE I.

TABLE I. THE COMPARISON OF REAL OPTION AND FINANCIAL OPTION

| Stock Option | Real Option |
|---|---|
| Current stock price S | Current value of project income P |
| Exercise price X | Investment cost V |
| Time to maturity of option T | Duration of investment opportunity T |
| Standard deviation $\sigma$ | Fluctuating rate of project's value $\theta$ |
| Risk free interest rate $r$ | Risk free interest rate $r$ |

The general pricing model of real option can be derived through: replacing current stock price $S$ with current value of project income $P$ ; exercise price $X$ with investment cost $V$ ; time to maturity of option in years $T$ with duration of investment opportunity $T$ ; and standard deviation $\sigma$ with fluctuating rate of project's value $\theta$.

When pricing the investment project by real option theory. Project income and investment cost do not happen as soon as the project begins, but happens in the next several years. So the cash flow should be discounted at a certain discount rate. Real option pricing model on the condition of changing discount rate is the one to derive the term structure of discount rate, and then to calculate the different discount rate in the corresponding terms and discount the cash flow.

## V. The Empirical Analysis

Company A plans to invest a research and development project on Sep 20th, 2006. The project is treated as a simplified typical model. The term of the cash flow is divided into two stages. The first stage, the first and second years, is about research and development of application technology. The second stage, from the third year to the sixth year, is about commercialization. So the products have a four-year life cycle. There is no residual value. The volatility is $\theta$=35%, discount rate $\mu$ and risk free interest rate r are unknown. Cash flow of the project is in TABLE II (unit: ten thousand dollars. The investment occurs in the beginning of each year, and return is obtained at the end of the year).

TABLE II. Cash flow of the two stages of the project

| Year | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Investment Cost | 50 | 40 | 600 | 450 | 400 | |
| Return | 0 | 0 | 150 | 500 | 650 | 300 |

Traditional method and improved method mentioned above are used separately to assess the value of real option as follows.

### *A. Based the constant risk-free rate and discount rate*

Let the overnight repurchase rates of Inter-bank Market on Sep 20th, 2006, r = 0.0195, be the risk free rate. Let 1.4 times of one-year Government Bond Yield in Shanghai Stock Exchange on Sep 20th, 2006, $\mu$ =0.0326, be the discount rate.

So the investment amount in the second stage is (ten thousand dollars):

600+450/1.0326+400/1.0326^2=1410.9

The return in the second stage is (ten thousand dollars):

150/1.0326+500/1.0326^2+650/1.0326^3+300/1.0326^4 =1468.4

Substitute exercise price X= 1410.9(ten thousand dollars), current stock price S=1468.4/1.0326^2=1377.1 (ten thousand dollars), r=0.0195， $\theta$ =35% into the Black-Scholes formula (24) to value the real option. We can get

$$d_1 = \frac{\ln(S/X) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$= \frac{\ln(1377.1/1410.9) + (0.0195 + 0.35^2/2)\cdot 2}{0.35\cdot\sqrt{2}} = 0.2773$$

$$d_2 = 0.2773 - 0.35 * \sqrt{2} = \text{-}0.2177$$

$$C = S\cdot N(d_1) - X\cdot e^{-rT}\cdot N(d_2)$$

$$= 1377.1\cdot N(0.2773) - 1410.9\cdot e^{-0.0195\cdot 2}\cdot N(\text{-}0.2177)$$

$$=280.2$$

Thus, the investment value of the project is (ten thousand dollars):

280.2-50-40/1.0326=191.5

### *B. Based on term structure of interest rate and discount rate*

Firstly, derive the term structure of discount rate on that day.

Select 22 government bonds in Shanghai Stock Exchange on Sep 20th, 2006. Exponential splines method mentioned in the first sector is used to fit the term structure of interest rate on Sep 20th, 2006.

