Impact of Variances of Random Weights and Biases on Extreme Learning Machine

Xiao Tao¹, Xu Zhou², Yu Lin He^{3*}, Rana Aamir Raza Ashfaq⁴

¹ College of Science, Agricultural University of Hebei, Baoding 071001, China.

² Department of Basic Courses, Agricultural University of Hebei, Huanghua 061100, China.

³ College of Computer Science & Software Engineering, Shenzhen University, Shenzhen 518060, China.

⁴ Department of Computer Science, Bahauddin Zakariya University, Multan, Pakistan.

* Corresponding author. Tel.: +86-18531315747; email: csylhe@126.com, yulinhe@szu.edu.cn

Manuscript submitted December 9, 2015; accepted March 12, 2016.

doi: 10.17706/jsw.11.5.440-454

Abstract: Although the uniform convergence of extreme learning machine (ELM) has been proved for any continuous probability distribution, the variances of random numbers initializing input-layer weights and hidden-layer biases indeed have the obvious impact on generalization performance of ELM. In this paper, we validate this effect by testing the classification accuracies of ELMs initialized by the random numbers with different variances. We select three commonly-used probability distributions (i.e., *Uniform*, *Gamma* and *Normal*) and 30 UCI data sets to conduct our comparative study. The experimental results present some important and valuable observations and instructions: (1) *Uniform* and *Gamma* distributions with the smaller variances usually make ELMs get the higher training and testing accuracies; (2) In comparison with *Normal* distribution, the variances of *Uniform* and *Gamma* distributions have the significant impact on classification performance of ELMs; (3) *Uniform* and *Gamma* distributions with the larger variances could seriously degrade the classification capability of ELMs; (4) ELMs initialized by *Uniform* and *Gamma* distributions with the larger variances generally need the more hidden-layer nodes to achieve the equivalent classification accuracies with ones having the smaller variances; and (5) *Normal* distribution are more easily lead to the over-fitting of ELMs.

Key words: Extreme learning machine, random initialization, probability distribution, variance.

1. Introduction

Extreme learning machine (ELM) [1] is a special Single hidden-Layer Feed-forward neural Network (SLFN) in which the input-layer weights and hidden-layer biases are randomly selected and the output-layer weights are analytically determined by solving *Moore-Penrose* generalized inverse [2] of hidden-layer output matrix. Because of avoiding the time-consuming iterations, ELMs obtain the extremely fast training speed. Meanwhile, the theoretical proof [3] guarantees the universal approximate capability of ELM. The lower computational complexity and acceptable generalization performance make ELM obtain a wide range of practical applications [4].

However, ELM has an obvious defect, i.e., the instability of predictive results. Although Huang *et al.* in [1] proved the uniform convergence of ELM for any continuous probability distribution, this defect indeed exists in the practical applications due to the inherent character of data set, e.g., noise and outlier, etc. The researchers have developed some representative works to improve the instability of ELM caused by random

initializations of input-layer weights and hidden-layer biases. Zhu *et al.* [5] proposed an Evolutionary ELM (E-ELM) which uses the differential evolutionary algorithm to select the input weights and hidden biases for ELM. Then, Cao *et al.* [6] improved E-ELM and developed a Self-adaptive E-ELM (SaE-ELM) to optimize the necessary parameters. Experimental results show that SaE-ELM outperforms E-ELM. An Optimized ELM (O-ELM) was designed by Matias *et al.* [7], which used three different optimization algorithms to optimize the input-layer weights, hidden-layer biases, and regularization factor simultaneously. Heeswijk *et al.* [8] proposed two weight initialization schemes, i.e., binary ELM based on $\{0, 1\}$ -weights and ternary ELM based on $\{-1, 0, 1\}$ -weights, to improve the diversity of neurons in the hidden layer. For binary/ternary ELMs, the necessary optimizations are also required to select the better parameters. From the above-mentioned description, we can find that E-ELM, SaE-ELM, O-ELM, and binary/ternary ELMs used the different optimization algorithms to tune the initially random input-layer weights and hidden-layer biases. These works confirmed the impact of random initialization on generalization capability of ELM.

The *Uniform* random numbers in interval $[0, 1]$ are commonly used to initialize the input-layer weights and hidden-layer biases, including ELM [1], E-ELM [5], SaE-ELM [6], O-ELM [7], binary/ternary ELMs [8], and other forms of improved ELM [3], [9]-[15]. Then, are *Uniform* random numbers in the larger intervals also suitable to initialize ELM? The main objective of this paper is to validate the impact of range of random numbers on generalization performance of ELM. Specifically, we want to clarify the influence of variances of random weights and biases on the training and testing accuracies of ELM. Three mostly-used continuous probability distributions [16], i.e., *Uniform*, *Gamma* and *Normal*, are employed in our study to initialize the input-layer weights and hidden-layer biases for ELMs. For each distribution, 6 different parameter pairs are initialized as shown in Fig. 1, which can be used to control the variance of random numbers. We select 30 UCI classification data sets which involve a wide range of real domains and data characteristics. In our experiment, the 10-fold cross-validation procedure is repeated 100 times. In every time of 10-fold cross-validation, the different ELMs are trained on the same training sets and evaluated on the same testing set. The training and testing accuracies on given data set are the average values of 100 times of 10-fold cross-validations. We further use Wilcoxon signed-ranks test and Friedman test [17] to compare the classification performances of different ELMs on 30 data sets. The experimental results and statistical analysis show that the variance of random weights and biases indeed have the obvious impact on the generalization performance of ELM. The obtained conclusions from experimental results can instruct users choose the appropriate weights and biases for ELM so as to reduce the possibility of instability of ELM caused by random initializations.

The rest of the paper is organized as follows: In Section 2, we summarize the basic extreme learning machine. In Section 3, we experimentally validate the influence of initializations obeying different probability distributions on generalization performance of ELM. We conclude this work with some remarks in the last section.

2. Extreme Learning Machine

Here, we consider using *extreme learning machine* (ELM) [1] to deal with the classification problem based on the training dataset $(X_{N \times D}, Y_{N \times C})$ and testing dataset $(X'_{M \times D}, Y'_{M \times C})$, where

$$X_{N \times D} = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} x_{11} & \cdots & x_{1D} \\ \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{ND} \end{bmatrix}, Y_{N \times C} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} y_{11} & \cdots & y_{1C} \\ \vdots & \ddots & \vdots \\ y_{N1} & \cdots & y_{NC} \end{bmatrix},$$

$$X'_{M \times D} = \begin{bmatrix} x'_1 \\ \vdots \\ x'_M \end{bmatrix} = \begin{bmatrix} x'_{11} & \cdots & x'_{1D} \\ \vdots & \ddots & \vdots \\ x'_{M1} & \cdots & x'_{MD} \end{bmatrix}, Y'_{M \times C} = \begin{bmatrix} y'_1 \\ \vdots \\ y'_M \end{bmatrix} = \begin{bmatrix} y'_{11} & \cdots & y'_{1C} \\ \vdots & \ddots & \vdots \\ y'_{M1} & \cdots & y'_{MC} \end{bmatrix},$$

Fig. 1. Three continuous probability distributions for ELM initialization: for *Uniform* distribution, its

probability density function (p.d.f) is $p(x) = \frac{1}{b-a}, x \in [a, b]$ and variance is $\frac{(b-a)^2}{12}$; for *Gamma*, its p.d.f. is

$p(x) = x^{k-1} \frac{e^{-\frac{x}{\theta}}}{\theta^k \Gamma(k)}, k > 0, \theta > 0, x \in [0, +\infty)$ and variance is $k\theta^2$, where $\Gamma(k) = \int_0^{+\infty} v^{k-1} e^{-v} dv$ is *Gamma* function;

for *Normal*, its p.d.f. is $p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \mu \in R, \sigma^2 > 0, x \in (-\infty, +\infty)$ and variance is σ^2 .

N is the number of training instances, D is the number of input variables, C is the number of output variables, and M is the number of testing instances. For the i -th training instance x_i ($i \in \{1, \dots, N\}$) which belongs to the c -th ($c \in \{1, \dots, C\}$) class, $y_{ic}=1$ and $y_{ic'}=0$ ($c' \in \{1, \dots, C\}$ and $c' \neq c$). $Y'_{M \times C}$ is unknown and needs to be predicted.

ELM is a single hidden-layer feed-forward neural network (SLFN) and does not require any iterative optimization to input-layer and output-layer weights. ELM determines $Y'_{M \times C}$ as follows:

$$Y'_{M \times C} = H'_{M \times L} \beta_{L \times C} = H'_{M \times L} H_{L \times N}^\dagger Y_{N \times C} = \begin{cases} H' (H^T H)^{-1} H^T Y & , \text{ if } N \geq L \\ H' H^T (H H^T)^{-1} Y & , \text{ if } N < L \end{cases}, \quad (1)$$

where $\beta_{L \times C}$ is the output-layer weight matrix, L is the number of hidden-layer nodes,

$$H_{N \times L} = \begin{bmatrix} h(x_1) \\ M \\ h(x_N) \end{bmatrix} = \begin{bmatrix} g(w_1 x_1 + b_1) & L & g(w_L x_1 + b_L) \\ M & O & M \\ g(w_1 x_N + b_1) & L & g(w_L x_N + b_L) \end{bmatrix} \quad (2)$$

is the hidden-layer output matrix for training data set, $H_{L \times N}^\dagger$ is *Moore-Penrose* generalized inverse of $H_{N \times L}$,

$$H'_{M \times L} = \begin{bmatrix} h(x'_1) \\ M \\ h(x'_M) \end{bmatrix} = \begin{bmatrix} g(w_1 x'_1 + b_1) & L & g(w_L x'_1 + b_L) \\ M & O & M \\ g(w_1 x'_M + b_1) & L & g(w_L x'_M + b_L) \end{bmatrix} \quad (3)$$

is the hidden-layer output matrix for testing data set, $g(v) = \frac{1}{1+e^{-v}}$, $v \in (-\infty, +\infty)$ is the sigmoid activation function, are the input-layer weight matrix and hidden-layer bias vector which are usually determined by assigning the random numbers obeying *Uniform* distribution. Our main work in this paper is to use the probability distributions in Fig. 1 to initialize $W_{D \times L}$ and b and compare the classification performances of corresponding ELMs.

$$W_{D \times L} = \begin{bmatrix} w_1 & \cdots & w_L \end{bmatrix} = \begin{bmatrix} w_{11} & \cdots & w_{L1} \\ \vdots & \ddots & \vdots \\ w_{1D} & \cdots & w_{LD} \end{bmatrix} \text{ and } b = \begin{bmatrix} b_1 \\ \vdots \\ b_L \end{bmatrix}$$

Table 1. The Details of 30 UCI Data Sets

	Data sets	Attributes	Classes	Class distribution	Instances
1	Auto Mpg	5	3	245/79/68	392
2	Blood Transfusion	4	2	570/178	748
3	Breast Cancer	10	2	458/241	699
4	Breast CancerW-D	30	2	357/212	569
5	Breast CancerW-P	33	2	151/47	198
6	Cleveland	13	5	160/54/35/35/13	297
7	Credit Approval	15	2	383/307	690
8	Cylinder Bands	20	2	312/228	540
9	Ecoli	5	8	143/77/52/35/20/5/2/2	336
10	Glass Identification	9	7	76/70/29/17/13/9/0	214
11	Haberman's Survival	3	2	225/81	306
12	Heart Disease	13	2	150/120	270
13	Image Segment	19	7	330×7	2310
14	Ionosphere	33	2	225/126	351
15	Iris	4	3	50×3	150
16	Libras Movement	90	15	24×15	360
17	Magic Telescope	10	2	12332/6688	19020 (10%)
18	Musk Version1	166	2	269/207	476
19	New Thyroid Gland	5	3	150/35/30	215
20	Page Blocks	10	5	4913/329/115/88/28	5473 (10%)
21	Parkinsons	22	2	147/48	195
22	Pima Indian Diabetes	8	2	500/268	768
23	Sonar	60	2	111/97	208
24	SPECTF Heart	44	2	212/55	267
25	Vehicle Silhouettes	18	4	218/217/212/199	846
26	Vowel Recognition	10	11	48×11	528
27	Wine	13	3	91/59/48	178
28	Wine Quality-Red	11	6	681/638/199/53/18/10	1599
29	WineQuality-White	11	6	2198/1457/880/175/163/20	4898 (10%)
30	Yeast	8	10	463/429/244/163/51/44/35/30/20/5	1484

3. Impact of Random Initialization on ELM's Generalization Performance

3.1. Data Preparation Guideline

In this experimental study, 30 UCI data sets are used to test the classification performances of ELMs corresponding to different random initializations. The detailed description of 30 data sets is summarized in Table 1. In order to use these data sets more efficiently and specifically, we preprocess them according to the following guidelines:

- 1) Delete nominal-value attributes. ELMs are mainly used to handle the classification with continuous-value attributes.
- 2) Replace missing attribute-values. The unsupervised filter named *ReplaceMissingValues* in Weka [18] is

used to replace all missing attribute-values in each data set.

- 3) Reduce large data sets. The unsupervised filter named Resample with the sample-SizePercent 10 in Weka is used to randomly reduce the sizes of three large data sets: *Magic Telescope*, *Page Blocks* and *Wine Quality-White*.