Discount factor $E(t)$ is gotten as follows:

$$E(t) = \begin{cases} 253.9402\text{-}783.2551e^{-0.0330t} + 807.4766e^{-0.0660t} \\ \text{-}277.1616e^{-0.0990t}, t\in[0,1] \\ 253.9402\text{-}783.2551e^{-0.0330t} + 807.4766e^{-0.0660t} \\ \text{-}277.1616e^{-0.0990t} + 301.2214(e^{-0.0330t} - e^{-0.0330})^3 \\ , t\in[1,4] \\ 253.9402\text{-}783.2551e^{-0.0330t} + 807.4766e^{-0.0660t} \\ \text{-}277.1616e^{-0.0990t} + 301.2214(e^{-0.0330t} - e^{-0.0330})^3 \\ \text{-}37.9259(e^{-0.0330t} - e^{-0.1320})^3, t\in[4,8] \\ 253.9402\text{-}783.2551e^{-0.0330t} + 807.4766e^{-0.0660t} \\ \text{-}277.1616e^{-0.0990t} + 301.2214(e^{-0.0330t} - e^{-0.0330})^3 \\ \text{-}37.9259(e^{-0.0330t} - e^{-0.1320})^3 \\ +16.3240(e^{-0.0330t} - e^{-0.2640})^3, t\in[8,21] \end{cases} \quad (45)$$

Convert $E(t)$ to annual risk free rate by the following formula:

$$r(t) = [\frac{1}{E(t)}]^{\frac{1}{t}} - 1 \quad (46)$$

Estimate return on risk investment and inflation rate according to macroeconomics environment, the developing prospect of industry, market and competition of similar enterprises. Suppose that both returns on risk investment and inflation rate 0.4 times risk free rate. The term structure of discount rate is

$$\mu(t) = 1.4r(t) = 1.4\{[\frac{1}{E(t)}]^{\frac{1}{t}} - 1\} \quad (47)$$

Long-term discount rate ${}_{t_i}\mu_{t_j}$ from $t_i$ to $t_j$ can be calculated on the basis of expectation theory.

$$ {}_{t_i}\mu_{t_j} = \left\{ \frac{[1+\mu(t_j)]^{t_j}}{[1+\mu(t_i)]^{t_i}} \right\}^{1/(t_j - t_i)} - 1 \qquad (48)$$

Secondly, calculate the investment amount and return in the second stage.

According to the formula (49), (50), we can get:

$$\mu(1) = 0.0326, \mu(2) = 0.0354,$$
$$\mu(3) = 0.0355, \mu(4) = 0.0365,$$
$$\mu(5) = 0.0387, \mu(6) = 0.0413,$$
$${}_2\mu_3 = 0.0357, {}_2\mu_4 = 0.0376,$$
$${}_2\mu_5 = 0.0409, {}_2\mu_6 = 0.0443.$$

Thus, the investment amount in the second stage (ten thousand dollar) is:

600+450/1.0357+400/1.0376^2= 1406

The return in the second stage (ten thousand dollar) is:
150/1.0357+500/1.0376^2+650/1.0409^3+300/1.0443^4
=1437.8

Thirdly, parameter estimation in the dynamic model of interest rate

On the basis of second sector, Assume the dynamic process of China's risk free rate is:

$$dr_t = \alpha \frac{1}{r_t}(\mu - r_t)dt + \sigma r^{1.5} dW_t \qquad (49)$$

Using repurchase interest rates of inter-bank market, estimate parameters in the dynamic model of interest rate. Selecting 527 overnight repurchase rates of inter-bank market from China's money market, from Aug 2nd, 2004 to Sep 20th, 2006, parameter values are estimated:

$\mu$ =0.015768， $\alpha$ =0.229082， $\sigma$ = 0.522946.

Fourthly, calculate the mathematical expectation of interest rates which comply with equation (49) at every time point. In order to exactness, the unit time is for one day (the shorter of the unit time, the more accurate of the result), that is $\Delta t = 1/365$ . The duration of the real option is t = 2, thus the number of the time points is $t/\Delta t$ =2/ (1/365) =730. By use of the method offered in section II, We can get the mathematical expectation of daily interest rates as shown in TABLE III.