3.2. Experimental Setup

Table 2. Training Accuracies of ELMs Corresponding to *Uniform* Distribution

<i>Uniform</i>	$a=0, b=5$	$a=0, b=10$	$a=0, b=15$	$a=0, b=20$	$a=0, b=25$	$a=0, b=30$
1	0.864±0.002	0.803±0.007	0.727±0.007	0.681±0.008	0.654±0.008	0.639±0.004
2	0.811±0.001	0.808±0.002	0.794±0.002	0.783±0.002	0.777±0.002	0.771±0.002
3	0.974±0.000	0.939±0.002	0.890±0.005	0.852±0.006	0.803±0.024	0.721±0.027
4	0.698±0.039	0.627±0.000	0.627±0.000	0.627±0.000	0.627±0.000	0.627±0.000
5	0.763±0.000	0.763±0.000	0.763±0.000	0.763±0.000	0.763±0.000	0.763±0.000
6	0.670±0.005	0.589±0.004	0.556±0.002	0.546±0.002	0.541±0.001	0.540±0.001
7	0.814±0.002	0.807±0.003	0.774±0.004	0.742±0.007	0.719±0.008	0.697±0.006
8	0.579±0.001	0.578±0.000	0.578±0.000	0.578±0.000	0.578±0.000	0.578±0.000
9	0.880±0.003	0.682±0.013	0.541±0.024	0.440±0.014	0.429±0.007	0.426±0.003
10	0.800±0.005	0.616±0.015	0.402±0.014	0.363±0.005	0.358±0.003	0.357±0.002
11	0.798±0.003	0.782±0.003	0.770±0.003	0.759±0.003	0.751±0.003	0.746±0.002
12	0.770±0.006	0.607±0.022	0.555±0.001	0.555±0.000	0.556±0.000	0.556±0.000
13	0.797±0.011	0.298±0.014	0.169±0.010	0.146±0.004	0.144±0.003	0.143±0.003
14	0.641±0.000	0.641±0.000	0.641±0.000	0.641±0.000	0.641±0.000	0.641±0.000
15	0.985±0.002	0.916±0.009	0.804±0.011	0.720±0.016	0.647±0.019	0.579±0.021
16	0.068±0.000	0.068±0.000	0.068±0.000	0.068±0.000	0.068±0.000	0.068±0.000
17	0.825±0.003	0.703±0.017	0.643±0.005	0.641±0.000	0.641±0.000	0.641±0.000
18	0.565±0.000	0.565±0.000	0.565±0.000	0.565±0.000	0.565±0.000	0.565±0.000
19	0.954±0.001	0.946±0.001	0.933±0.002	0.923±0.002	0.918±0.002	0.916±0.001
20	0.967±0.001	0.923±0.004	0.904±0.002	0.898±0.001	0.895±0.001	0.892±0.001
21	0.873±0.007	0.756±0.002	0.754±0.000	0.754±0.000	0.754±0.000	0.754±0.000
22	0.772±0.003	0.677±0.007	0.656±0.001	0.652±0.001	0.651±0.000	0.651±0.000
23	0.534±0.000	0.534±0.000	0.534±0.000	0.534±0.000	0.534±0.000	0.534±0.000
24	0.794±0.000	0.794±0.000	0.794±0.000	0.794±0.000	0.794±0.000	0.794±0.000
25	0.676±0.007	0.450±0.008	0.360±0.010	0.298±0.009	0.275±0.005	0.267±0.003
26	0.582±0.014	0.107±0.008	0.093±0.001	0.093±0.001	0.093±0.001	0.093±0.000
27	0.825±0.009	0.424±0.018	0.399±0.000	0.399±0.000	0.399±0.000	0.399±0.000
28	0.614±0.003	0.518±0.008	0.438±0.006	0.426±0.001	0.426±0.000	0.426±0.000
29	0.629±0.005	0.488±0.006	0.450±0.002	0.446±0.001	0.446±0.000	0.446±0.000
30	0.602±0.002	0.485±0.012	0.352±0.010	0.323±0.004	0.315±0.002	0.313±0.001

Fig. 2. Training accuracies of 6 ELMs corresponding to 6 *Uniform* distributions on 9 representative data sets.

The parameter pairs (a, b) , (k, θ) and (μ, σ) in *Uniform*, *Gamma*, and *Normal* distributions are respectively initialized as 6 different two-tuples: $(0, 5)$, $(0, 10)$, $(0, 15)$, $(0, 20)$, $(0, 25)$ and $(0, 30)$ for *Uniform*; $(9, 0.1)$, $(9,$

0.3), (9, 0.5), (9, 0.7), (9, 0.9) and (9, 1.1) for *Gamma*; and (0, 0.05), (0, 0.1), (0, 0.15), (0, 0.2), (0, 0.25) and (0, 0.3) for *Normal*. For each distribution, we use 10-fold cross-validation to get the training and testing accuracies corresponding to 6 ELMs (i.e., initializing the input-layer weights and hidden-layer biases with random numbers corresponding to 6 different variances respectively) on every data set. In order to make the experimental results more reliable, 10-fold cross-validation on each data set is repeated 100 times for each ELM and the average values of 100 training and testing accuracies are used as finally experimental results. In every time of 10-fold cross-validation, the different ELMs are trained on the same training sets and evaluated on same testing set and the random initializations to input-layer weights and hidden-layer biases are independent.

Table 3. Testing Accuracies of ELMs Corresponding to *Uniform* Distribution

<i>Uniform</i>	$a=0, b=5$	$a=0, b=10$	$a=0, b=15$	$a=0, b=20$	$a=0, b=25$	$a=0, b=30$
1	0.804±0.012	0.759±0.012	0.704±0.012	0.671±0.012	0.646±0.011	0.634±0.006
2	0.797±0.005	0.796±0.004	0.785±0.005	0.774±0.006	0.772±0.005	0.764±0.005
3	0.965±0.003	0.929±0.006	0.882±0.008	0.845±0.010	0.797±0.023	0.715±0.030
4	0.684±0.039	0.627±0.000	0.627±0.000	0.627±0.000	0.627±0.000	0.627±0.000
5	0.761±0.003	0.763±0.000	0.763±0.000	0.763±0.000	0.763±0.000	0.763±0.000
6	0.529±0.014	0.526±0.010	0.528±0.007	0.532±0.006	0.536±0.003	0.538±0.003
7	0.775±0.008	0.774±0.009	0.748±0.008	0.724±0.011	0.704±0.013	0.684±0.015
8	0.579±0.002	0.578±0.000	0.578±0.000	0.578±0.000	0.578±0.000	0.578±0.000
9	0.830±0.013	0.655±0.014	0.530±0.022	0.437±0.013	0.428±0.006	0.426±0.002
10	0.659±0.019	0.569±0.029	0.381±0.026	0.359±0.011	0.357±0.004	0.355±0.003
11	0.715±0.011	0.730±0.012	0.739±0.011	0.734±0.011	0.733±0.008	0.731±0.008
12	0.698±0.022	0.589±0.025	0.553±0.003	0.555±0.001	0.556±0.000	0.556±0.000
13	0.791±0.013	0.298±0.015	0.169±0.010	0.146±0.004	0.144±0.003	0.142±0.003
14	0.641±0.000	0.641±0.000	0.641±0.000	0.641±0.000	0.641±0.000	0.641±0.000
15	0.943±0.009	0.867±0.022	0.773±0.020	0.698±0.021	0.622±0.028	0.563±0.028
16	0.056±0.000	0.056±0.000	0.056±0.000	0.056±0.000	0.056±0.000	0.056±0.000
17	0.819±0.004	0.701±0.017	0.643±0.004	0.641±0.000	0.641±0.000	0.641±0.000
18	0.565±0.000	0.565±0.000	0.565±0.000	0.565±0.000	0.565±0.000	0.565±0.000
19	0.940±0.003	0.937±0.003	0.927±0.003	0.920±0.004	0.916±0.003	0.914±0.002
20	0.939±0.005	0.900±0.006	0.890±0.005	0.889±0.003	0.889±0.002	0.888±0.002
21	0.822±0.017	0.754±0.005	0.754±0.000	0.754±0.000	0.754±0.000	0.754±0.000
22	0.744±0.010	0.661±0.010	0.648±0.004	0.648±0.003	0.649±0.002	0.650±0.001
23	0.534±0.000	0.534±0.000	0.534±0.000	0.534±0.000	0.534±0.000	0.534±0.000
24	0.794±0.000	0.794±0.000	0.794±0.000	0.794±0.000	0.794±0.000	0.794±0.000
25	0.616±0.014	0.423±0.011	0.345±0.012	0.289±0.012	0.272±0.007	0.265±0.004
26	0.521±0.022	0.092±0.011	0.076±0.002	0.076±0.002	0.076±0.001	0.076±0.000
27	0.758±0.022	0.420±0.022	0.399±0.000	0.399±0.000	0.399±0.000	0.399±0.000
28	0.589±0.006	0.512±0.013	0.436±0.009	0.426±0.002	0.426±0.000	0.426±0.000
29	0.526±0.015	0.457±0.011	0.444±0.005	0.446±0.002	0.446±0.001	0.446±0.000
30	0.579±0.004	0.478±0.014	0.349±0.009	0.322±0.006	0.314±0.002	0.313±0.002

3.3. Experimental Result and Analysis

The training and testing accuracies of 18 ELMs (The numbers of hidden-layer nodes are 50) corresponding to 3 different kinds of probability distributions on 30 UCI data sets are summarized in Tables 2-3, 4-5, and 6-7, respectively. For each distribution, we use Wilcoxon signed ranks test and Friedman test [17] to conduct the statistical analysis to the above-mentioned experimental results. The former is to assess whether the classification accuracies of two ELMs differ on a single data set, while the latter is to test whether or not the classification accuracies of 6 ELMs are equal on all data sets. For a given distribution, we only give the detailed procedures of these two statistical tests based on training accuracies of 6 ELMs. The testing accuracies can be handled in the same way. The compared objects of Wilcoxon signed-ranks test are the respective 100 training accuracies of two ELMs (ELM_1 and ELM_2) on a single data set.

Fig. 3. Testing accuracies of 6 ELMs corresponding to 6 *Uniform* distributions on 9 representative data sets.

Table 4. Training Accuracies of ELMs Corresponding to *Gamma* Distribution

<i>Gamma</i>	$k=9, \theta=0.1$	$k=9, \theta=0.3$	$k=9, \theta=0.5$	$k=9, \theta=0.7$	$k=9, \theta=0.9$	$k=9, \theta=1.1$
1	0.863±0.002	0.851±0.003	0.789±0.006	0.705±0.006	0.647±0.004	0.628±0.002
2	0.806±0.001	0.809±0.001	0.804±0.002	0.791±0.002	0.779±0.002	0.773±0.002
3	0.979±0.001	0.972±0.000	0.941±0.002	0.900±0.002	0.855±0.004	0.830±0.020
4	0.935±0.002	0.626±0.000	0.627±0.000	0.627±0.000	0.627±0.000	0.627±0.000
5	0.817±0.003	0.763±0.000	0.763±0.000	0.763±0.000	0.763±0.000	0.763±0.000
6	0.698±0.004	0.652±0.004	0.581±0.003	0.546±0.002	0.539±0.001	0.539±0.000
7	0.805±0.002	0.816±0.002	0.809±0.002	0.781±0.003	0.744±0.004	0.711±0.004
8	0.688±0.005	0.578±0.000	0.578±0.000	0.578±0.000	0.578±0.000	0.578±0.000
9	0.899±0.002	0.851±0.004	0.639±0.005	0.483±0.029	0.426±0.000	0.426±0.000
10	0.819±0.005	0.785±0.005	0.521±0.021	0.355±0.002	0.355±0.000	0.355±0.000
11	0.782±0.002	0.785±0.002	0.778±0.003	0.762±0.002	0.750±0.002	0.743±0.001
12	0.857±0.004	0.738±0.004	0.554±0.001	0.556±0.000	0.556±0.000	0.556±0.000
13	0.960±0.002	0.667±0.018	0.232±0.015	0.178±0.000	0.178±0.000	0.178±0.000
14	0.693±0.003	0.641±0.000	0.641±0.000	0.641±0.000	0.641±0.000	0.641±0.000
15	0.985±0.001	0.979±0.002	0.890±0.008	0.785±0.008	0.671±0.011	0.563±0.013
16	0.068±0.002	0.068±0.000	0.068±0.000	0.068±0.000	0.068±0.000	0.068±0.000
17	0.851±0.001	0.807±0.002	0.646±0.005	0.641±0.000	0.641±0.000	0.641±0.000
18	0.565±0.000	0.565±0.000	0.565±0.000	0.565±0.000	0.565±0.000	0.565±0.000
19	0.953±0.001	0.951±0.001	0.946±0.001	0.930±0.001	0.917±0.001	0.914±0.000
20	0.969±0.001	0.970±0.001	0.911±0.002	0.901±0.001	0.897±0.001	0.892±0.001
21	0.981±0.003	0.790±0.005	0.754±0.000	0.754±0.000	0.754±0.000	0.754±0.000
22	0.799±0.001	0.763±0.002	0.666±0.006	0.652±0.001	0.651±0.000	0.651±0.000
23	0.571±0.005	0.534±0.000	0.534±0.000	0.534±0.000	0.534±0.000	0.534±0.000
24	0.794±0.000	0.794±0.000	0.794±0.000	0.794±0.000	0.794±0.000	0.794±0.000
25	0.780±0.003	0.642±0.006	0.437±0.005	0.341±0.010	0.267±0.002	0.259±0.001
26	0.858±0.004	0.357±0.014	0.093±0.000	0.093±0.000	0.092±0.001	0.093±0.000
27	0.988±0.001	0.728±0.007	0.399±0.000	0.399±0.000	0.399±0.000	0.399±0.000
28	0.629±0.002	0.606±0.002	0.504±0.007	0.426±0.001	0.426±0.000	0.426±0.000
29	0.652±0.004	0.613±0.004	0.513±0.002	0.490±0.002	0.486±0.000	0.486±0.000
30	0.622±0.001	0.585±0.002	0.454±0.008	0.325±0.002	0.313±0.001	0.312±0.000