TABLE III. THE SIMULATED MATHEMATICAL EXPECTATION OF DAILY INTEREST RATES IN TWO YEARS (730 DAYS)

| Time i | 1 | 2 | 3 | … | 728 | 729 | 730 |
|---|---|---|---|---|---|---|---|
| $E(r_i)$ | 0.01936 | 0.01924 | 0.0191 | … | 0.01575 | 0.01575 | 0.01576 |

Thus, we can get

$$\sum_{i=1}^{k} E(r_i)\Delta t = \sum_{i=1}^{730} E(r_i)/365 = 0.031819$$

Fifthly, substitute parameter values into formula (44) to value the real option.

The enterprise has a call option. Its Exercise price X= 1406 ten thousand dollar, Current stock price S =1437.8/1.0354^2=1341.2 ten thousand dollar, substitute them into the formula (44). We can get

$$d_1 = \frac{\ln(S/X) + (\sum_{i=1}^{k} E(r_i)\Delta t + \frac{\sigma^2}{2} t)}{\sigma\sqrt{t}}$$

$$= \frac{\ln(1341.2\ /1406) + (0.031819 + \frac{0.35^2}{2} * 2)}{0.35 * \sqrt{2}} = 0.2166,$$

$$d_2 = d_1 - \sigma\sqrt{T} = -0.35\sqrt{2} = -0.2784,$$

$$c(S,t) = SN(d_1) - Xe^{-\sum_{i=1}^{k} E(r_i)\Delta t} N(d_2)$$

$$= 1341.2 * N(0.2166) - 1406 * e^{-0.31819} * N(-0.2784)$$

=256.7 (ten thousand dollar).

Thus, the value of investment project is (ten thousand dollar):

256.7-50-40/1.0326=168.0

The estimated outcome indicates that if the changes of risk-free rate and discount rate are not considered, the value of investment project is overvalued (ten thousand dollar):

191.5-168.0= 23.5

The improved method, taking consider of those changes, make estimation on the real option of the project better.

Decision-making for investment based on option and term structure is unnecessary inaccurate. The development of real option valuation methods has in theory the potential to improve the valuation process substantially. This would benefit also the national economy, as a better valuation on one hand steers investments to business ideas that valued without flexibility might not get funded, and, on the other hand, steers away investments from businesses that are currently overvalued(Niklas Kari,2002)[11].

## VI. CONCLUSIONS

On the basis of former researches, risk-free interest rate and discount rate are distinguished and dynamic and static term structures of interest rates are studied according to the need for using. As for the static model, exponential splines model is studied and every cash flow of the project is discounted relatively accurately by getting the model of discount rate. As for the dynamic model, a basic model is studied, and the option pricing formula under changing risk-free rate is gotten by bringing it into the option pricing formula. And then a complex model which is more approaching the fact is used to describe the moving pattern of risk-free interest rates and the former formula that we have educed is modified. Finally, Empirical research is carried on based on the data of Chinese financial market. Both dynamic and static term structure model of interest rates are estimated by use of the data of buy-back rate and Shanghai Stock Exchange, and an example is given to compare the difference between the traditional method and the method based on the term structure of the interest rate and discount rate.

The paper has only studied the European option based on term structure of interest rate and discount rate. In fact, there are many new options and opponent's behavior should also be considered when investors making policy decisions. Therefore the next step should combine new options with game theory to study the real option pricing.

While the potential for decision-making for investment based on option and term structure are big so are also the challenges. One of the challenges is that project capitalists and managers in general have not widely been trained in identifying and analyzing real options[11], thus how to enhance the practicality of real option is also a problem.

In the next, there are two directions to study in depth. One of the two directions is to combine new options with game theory and term structure to study the real option pricing. The second one is to develop the standardized procedure which is easy for manage staff's decision-making and enhance the practical application capacity of real option theory.

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