- 1) $a_i^{(1)}$ and $a_i^{(2)}$ are the training accuracies of ELM₁ and ELM₂ in the i -th ($i \in \{1, \dots, 100\}$) run of 10-fold cross-validation, respectively.
- 2) $d_i = a_i^{(1)} - a_i^{(2)}$ is the difference between the training accuracies of ELM₁ and ELM₂ in the i -th run of 10-fold cross-validation.
- 3) Increasingly order d_i ($i = 1, \dots, 100$) starting with the smallest as 1. For d_i with same absolute values, the average of their ranks is assigned as the new rank. Let $rank(d_i)$ denote the rank of d_i .
- 4) Let R_+ and R_- denote the sums of ranks for the runs on which ELM₁ is better and worse than ELM₂

respectively. R_+ and R_- can be calculated as follows:

$$R_+ = \sum_{i=1}^{100} \text{rank}(d_i) + \frac{1}{2} \sum_{i=1}^{100} \text{rank}(d_i)$$

and

- 5) Construct the statistic $z = \frac{\min(R_+, R_-) - \frac{1}{4}m(m+1)}{\sqrt{\frac{1}{24}m(m+1)(2m+1)}}$ based on the null-hypothesis which indicates that

ELM₁ and ELM₂ have the equal training accuracy on a given data set, m is the times of 10-fold cross-validations. In our study, $m=100$. When $m \geq 30$, the distribution of z is approximately normal.

- 6) Set the significance level $\alpha=0.1$. If $z > z_{\alpha/2}$ or $z < z_{-\alpha/2}$, the null-hypothesis can be rejected, i.e., there are some differences between ELM1 and ELM2; else, we accept the null-hypothesis. The value $z_{0.1/2} = z_{0.05} = 1.645$ can be found from any statistical textbook.

$$R_- = \sum_{i=1}^{100} \text{rank}(d_i) + \frac{1}{2} \sum_{i=1}^{100} \text{rank}(d_i)$$

Fig. 4. Training accuracies of 6 ELMs corresponding to 6 Gamma distributions on 9 representative data sets

Fig. 5. Testing accuracies of 6 ELMs corresponding to 6 Gamma distributions on 9 representative data sets.

Table 5. Testing Accuracies of ELMs Corresponding to *Gamma* Distribution

<i>Gamma</i>	$k=9, \theta=0.1$	$k=9, \theta=0.3$	$k=9, \theta=0.5$	$k=9, \theta=0.7$	$k=9, \theta=0.9$	$k=9, \theta=1.1$
1	0.806±0.009	0.798±0.011	0.754±0.011	0.688±0.013	0.640±0.008	0.627±0.002
2	0.795±0.004	0.797±0.005	0.794±0.005	0.783±0.005	0.774±0.005	0.770±0.004
3	0.965±0.003	0.965±0.003	0.929±0.006	0.892±0.005	0.849±0.008	0.825±0.022
4	0.916±0.007	0.623±0.003	0.627±0.000	0.627±0.000	0.627±0.000	0.627±0.000
5	0.707±0.018	0.763±0.000	0.763±0.000	0.763±0.000	0.763±0.000	0.763±0.000
6	0.548±0.010	0.532±0.013	0.530±0.008	0.532±0.005	0.537±0.002	0.539±0.000
7	0.770±0.007	0.779±0.008	0.775±0.007	0.755±0.010	0.724±0.008	0.699±0.010
8	0.616±0.013	0.578±0.000	0.578±0.000	0.578±0.000	0.578±0.000	0.578±0.000
9	0.854±0.008	0.809±0.010	0.620±0.010	0.480±0.031	0.426±0.000	0.426±0.000
10	0.648±0.021	0.679±0.014	0.488±0.031	0.355±0.005	0.355±0.000	0.355±0.000
11	0.735±0.008	0.732±0.010	0.738±0.011	0.738±0.010	0.736±0.006	0.732±0.006
12	0.775±0.014	0.691±0.016	0.549±0.006	0.556±0.000	0.556±0.000	0.556±0.000
13	0.924±0.008	0.650±0.022	0.231±0.015	0.178±0.000	0.178±0.000	0.178±0.000
14	0.686±0.007	0.641±0.000	0.641±0.000	0.641±0.000	0.641±0.000	0.641±0.000
15	0.956±0.009	0.946±0.010	0.856±0.018	0.765±0.015	0.655±0.021	0.548±0.015
16	0.056±0.005	0.056±0.000	0.056±0.000	0.056±0.000	0.056±0.000	0.056±0.000
17	0.842±0.003	0.803±0.005	0.646±0.005	0.641±0.000	0.641±0.000	0.641±0.000
18	0.565±0.000	0.565±0.000	0.565±0.000	0.565±0.000	0.565±0.000	0.565±0.000
19	0.941±0.002	0.941±0.003	0.939±0.003	0.926±0.002	0.915±0.002	0.913±0.001
20	0.939±0.004	0.941±0.005	0.892±0.005	0.889±0.003	0.887±0.003	0.884±0.002
21	0.894±0.019	0.766±0.013	0.754±0.001	0.754±0.000	0.754±0.000	0.754±0.000
22	0.767±0.007	0.742±0.007	0.655±0.007	0.648±0.002	0.650±0.001	0.651±0.000
23	0.573±0.009	0.534±0.000	0.534±0.000	0.534±0.000	0.534±0.000	0.534±0.000
24	0.794±0.000	0.794±0.000	0.794±0.000	0.794±0.000	0.794±0.000	0.794±0.000
25	0.718±0.011	0.586±0.011	0.413±0.010	0.329±0.015	0.263±0.004	0.259±0.002
26	0.776±0.012	0.322±0.019	0.076±0.000	0.076±0.001	0.076±0.001	0.076±0.000
27	0.943±0.014	0.707±0.016	0.399±0.000	0.399±0.000	0.399±0.000	0.399±0.000
28	0.599±0.006	0.583±0.008	0.500±0.011	0.426±0.002	0.426±0.000	0.426±0.000
29	0.549±0.014	0.533±0.011	0.503±0.007	0.489±0.004	0.486±0.000	0.486±0.000
30	0.594±0.004	0.567±0.005	0.450±0.012	0.324±0.003	0.312±0.001	0.312±0.000

Table 6. Training Accuracies of ELMs Corresponding to *Normal* Distribution

<i>Normal</i>	$\mu=0, \sigma=0.05$	$\mu=0, \sigma=0.1$	$\mu=0, \sigma=0.15$	$\mu=0, \sigma=0.2$	$\mu=0, \sigma=0.25$	$\mu=0, \sigma=0.3$
1	0.812±0.003	0.838±0.003	0.850±0.002	0.852±0.002	0.854±0.002	0.855±0.002
2	0.791±0.001	0.795±0.001	0.795±0.001	0.796±0.001	0.798±0.001	0.800±0.001
3	0.972±0.001	0.972±0.001	0.972±0.001	0.972±0.001	0.972±0.001	0.972±0.001
4	0.971±0.001	0.971±0.001	0.971±0.001	0.972±0.001	0.973±0.001	0.974±0.001
5	0.895±0.005	0.893±0.004	0.895±0.004	0.895±0.004	0.894±0.004	0.893±0.004
6	0.636±0.003	0.682±0.004	0.699±0.003	0.706±0.003	0.707±0.003	0.707±0.003
7	0.772±0.002	0.794±0.002	0.801±0.002	0.802±0.002	0.803±0.002	0.803±0.002
8	0.722±0.005	0.721±0.006	0.722±0.005	0.722±0.005	0.722±0.005	0.722±0.004
9	0.890±0.002	0.901±0.002	0.904±0.002	0.906±0.002	0.906±0.002	0.906±0.002
10	0.779±0.004	0.817±0.005	0.817±0.005	0.817±0.005	0.819±0.005	0.816±0.004
11	0.772±0.002	0.782±0.002	0.782±0.002	0.783±0.002	0.785±0.003	0.788±0.002
12	0.888±0.004	0.888±0.003	0.887±0.004	0.888±0.004	0.889±0.003	0.889±0.003
13	0.946±0.003	0.950±0.003	0.950±0.002	0.951±0.002	0.951±0.003	0.951±0.002
14	0.929±0.003	0.930±0.003	0.930±0.003	0.929±0.003	0.931±0.003	0.932±0.004
15	0.984±0.002	0.986±0.001	0.986±0.001	0.986±0.001	0.986±0.001	0.986±0.001
16	0.806±0.004	0.832±0.004	0.851±0.005	0.863±0.004	0.870±0.004	0.873±0.004
17	0.843±0.001	0.846±0.001	0.845±0.001	0.846±0.001	0.846±0.001	0.846±0.001
18	0.844±0.005	0.843±0.005	0.841±0.005	0.838±0.006	0.837±0.006	0.834±0.006
19	0.938±0.001	0.945±0.001	0.949±0.001	0.951±0.001	0.953±0.001	0.954±0.001
20	0.951±0.001	0.962±0.001	0.965±0.001	0.966±0.001	0.967±0.001	0.967±0.001
21	0.934±0.003	0.937±0.003	0.938±0.004	0.937±0.003	0.937±0.004	0.938±0.004
22	0.799±0.002	0.802±0.002	0.802±0.002	0.803±0.002	0.803±0.002	0.803±0.002
23	0.886±0.005	0.887±0.005	0.888±0.005	0.890±0.005	0.893±0.005	0.895±0.005
24	0.855±0.003	0.854±0.004	0.853±0.004	0.852±0.004	0.851±0.004	0.850±0.005
25	0.742±0.004	0.770±0.003	0.769±0.003	0.769±0.005	0.770±0.004	0.771±0.003
26	0.860±0.005	0.863±0.005	0.863±0.005	0.862±0.004	0.862±0.005	0.863±0.005

27	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000
28	0.624±0.002	0.627±0.002	0.627±0.003	0.627±0.002	0.627±0.002	0.627±0.002
29	0.627±0.005	0.650±0.004	0.649±0.004	0.648±0.004	0.648±0.005	0.649±0.004
30	0.617±0.002	0.624±0.002	0.624±0.001	0.623±0.002	0.624±0.002	0.624±0.002

Table 7. Testing Accuracies of ELMs Corresponding to Normal Distribution

Normal	$\mu=0, \sigma=0.05$	$\mu=0, \sigma=0.1$	$\mu=0, \sigma=0.15$	$\mu=0, \sigma=0.2$	$\mu=0, \sigma=0.25$	$\mu=0, \sigma=0.3$
1	0.781±0.009	0.800±0.009	0.798±0.009	0.797±0.009	0.798±0.009	0.799±0.010
2	0.786±0.004	0.788±0.002	0.788±0.003	0.787±0.003	0.788±0.003	0.788±0.004
3	0.962±0.003	0.963±0.003	0.963±0.003	0.963±0.003	0.963±0.003	0.963±0.003
4	0.955±0.004	0.955±0.005	0.955±0.004	0.955±0.005	0.957±0.005	0.958±0.005
5	0.783±0.018	0.779±0.020	0.771±0.016	0.775±0.021	0.774±0.022	0.777±0.018
6	0.557±0.011	0.550±0.012	0.554±0.014	0.552±0.011	0.552±0.013	0.550±0.013
7	0.753±0.006	0.762±0.007	0.764±0.008	0.764±0.007	0.766±0.008	0.763±0.007
8	0.644±0.018	0.646±0.013	0.644±0.011	0.642±0.016	0.647±0.016	0.647±0.017
9	0.858±0.007	0.855±0.008	0.850±0.007	0.847±0.007	0.846±0.009	0.846±0.007
10	0.663±0.019	0.656±0.021	0.650±0.018	0.651±0.018	0.648±0.020	0.651±0.017
11	0.741±0.011	0.748±0.008	0.747±0.007	0.743±0.008	0.739±0.008	0.736±0.008
12	0.806±0.014	0.799±0.015	0.801±0.014	0.804±0.018	0.805±0.012	0.804±0.016
13	0.917±0.008	0.920±0.007	0.920±0.009	0.920±0.007	0.921±0.008	0.920±0.007
14	0.871±0.011	0.870±0.012	0.869±0.010	0.872±0.011	0.875±0.013	0.875±0.012
15	0.969±0.010	0.963±0.007	0.960±0.007	0.957±0.009	0.955±0.009	0.950±0.009
16	0.623±0.018	0.666±0.016	0.706±0.016	0.734±0.014	0.747±0.014	0.760±0.014
17	0.835±0.003	0.836±0.003	0.837±0.005	0.837±0.003	0.837±0.003	0.837±0.003
18	0.780±0.014	0.780±0.015	0.779±0.017	0.776±0.016	0.775±0.015	0.767±0.014
19	0.935±0.002	0.938±0.002	0.938±0.002	0.939±0.003	0.941±0.003	0.941±0.003
20	0.935±0.005	0.938±0.003	0.937±0.005	0.938±0.005	0.937±0.005	0.937±0.004
21	0.867±0.015	0.871±0.016	0.871±0.015	0.868±0.015	0.871±0.016	0.872±0.018
22	0.766±0.007	0.766±0.007	0.765±0.006	0.764±0.008	0.765±0.008	0.766±0.007
23	0.746±0.021	0.745±0.025	0.752±0.021	0.750±0.021	0.753±0.023	0.755±0.023
24	0.783±0.016	0.779±0.012	0.780±0.013	0.780±0.017	0.780±0.015	0.775±0.014
25	0.698±0.010	0.714±0.009	0.710±0.010	0.715±0.009	0.715±0.011	0.714±0.010
26	0.773±0.014	0.776±0.011	0.779±0.011	0.777±0.013	0.775±0.010	0.774±0.015
27	0.969±0.010	0.971±0.012	0.971±0.011	0.970±0.011	0.971±0.009	0.972±0.010
28	0.595±0.005	0.595±0.006	0.597±0.006	0.596±0.006	0.596±0.007	0.596±0.006
29	0.551±0.014	0.549±0.015	0.548±0.015	0.547±0.012	0.548±0.013	0.547±0.012
30	0.598±0.005	0.597±0.004	0.596±0.006	0.596±0.005	0.596±0.004	0.595±0.005

After we compare the training accuracies of two ELMs on a single data set, Friedman test is used to analyze the difference of training accuracies corresponding to 6 ELMs on 30 data sets. The analyzed object is the experimental results in Tables 2, 4, or 6.

- 1) Increasingly order the average training accuracies of 6 ELMs on every data set. Then, the average value of ranks is assigned to the same training accuracy (This depends on the results obtained by previous Wilcoxon signed-ranks test: if the null-hypothesis is accepted, we say the training accuracies of two ELMs are same and thus have the same rank).
- 2) For 6 ELMs, the average value of ranks R_i ($i = 1, \dots, 6$) on 30 data sets is calculated respectively.
- 3) Construct the statistic $F_F = \frac{(m-1)\chi_F^2}{m(k-1)-\chi_F^2}$ based on the null-hypothesis which indicates that all models

have the equal training accuracy on all data sets, where $\chi_F^2 = \frac{12m}{k(k+1)} \left[\sum_{i=1}^6 R_i^2 - \frac{k(k+1)^2}{4} \right]$ is the

Friedman statistic, k is the number of learning models, m is the number of data sets. In this study, $k=6$ and $m=30$.

- 4) The statistic F_F obeys F-distribution with $(k-1)$ and $(k-1) \times (m-1)$ freedom degrees. Under significance

level $\alpha = 0.1$, we can get $F_{\alpha}[(k-1), (k-1) \times (m-1)] = F_{0.1}(5, 145) = 1.887$.

- 5) If $F_F > F_{0.1}(5, 145)$, the null-hypothesis can be rejected. That is to say the training accuracies of these 6 ELMs are different on the 30 data sets.

Based on the statistical analysis results with Wilcoxon signed-ranks test and Friedman test, we select the 9 representative data sets and compare the training and testing accuracies of ELMs corresponding to random initialization with different variances for each distribution (Figs. 2 and 3 are training and testing accuracies for different *Uniform* distributions; Figs. 4 and 5 are training and testing accuracies for different *Gamma* distributions; and Figs. 6 and 7 are training and testing accuracies for different *Normal* distributions). From Tables 2-7 and Figs. 2-7, we can find the following experimental observations and further summarize some usefully empirical conclusions.

Fig. 6. Training accuracies of 6 ELMs corresponding to 6 *Normal* distributions on 9 representative data sets.

Fig. 7. Testing accuracies of 6 ELMs corresponding to 6 *Normal* distributions on 9 representative data sets.

- 1) *Uniform* and *Gamma* distributions with the smaller variances usually make ELMs get the higher training and testing accuracies. From Figs. 2 and 3, we can find the training and testing accuracies all obviously (confirmed by above-mentioned statistical analysis) decrease with the increase of variances of *Uniform* random numbers. The similar observation occurs in Figs. 4 and 5 for *Gamma* random numbers.
- 2) Compared with *Normal* distribution, the variances of *Uniform* and *Gamma* distributions have significant impact on the classification performance of ELMs. From Figs. 6 and 7, we can find with the increase of variances of *Normal* random numbers, the training and testing accuracies don't change significantly. We try to give the analysis to this experimental result. Although $x \in (-\infty, +\infty)$ for *Normal* distribution $p(x)$, the large percentage (i.e., 99.7%) of random numbers locate in interval $[\mu - 3\sigma, \mu + 3\sigma]$. From $[-0.15, 0.15]$ to $[-0.9, 0.9]$, even though the standard deviation of *Normal*

distribution is 0.3, a large number of *Normal* random numbers in interval $[-0.15, 0.15]$ will be selected to initialize the weights and biases.

- 3) *Uniform* and *Gamma* distributions with larger variances could seriously deteriorate the classification capability of ELMs. From Tables 2-5, we can find the training and testing accuracies on *Image Segment*, *Libras Movement*, and *Vowel Recognition* data sets are severely damaged by *Uniform* and *Gamma* random numbers with larger variances. However, this phenomenon doesn't occur for *Normal* random numbers. This is because the random numbers with the larger variances for *Uniform* and *Gamma* distributions on *Image Segment*, *Libras Movement* and *Vowel Recognition* data sets make the solution to *Moore-Penrose* generalized inverse of hidden-layer output matrix H more inaccurate and we can't get $H^\dagger = (H^T H)^{-1} H^T$ or $H^\dagger = H^T (H H^T)^{-1}$ by using the orthogonal projection method [19].
- 4) ELMs initialized by *Uniform* and *Gamma* distributions with larger variances generally needs more hidden-layer nodes to achieve equivalent classification accuracies with ones having the smaller variances. We conduct an additional experiment to compare the learning capabilities of ELMs corresponding to.

Fig. 8. Learning capability of ELMs with *Uniform* initialization in *Iris* data set.

Fig. 9. Learning capability of ELMs with *Gamma* initialization in *Iris* data set.

Fig. 10. Learning capability of ELMs with *Normal* initialization in *Iris* data set.

random numbers with different variances. Let the number of hidden-layer nodes range from 10 to 300 in step of 10. On *Iris* data set we plot the learning curves of different ELMs (See Figs. 8-10). For *Uniform* and *Gamma* random numbers, we can see that with the increase of variance, ELM needs the more hidden-layer nodes to obtain an acceptable learning capability. This is because a number of hidden-layer nodes will makes orthogonal projection method more effective for random numbers with the larger variance.

- 5) By comparing Figs. 8-9 with Fig. 10, we can find that *Normal* distribution is more easily lead to the over-fitting of ELMs. On *Normal* distribution, we can use the orthogonal projection method to get the more accurate solution to *Moore-Penrose* generalized inverse of H , i.e., $H^+ = H^T (HH^T)^{-1}$. Furthermore, the following equation can be derived:

$$H\beta = H(H^+Y) = H\left[H^T(HH^T)^{-1}Y\right] = (HH^T)(HH^T)^{-1}Y = Y.$$

4. Conclusion

In this paper, we experimentally validate the impact of variance of random weights and biases on the classification performance of ELMs and give some important and useful enlightenments regarding ELM initialization. By initializing the input-layer weights and hidden-layer biases with random numbers having different variances, we compare the training and testing accuracies of ELMs on 30 UCI data sets. The experimental results and statistical analysis reflect that (1) *Uniform* and *Gamma* distributions with the smaller variances usually make ELMs get the higher training and testing accuracies and *Uniform* and *Gamma* distributions with the larger variances could seriously deteriorate the classification capability of ELMs; (2) Compared with *Normal* distribution, the variances of *Uniform* and *Gamma* distributions have the significant impact on the classification performance of ELMs; and (3) ELMs initialized by *Uniform* and *Gamma* distributions with the larger variances generally needs the more hidden-layer nodes to achieve the equivalent classification accuracies with ones having the smaller variances. In addition, we also discuss the influence of initialization's variance on over-fitting of ELMs.

Acknowledgment

We thank Editor-in-Chief and anonymous reviewers, whose valuable comments and suggestions help us to improve this paper significantly. The corresponding author, Dr. Yulin He, is supported by China Postdoctoral Science Foundation (2015M572361) and National Natural Science Foundation of China (61503252).

References

- [1] Huang, G. B., Zhu, Q. Y., & Siew, C. K. (2006). Extreme learning machine: Theory and applications. *Neurocomputing*, 70(1), 489-501.
- [2] Serre, D. (2002). *Matrices: Theory and Applications*. Springer, New York.
- [3] Huang, G. B., Chen, L., & Siew C. K. (2006). Universal approximation using incremental constructive feedforward networks with random hidden nodes. *IEEE Transactions on Neural Networks*, 17(4), 879-892.
- [4] Huang, G. B., Wang, D. H., & Lan Y. (2011). Extreme learning Machines: A survey. *International Journal of Machine Learning and Cybernetics*. 2(2), 107-122.
- [5] Zhu, Q. Y., Qin, A., Suganthan, P., & Huang, G. B. (2005). Evolutionary extreme learning machine. *Pattern Recognition*, 38(10), 1759-1763.

- [6] Cao, J., Lin, Z., & Huang, G. B. (2012). Self-adaptive evolutionary extreme learning machine. *Neural Processing Letters*, 36(3), 285-305.
- [7] Matias, T., Souza, F., Araújo, R., & Antunes, C. H. (2014). Learning of a single-hidden layer feedforward neural network using an optimized extreme learning machine. *Neurocomputing*, 129, 428-436.
- [8] Heeswijk, M. V., & Miche, Y. (2015). Binary/ternary extreme learning machines. *Neurocomputing*, 149, 187-197.
- [9] Balasundaram, S., & Gupta, D. (2014) On optimization based extreme learning machine in primal for regression and classification by functional iterative method. *International Journal of Machine Learning and Cybernetics*.
- [10] Deng, W., Zheng, Q., & Chen, L. (2009). Regularized extreme learning machine. *Proceedings of 2009 IEEE Symposium on Computational Intelligence and Data Mining* (pp. 389-395).
- [11] Horata, P., Chiewchanwattana, S., & Sunat, K. (2013). Robust extreme learning machine. *Neurocomputing*, 102, 31-44.
- [12] Huang, G. B., Li, M. B., Chen, L., & Siew, C. K. (2008). Incremental extreme learning machine with fully complex hidden nodes. *Neurocomputing*, 71(4-6), 576-583.
- [13] Miche, Y., Sorjamaa, A., Bas, P., Simula, O., Jutten, C., & Lendasse, A. (2010). OP-ELM: Optimally pruned extreme learning machine. *IEEE Transactions on Neural Networks*, 21(1), 158-162.
- [14] Wang, X. Z., Shao, Q. Y., Miao, Q., & Zhai, J. H., (2013). Architecture selection for networks trained with extreme learning machine using localized generalization error model. *Neurocomputing*, 102, 3-9.
- [15] Zhang, K., & Luo, M. X. (2014). Outlier-robust extreme learning machine for regression problems. *Neurocomputing*.
- [16] He, Y. L., Liu, J. N. K., Wang, X. Z., & Hu, Y. X. (2012). Optimal bandwidth selection for resubstitution entropy estimation. *Applied Mathematics and Computation*, 219(8), 3425-3460.
- [17] Demšar, J. (2006). Statistical comparisons of classifiers over multiple data sets. *Journal of Machine Learning Research*, 7(1), 1-30.
- [18] Hall, M., Frank, E., Holmes, G., Pfahringer, B., Reutemann, P., & Witten, I. H. (2009). The WEKA data mining software: An update. *SIGKDD Explorations*, 11(1), 10-18.
- [19] Huang, G. B., Zhou, H., Ding, X., & Zhang, R. (2012). Extreme learning machine for regression and multiclass classification. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 42(2), 513-529.

Tao Xiao received her bachelor's degree in mathematics and mathematics education from Hebei Normal University in June 2003 and received her master's degree in applied mathematics from Hebei University in June 2009. Since July 2003, she has been a lecturer of College of Science in Hebei Agricultural University, engaging in mathematics teaching. Her research interests include fuzzy mathematics and uncertain information processing.

Xu Zhou received his bachelor's degree in mathematics and application from Agricultural University of Hebei, Baoding, China, in June 2010. He received his master degree in application mathematics from Hebei University of Hebei, Baoding, China, in June 2013. He is currently a mathematics teacher in Agricultural University of Hebei, Baoding, China. His research interests include artificial neural networks, machine learning, extreme learning machine, and support vector machine.

Yu Lin He received both his master degrees in computer science and Ph.D. degree in optical engineering from Hebei University in June 2009 and June 2014, respectively. He is currently a post-doctoral fellow in College of Computer Science and Software Engineering, Shenzhen University, Shenzhen, Guangdong, China. From February 2011 to January 2012 and from August 2013 to August 2014, he has served as a research assistant in the Department of Computing, Hong Kong Polytechnic University, Hong Kong. His research interests include bayesian network, artificial neural networks, evolutionary optimization, probability density estimation, and approximate reasoning. He is the assistant of editor-in-chief of International Journal of Machine Learning and Cybernetics (Science Citation Index Expanded).

Rana Aamir Raza Ashfaq received his master degree in computer science from Blekinge Tekniska Hgskola (BTH), Sweden. He also received his bachelor and master degrees in computer science from Bahauddin Zakariya University, Multan, Pakistan. Since 2010 he is working as an assistant professor in the Department of Computer Science, Bahauddin Zakariya University, Multan, Pakistan. He is currently a Ph.D. student in College of Computer Science and Software Engineering, Shenzhen University, Shenzhen, Guangdong, China. His main research interests include machine learning and big data mining.